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Original published in:	Journal of evolutionary economics. - Berlin : Springer. - 33 (2023), 3, p. 797-835.
Original published:	2023-08-05
ISSN:	1432-1386, 0936-9937
DOI:	10.1007/s00191-023-00816-8
[*Visited:*	2024-08-01]

Journal of Evolutionary Economics (2023) 33:797–835
https://doi.org/10.1007/s00191-023-00816-8

REGULAR ARTICLE

The fallacy in productivity decomposition

Simon Bruhn[1] · Thomas Grebel[1] · Lionel Nesta[2]

Accepted: 27 February 2023 / Published online: 5 August 2023

Abstract
This paper argues that the typical practice of performing growth decompositions based on log-transformed productivity values induces fallacious conclusions: using logs may lead to an inaccurate aggregate growth rate, an inaccurate description of the micro sources of aggregate growth, or both. We identify the mathematical sources of this log-induced fallacy in decomposition and analytically demonstrate the questionable reliability of log results. Using firm-level data from the French manufacturing sector during the 2009–2018 period, we empirically show that the magnitude of the log-induced distortions is substantial. We find that around 60–80% of four-digit industry results are prone to mismeasurement depending on the definition of accurate log measures. We further find significant correlations of this mismeasurement with commonly deployed industry characteristics, indicating, among other things, that less competitive industries are more prone to log distortions. Evidently, these correlations also affect the validity of studies investigating industry characteristics' role in productivity growth.

Keywords Productivity decomposition · Growth · Log approximation · Geometric mean · Arithmetic mean

JEL Classification C18 · L22 · L25 · O47

1 Introduction

This paper questions the common practice of performing growth decompositions based on productivity growth rates proxied by log-differenced productivity values. We argue that this procedure leads to an inaccurate aggregate growth rate, an inaccurate description of the micro sources of aggregate growth, or both. These three cases of potential

✉ Simon Bruhn
simon.bruhn@posteo.de

1 Technische Universität Ilmenau, Ehrenbergstraße 29, 98693 Ilmenau, Germany

2 Université Côte d'Azur, CNRS, GREDEG and SKEMA Business School (France), Sciences Po OFCE, 250 rue Albert Einstein, Sophia Antipolis 06560, France

misconceptions comprise what we refer to as the fallacy in productivity decomposition. Therefore, policy recommendations stemming from log-based decomposition exercises may prove inappropriate.

Productivity decomposition methods are useful tools to shed light on the underlying causes of aggregate productivity movements. The most commonly used shift-share decomposition methods include those proposed by Griliches and Regev (1995), Foster, Haltiwanger and Krizan (2001) and Melitz and Polanec (2015). While the former two are time-series approaches and refinements of the seminal contribution by Baily, Hulten and Campbell (1992), the latter is based on the cross-sectional methodology by Olley and Pakes (1996). Despite their technical differences, they all use the weighted average of firm-level productivity and decompose aggregate productivity growth according to its underlying micro sources. These typically include firm learning, resource reallocation, and net entry of firms.

The use of productivity decomposition methods differs in various ways. Some studies use labor productivity, whereas others use total factor productivity. Some use inputs, whereas others choose output shares as weights (see, e.g., Fagerberg 2000; Foster et al. 2001; Melitz and Polanec 2015; Decker et al. 2017). They also differ with respect to the analyzed time length, which, in the case of short periods, leads to larger contributions of firm learning (Brown et al. 2018). All of these methodological differences affect the comparability of productivity decomposition studies. Nevertheless, there are good reasons behind the specific choice of the respective methodology. What many of these studies have in common, however, is the use of log-based productivity growth rates – the technical procedure that we criticize in this paper.

There are numerous good reasons for using logs in a wide range of applications in economics. To begin with, log differences are symmetrical, which is a useful characteristic in the computation of job flows (see, e.g., Davis and Haltiwanger 1999; Haltiwanger et al. 2013). Furthermore, they are additive and therefore facilitate the seamless calculation of compound interest rates (Törnqvist et al. 1985). In growth accounting, the log-linearization of production functions allows the application of simple OLS regression, which is also the standard procedure to estimate total factor productivity (TFP).[1] The estimated coefficients can be interpreted as production factors' output elasticity, and last but not the least, the use of logs simultaneously reduces the impact of outliers. Hence, the logarithm is and continues to remain a valuable tool and all the described practices are untouched by our criticism.

However, we argue that no well-grounded reasoning exists for using logs to decompose productivity growth. As our literature review shows, it is typical to base such growth decomposition on log-transformed productivity values (see Appendix A). The papers included in this literature review do not provide any reasons as to why they use logs. According to Van Biesebroeck (2008), the main reason is that logs linearize and facilitate the decomposition exercise. This linearization is achieved because log differences can be considered an approximation to growth rates in percent, that is, in

[1] In this paper, we confine our investigation to labor productivity. However, when decomposing TFP estimated in logs, the same log-induced misinterpretations will occur. Hence, our recommendation is to exponentiate the log-based TFP measure and to run the decomposition on levels of TFP.

levels.[2] This practice, however, can lead to severe misrepresentations because using log-approximated growth rates in productivity decomposition generates inaccuracies beyond the well-known log approximation error.

More fundamentally, we show that log-based productivity growth decomposition leads to the following three sources of deviations from results calculated in levels: (i) the log approximation error, as a consequence of the logarithm's concavity; (ii) the reference deviation, arising from a different reference assumption implicit in log differences; and (iii), the mean deviation, caused by the difference in the deployed benchmark productivity. The consequences of these three distortions significantly outweigh the facilitating impact of log linearization in decomposition exercises. Therefore, we advocate performing productivity decompositions in levels.

Our paper makes several contributions to the literature. We demonstrate the fallacy in decomposition: using logs may lead to either an inaccurate aggregate growth rate, an inaccurate description of the contribution of the micro sources of productivity growth, or both. We enrich the case proposed by Dias and Marques (2021a) who shed light on the potential misconceptions in productivity decompositions propelled by the fact that the aggregation of logs leads to a geometric instead of an arithmetic mean. In contrast to their paper, however, we analyze the impact of logs from the perspective of an individual firm's contribution to aggregate growth instead of adopting the aggregate perspective. Adopting the firm-level perspective reveals that the discrepancy between level and log results for individual productivity components can be traced back to three sources of distortions: the log approximation error, the reference deviation, and the mean deviation. The separation of the three log distortions provides a straightforward analytical framework to determine the circumstances under which firm-level contributions are overestimated or underestimated by logs.

In addition, our study is the first to quantify the fallacy in decomposition by exploiting data on four-digit industries. We show that around 60–80% of four-digit industry results are subject to mismeasurement depending on the definition of accurate log measures.

Moreover, we document those log distortions associated with industry characteristics, revealing, in particular, that the lack of competition and a high degree of industry openness reinforce log distortions. We thereby take an important step toward evaluating the extent to which past and future studies of industry dynamics may be affected by the use of logs and how they compare with one another.

Furthermore, unlike Dias and Marques (2021a), who use a modified version of the decomposition method outlined by Melitz and Polanec (2015), we investigate the consequences of using log-transformed productivity measures in the widely applied decomposition method of Foster et al. (2001) (hereafter: FHK). We further show that our findings also hold true for the method proposed by Griliches and Regev (1995) and the decomposition method by Melitz and Polanec (2015).

The remainder of the paper is structured as follows. Section 2 defines and formalizes the first two distortions, namely, the log approximation error and the reference deviation. Section 3 discusses the use of logs in the productivity components of the FHK

[2] See, for instance, Melitz and Polanec (2015), who remarked that "... [a]ll productivity changes are reported as log percents (or log points) – and can thus be interpreted as percentage point changes" (p. 371 Melitz and Polanec 2015).

decomposition method, which reveals the mean deviation as the third log-induced distortion. Section 4 shows the magnitude of the log distortions using firm-level data derived from the French manufacturing sector. Section 5 quantifies the fallacy in productivity decomposition, whereas Section 6 addresses how the log-induced discrepancies correlate with certain industry characteristics. Section 7 concludes this paper.

2 Log approximation error and reference deviation

It is common practice in productivity decompositions to represent firm-level productivity in logs. The main motive lies in the linearization of the decomposition exercise (Van Biesebroeck 2008), which is achieved by using a log difference as an approximation to a productivity growth rate as follows:

$$\frac{\varphi_{i2} - \varphi_{i1}}{\varphi_{i1}} \approx \ln(\varphi_{i2}) - \ln(\varphi_{i1}) \tag{1}$$

where φ_{i1} and φ_{i2} denote productivity levels of firm i in two successive periods. Due to the concavity of the logarithmic function, logged values underestimate productivity growth, that is, $(\varphi_{i2} - \varphi_{i1})/\varphi_{i1} - (\ln(\varphi_{i2}) - \ln(\varphi_{i1})) \geq 0$. This is the well-known *log approximation error* – the discrepancy between the growth rate measured as a percentage change of absolute productivity and the growth rate measured by a log difference. This leads to the first proposition:

Proposition 1 *The use of logs introduces a log approximation error, that is, a systematic underestimation of productivity growth rates.*

It is usually argued that the approximation error can be restricted within reasonable limits as long as the growth rates fluctuate within a range of approximately ±10%, as is well known among economists. However, we observe that this problem is not addressed in many studies. For instance, the decomposition studies conducted by Foster et al. (2001), Scarpetta et al. (2002), and Melitz and Polanec (2015) report average growth rates that go well beyond the conventional ±10% threshold.[3]

The second distortion takes into consideration the discrepancy that arises between levels and logs when aggregating firm-level growth rates. In contrast to levels, where absolute changes in firm-level productivity are measured relative to some reference productivity – usually, the aggregate productivity of the previous period – the reference of a productivity growth rate, calculated with log-transformed values, is implicit in the (firm-individual) log difference. This is what we refer to as *reference deviation*. For each firm-specific productivity growth rate, the reference productivity will differ from the reference productivity calculated with levels.

Proposition 2 *Aggregating log differences as a proxy for growth rates induces a reference deviation that arises from the idiosyncratic reference productivity implicit in the log difference.*

[3] To make things worse, a ±10% threshold on the aggregate growth rate does not guarantee that the productivity components will not exceed this threshold.

To illustrate the two log distortions mentioned above, we start with the standard textbook definition of aggregate productivity as the sum of firm-level output over the sum of firm-level input(s). In the case of aggregate labor productivity, this reads as follows:

$$\Phi_t = \frac{\sum Y_{it}}{\sum L_{it}} \tag{2}$$

where Y_{it} denotes the output (e.g., value-added) and L_{it} indicates the input (e.g., working hours) of firm i at time t. We further define growth in aggregate productivity as the difference between two aggregate productivity values in two successive periods, say Φ_2 and Φ_1, divided by its initial value Φ_1. Representing aggregate productivity as the share-weighted mean of firm-level productivity, this implies for aggregate productivity growth in levels, $\hat{\Phi}_{lev}$:

$$\hat{\Phi}_{lev} = \left(\sum s_{i2} \cdot \varphi_{i2} - \sum s_{i1} \cdot \varphi_{i1}\right) \frac{1}{\Phi_1} \tag{3}$$

where φ_{i1} and φ_{i2} denote productivity levels of firm i in two successive periods, and s_{i1} and s_{i2} share weights. To ensure that the aggregation of firm-level data corresponds to the industry aggregate as defined in Eq. (2), we use input shares for weighing individual firm productivity, that is, $s_{it} = \frac{L_{it}}{\sum L_{jt}}$. In using input shares, we follow the 'denominator rule in share-weighting aggregation' of Färe and Karagiannis (2017), who show that, when aggregating, consistent results are achieved only by using the denominator of the productivity measure as weights.

If, instead, firm-level productivity is measured in logs, then the share-weighted industry aggregate is defined as $\Phi_{t,log} = \sum s_{it} \cdot \ln \varphi_{it}$ (see, e.g., Van Biesebroeck 2008; Bartelsman et al. 2013; Melitz and Polanec 2015; Decker et al. 2017). As the difference between two log aggregates corresponds to a percentage change, the aggregate productivity growth in logs can be expressed as follows:

$$\hat{\Phi}_{log} = \sum s_{i2} \cdot \ln \varphi_{i2} - \sum s_{i1} \cdot \ln \varphi_{i1} \tag{4}$$

which is equivalent to the log difference of two geometric means:

$$\hat{\Phi}_{log} = \ln \prod \varphi_{i2}^{s_{i2}} - \ln \prod \varphi_{i1}^{s_{i1}} \tag{5}$$

In other words, instead of a growth rate between two share-weighted arithmetic means, logs approximate a growth rate between two share-weighted geometric means (see, e.g., Van Biesebroeck 2008; Brown et al. 2018; Dias and Marques 2021a). This procedure affects the resulting productivity growth rate because a geometric mean is more sensitive to smaller than larger numbers. It mitigates the impact of high values while reinforcing the impact of low values. Moreover, as is well known from Jensen's inequality, the (weighted) geometric mean is always smaller than the (weighted) arithmetic mean unless all numbers constituting the means are equal (Casella and Berger 2002).

This, however, does not imply that the growth rate between two geometric means is always smaller than the growth rate between two arithmetic means. As shown by Dias and Marques (2021a), this hinges on the composition of the respective means and their changes over time. Specifically, it depends on each firm's productivity, input share, its initial position in the industry's productivity distribution, and the changes of these values over time. To elaborate on this point, let us adopt an individual firm's perspective. In Eqs. (3) and (4), we can extract the individual firm productivity contribution C_i to aggregate productivity growth measured in levels and logs, respectively:

$$C_{i,lev} = (s_{i2}\varphi_{i2} - s_{i1}\varphi_{i1}) \cdot \frac{1}{\Phi_1} \tag{6}$$

$$C_{i,log} = s_{i2} \cdot \ln \varphi_{i2} - s_{i1} \cdot \ln \varphi_{i1} \tag{7}$$

Comparing $C_{i,lev}$ and $C_{i,log}$ will reveal the two log distortions we identified in Propositions (1) and (2). In Appendix B, we provide a general derivation of the two distortions that result from subtracting Eq. (7) from Eq. (6). The endeavor to identify the log approximation error is trivial. Suppose all weights $s_{i\cdot}$ were equal to one, then we, in principle, end up with Eq. (1), which substantiates the log approximation with one difference: the reference productivity Φ_1 from Eq. (6). This leads us directly to the next distortion, that is, the reference deviation. In contrast to the reference productivity Φ_1 for levels, the implicit reference in Eq. (7) is φ_{i1}.[4]

Due to the reference deviation, logged firm-level productivity growth rates can either underestimate, that is, $C_{i,lev} > C_{i,log}$, or overestimate, that is, $C_{i,lev} < C_{i,log}$, the contribution to aggregate growth when compared to levels. To illustrate this point, let us assume two firms with constant input shares $s_{it} = 10\%$ for i={1,2}, productivity levels $\varphi_{11} = 50$ and $\varphi_{21} = 150$, and an initial aggregate productivity $\Phi_1 = 100$. Suppose both firms increase their productivity by 10%. With logs, the contribution of both firms to aggregate productivity growth will be the same, namely, $0.1 \cdot \ln(1.1) \approx 0.953\%$. However, when calculated in levels, the impact of firm 1 will be smaller than the impact of firm 2, namely, $0.1 \cdot \frac{55-50}{100} = 0.5\%$ for firm 1 and $0.1 \cdot \frac{165-150}{100} = 1.5\%$ for firm 2. Hence, logs overestimate the impact of firm 1 and underestimate the impact of firm 2.

Following this line of reasoning, when aggregating logged firm contributions to aggregate productivity growth, the resulting aggregate growth rate is susceptible to bias. The industry structure and its change over time determine the extent to which productivity growth contributions are underestimated or overestimated. This raises doubts about the reliability and comparability of productivity measures based on logs.

In the following section, we flesh out our propositions by decomposing aggregate productivity growth according to the FHK method. For each productivity component in the FHK method, we propose a separation of the different log distortions, which

[4] Note that we draw this analogy based on the approach that a log difference between φ_{i2} and φ_{i1} approximates the ratio of the absolute difference $(\varphi_{i2} - \varphi_{i1})$ and the initial productivity φ_{i1}, while we isolate the inaccuracy caused by the log approximation in the approximation error. Hence, our analogy is not opposed to Törnqvist et al. Törnqvist et al. (1985), who stated that a log difference equals the ratio of the absolute difference $(\varphi_{i2} - \varphi_{i1})$ and the logarithmic mean $L(\varphi_{i1}, \varphi_{i2})$, with $L(\varphi_{i1}, \varphi_{i2}) = (\varphi_{i2} - \varphi_{i1})/\ln(\varphi_{i2}/\varphi_{i1})$ and $(\varphi_{i1}\varphi_{i2})^{1/2} < L(\varphi_{i1}, \varphi_{i2}) < (\varphi_{i1} + \varphi_{i2})/2$ for $\varphi_{i1} \neq \varphi_{i2}$.

provides a straightforward analytical approach to determine the circumstances under which firms' productivity contributions are underestimated or overestimated by logs.

3 Decomposing the log distortions in the FHK decomposition

The FHK decomposition distinguishes three groups of firms, that is, surviving (S), entering (N), and exiting firms (X). The contribution of surviving firms is further broken down into three subcomponents, which they label as the within-firm effect (*WFE*), the between-firm effect (*BFE*), and the cross-firm effect (*CFE*). When expressed in levels, the decomposition reads as follows:

$$\begin{aligned}\hat{\Phi}_{lev} = &\sum_{i\in S} \underbrace{\frac{1}{\Phi_1}\cdot s_{i1}\cdot \triangle\varphi_i}_{WFE_{i,lev}} + \sum_{i\in S} \underbrace{\frac{1}{\Phi_1}\cdot \triangle s_i \cdot (\varphi_{i1}-\Phi_1)}_{BFE_{i,lev}} \\ &+ \sum_{i\in S} \underbrace{\frac{1}{\Phi_1}\cdot \triangle s_i \cdot \triangle\varphi_i}_{CFE_{i,lev}} + \sum_{i\in N} \underbrace{\frac{1}{\Phi_1}\cdot s_{i2}\cdot (\varphi_{i2}-\Phi_1)}_{N_{i,lev}} \\ &+ \sum_{i\in X} \underbrace{\frac{1}{\Phi_1}\cdot s_{i1}\cdot (\Phi_1-\varphi_{i1})}_{X_{i,lev}} \end{aligned} \tag{8}$$

In logs, the individual productivity components are expressed as follows:

$$\begin{aligned}\hat{\Phi}_{log} = &\sum_{i\in S} \underbrace{s_{i1}\cdot \triangle \ln\varphi_i}_{WFE_{i,log}} + \sum_{i\in S} \underbrace{\triangle s_i \cdot (\ln\varphi_{i1}-\Phi_{1,log})}_{BFE_{i,log}} \\ &+ \sum_{i\in S} \underbrace{\triangle s_i \cdot \triangle \ln\varphi_i}_{CFE_{i,log}} + \sum_{i\in N} \underbrace{s_{i2}\cdot (\ln\varphi_{i2}-\Phi_{1,log})}_{N_{i,log}} \\ &+ \sum_{i\in X} \underbrace{s_{i1}\cdot (\Phi_{1,log}-\ln\varphi_{i1})}_{X_{i,log}} \end{aligned} \tag{9}$$

Note that in the *BFE* as well as in the components of entering and exiting firms, firm-level productivity is set in relation to aggregate productivity as a benchmark, which is Φ_1 for levels and $\Phi_{1,log}$ for logs. As noted in Section 2, we can rewrite the log aggregate as $\Phi_{1,log} = \sum s_{i1} \ln\varphi_{i1} = \ln\prod\varphi_{i1}^{s_{i1}}$. In turn, we will denote the geometric mean of firm-level productivity as Π_1, that is, $\Phi_{1,log} = \ln\prod\varphi_{i1}^{s_{i1}} = \ln\Pi_1$, distinguishing it from the arithmetic mean used in case of levels, Φ_1. Self-evidently, the discrepancy in means will induce a further distortion in the computation of productivity growth, which we term as *mean deviation*.

Proposition 3 *Logs introduce a mean deviation in the between-firm effect and the components of entering and exiting firms, as logs use a geometric mean instead of an arithmetic mean as benchmark productivity.*

In contrast to the first two propositions, Proposition (3) does not emerge in computations of aggregate growth rates (as in Eqs. 3 and 4) but only appears due to the introduction of a benchmark productivity when decomposing productivity growth. There are two reasons behind deploying a benchmark productivity in decomposition methods. First, it ensures that in the *BFE,* the impact of input share fluctuations depends on the individual firm's productivity relative to the aggregate. Second, it allows one to assess the growth contribution of entering and exiting firms relative to surviving firms.

As follows from Eqs. (8) and (9) and as Proposition (3) emphasizes, the mean deviation will only appear in the *BFE* and in the components of entering and exiting firms. Conversely, the impact of the mean deviation on the discrepancy between the log and the level *aggregate* growth rate will always be zero, as input shares sum to one in each period, that is, $\sum s_{i1} = \sum s_{i2} = 1$ (see, e.g., Melitz and Polanec 2015).

Not all productivity decomposition methods deploy a benchmark productivity. Take, for instance, the decomposition proposed in the seminal contribution by Baily et al. (1992), which is exactly identical to the FHK method with the exception of a benchmark productivity. Therefore, this method will not be subject to a mean deviation. Nonetheless, the use of a benchmark productivity, as shown in Eqs. (8) and (9), is not unique to the FHK decomposition method but rather a typical feature for the aforementioned reasons. The resulting mean deviation will vary with the decomposition method and the deployed benchmark productivity. Griliches and Regev (1995), for instance, use the average aggregate productivity as benchmark. In addition, Melitz and Polanec (2015) use productivity benchmarks albeit in a slightly different way. Irrespective of which benchmark productivity is used in the decomposition, a mean deviation is inevitable between level-based and log-based growth rates. Appendix G demonstrates that our main conclusions also hold true for these related decomposition methods.

The following sections show the extent to which the three identified log distortions, that is, the log approximation error, the reference deviation, and the mean deviation, affect the individual components of the FHK decomposition method. In Appendix B, we generalize our approach and show the impact of the three propositions on a non-decomposed, aggregate productivity growth rate.

3.1 Log distortions in the within-firm effect

The within-firm effect (*WFE*) denotes the contribution of firm learning to aggregate productivity growth. As Eqs. (8) and (9) show, it is calculated as the input share-weighted change in firm productivity, either calculated with levels or logs, respectively.

By subtracting the *WFE* in Eq. (9) from the *WFE* in Eq. (8), we can decompose the log distortion as follows:

$$\begin{aligned}\varepsilon_{i,W} &= WFE_{i,lev} - WFE_{i,log}\\ &= s_{i1}\left(\frac{\triangle\varphi_i}{\Phi_1} - \triangle \ln \varphi_i\right)\\ &= \underbrace{s_{i1}\left(\frac{\triangle\varphi_i}{\varphi_{i1}} - \triangle \ln \varphi_i\right)}_{\varepsilon_{i,W,appr}} + \underbrace{s_{i1}\left(\frac{\triangle\varphi_i}{\Phi_1} - \frac{\triangle\varphi_i}{\varphi_{i1}}\right)}_{\varepsilon_{i,W,ref}}\end{aligned} \tag{10}$$

Note that the input shares s_{i1} simply function as a scaling factor of the two log distortions. The magnitude of the log approximation error ($\varepsilon_{i,W,appr}$) increases in $|\Delta\varphi_i|$. Therefore, industries that experience high fluctuations in firm-level productivity will be subject to large log approximation errors. Due to the concavity of the logarithm, the log approximation error is always greater than or equal to zero ($\varepsilon_{i,W,appr} \geq 0$). Hence, it introduces a systematic underestimation of the *WFE*. The sign and magnitude of the reference deviation ($\varepsilon_{i,W,ref}$) depend on the position of the firm's productivity and its development within the industry's productivity distribution. More precisely, it depends on the relationship between φ_{i1} and Φ_1 and the directional change in $\Delta\varphi_i$. It is positive, if $(\varphi_{i1} > \Phi_1 \wedge \Delta\varphi_i > 0) \vee (\varphi_{i1} < \Phi_1 \wedge \Delta\varphi_i < 0)$, and negative if $(\varphi_{i1} > \Phi_1 \wedge \Delta\varphi_i < 0) \vee (\varphi_{i1} < \Phi_1 \wedge \Delta\varphi_i > 0)$. This implies that an industry that experiences a widening productivity gap between the most and the least productive firms is likely to be subject to a positive reference deviation and vice versa.

Summarizing the log distortions in the *WFE,* the reference deviation will either add to the consistently positive log approximation error, compensate or even overcompensate it. As noted by Dias and Marques (2021a), the described tendencies in the two error terms anticipate a mostly positive log distortion in the *WFE*. This is also reflected in our empirical findings in Section 4.

3.2 Log distortions in the between-firm effect

The between-firm effect (*BFE*) captures the change in productivity growth induced by the reallocation of inputs within an industry. In its calculation, productivity remains constant, and only input shares may change. To motivate Propositions (1) to (3), we subtract the *BFE* in logs from the *BFE* in levels (see Eqs. 8 and 9) and decompose the log distortion as follows:

$$\begin{aligned}\varepsilon_{i,B} &= BFE_{i,lev} - BFE_{i,log}\\ &= \triangle s_i\left(\frac{\varphi_{i1} - \Phi_1}{\Phi_1} - (\ln \varphi_{i1} - \ln \Pi_1)\right)\end{aligned}$$

$$= \underbrace{\triangle s_i \left(\frac{\varphi_{i1} - \Pi_1}{\Pi_1} - (\ln \varphi_{i1} - \ln \Pi_1) \right)}_{\varepsilon_{i,B,appr}} + \underbrace{\triangle s_i \left(\frac{\varphi_{i1} - \Pi_1}{\Phi_1} - \frac{\varphi_{i1} - \Pi_1}{\Pi_1} \right)}_{\varepsilon_{i,B,ref}}$$

$$+ \underbrace{\triangle s_i \left(\frac{\Pi_1 - \Phi_1}{\Phi_1} \right)}_{\varepsilon_{i,B,\triangle mean}} \tag{11}$$

We define the log approximation error ($\varepsilon_{i,B,appr}$) as the difference between the growth rate $(\varphi_{i1} - \Pi_1)/\Pi_1$ and its approximation via the log difference $(\ln \varphi_{i1} - \ln \Pi_1)$.[5]. In line with Proposition (2), the reference productivity embodied in the log difference is Π_1, whereas the reference productivity deployed by the *BFE* in levels is Φ_1. This introduces the reference deviation ($\varepsilon_{i,B,ref}$). The remaining discrepancy is rooted in the mean deviation ($\varepsilon_{i,B,\triangle mean}$). In levels, firm productivity is measured against the arithmetic mean; for logs, it is compared to the geometric mean.

Let us now look at the potential magnitude of the distortions. The magnitude of $\varepsilon_{i,B,appr}$ increases with the difference $(\varphi_{i1} - \Pi_1)$ and is simultaneously scaled by the change in input share $\triangle s_i$. In line with Proposition (2), $\varepsilon_{i,B,appr}$ is positive for $\triangle s_i > 0$ and negative for $\triangle s_i < 0$. Therefore, the log approximation error will have a considerable impact on firms that are located at the fringes of the productivity distribution and that experience high changes in their input share. The impact of $\varepsilon_{i,B,ref}$ depends on the sign and magnitude of the change in input share $\triangle s_i$ and the position of firm productivity φ_{i1} relative to the geometric mean Π_1. According to Jensen's inequality, we can state that $\Phi_1 > \Pi_1$, so that $\varepsilon_{i,B,ref}$ is positive if $(\varphi_{i1} > \Pi_1 \wedge \triangle s_i < 0) \vee (\varphi_{i1} < \Pi_1 \wedge \triangle s_i > 0)$ and negative if $(\varphi_{i1} > \Pi_1 \wedge \triangle s_i > 0) \vee (\varphi_{i1} < \Pi_1 \wedge \triangle s_i < 0)$. Hence, if we assume an industry with a positive *BFE*, which implies that input shares tend to be allocated away from the least to the most productive firms, the two conditions for a negative reference deviation are likely to be dominant in this industry. Therefore, we would expect a negative aggregate reference deviation in such a case. The sign and magnitude of $\varepsilon_{i,B,\triangle mean}$ depend on the magnitude of the difference between the two means, $(\Pi_1 - \Phi_1)$, and the change in input share $\triangle s_i$. Provided that $\Phi_1 > \Pi_1$, the sign of the distortion for an individual firm will take the opposite sign of the change in input share $\triangle s_i$, scaled by its absolute magnitude. In the aggregate, the direction of the mean deviation depends on the changes in input shares of surviving firms relative to exiting and entering firms. It will be positive, if the sum of changes in shares of surviving firms is negative, that is, $\sum_{i \in S} \triangle s_i < 0$, which implies that the share of entering firms exceeds the share of exiting firms.

Overall, each of the three distortions in the *BFE* can be either positive or negative, both at the individual firm level and at the aggregate level.

[5] Note that even though the *BFE* holds the productivity measure constant, it is still subject to a log approximation error. This is because normalizing the impact of an individual firm using the weighted mean corresponds to the mathematical equivalent of a growth rate (see Eqs. 8 and 9)

3.3 Log distortions in the cross-firm effect

The cross-firm effect (*CFE*) is the interaction between the two previous components, that is, between the *WFE* and the *BFE*. We derive the log distortions in the *CFE* component from Eqs. (8) and (9) as follows:

$$\begin{aligned}\varepsilon_{i,C} &= CFE_{i,lev} - CFE_{i,log} \\ &= \triangle s_i \left(\frac{\triangle \varphi_i}{\Phi_1} - \triangle \ln \varphi_i \right) \\ &= \underbrace{\triangle s_i \left(\frac{\triangle \varphi_i}{\varphi_{i1}} - \triangle \ln \varphi_i \right)}_{\varepsilon_{i,C,appr}} + \underbrace{\triangle s_i \left(\frac{\triangle \varphi_i}{\Phi_1} - \frac{\triangle \varphi_i}{\varphi_{i1}} \right)}_{\varepsilon_{i,C,ref}} \end{aligned} \tag{12}$$

The log approximation error in the *CFE* ($\varepsilon_{i,C,appr}$) increases with $|\Delta\varphi_i|$. It is weighed and scaled by $\triangle s_i$ and follows this scaling factor in terms of sign and magnitude. The sign and magnitude of the reference deviation ($\varepsilon_{i,C,ref}$) depends on the development of $\triangle s_i$, the position of φ_{i1} relative to Φ_1, and the change in firm-level productivity $\triangle\varphi_i$. All these determinants of $\varepsilon_{i,C,ref}$ lead to a variety of biased results that either arbitrarily overestimate or underestimate productivity growth contributions.[6]

Overall, the approximation error and the reference deviation may induce a positive or a negative bias in the *CFE*. It is also conceivable that the distortions balance out in the aggregate.

3.4 Log distortions in entry and exit

The contribution of entering or exiting of a firms to aggregate productivity growth can be positive or negative, depending on the entering or exiting firm's position relative to the industry's benchmark productivity. In the case of the FHK decomposition, the industry benchmark is Φ_1 for levels and Π_1 for logs. Analogous to the *BFE*, we can decompose the log-induced distortion to isolate three different distortions in the entry and exit components: the log approximation error ($\varepsilon_{i,\cdot,appr}$), the reference deviation ($\varepsilon_{i,\cdot,ref}$), and the mean deviation ($\varepsilon_{i,\cdot,\Delta mean}$). For entering firms, this leads to the following equation:

$$\begin{aligned}\varepsilon_{i,N} &= N_{i,lev} - N_{i,log} \\ &= s_{i2}\left(\frac{\varphi_{i2} - \Phi_1}{\Phi_1} - (\ln \varphi_{i2} - \ln \Pi_1) \right)\end{aligned}$$

[6] For completeness: $\varepsilon_{i,C,ref}$ is positive for ($\varphi_{i1} > \Phi_1 \wedge \Delta s_i > 0 \wedge \Delta\varphi_i > 0$) or ($\varphi_{i1} > \Phi_1 \wedge \Delta s_i < 0 \wedge \Delta\varphi_i < 0$) or ($\varphi_{i1} < \Phi_1 \wedge \Delta s_i > 0 \wedge \Delta\varphi_i < 0$) or ($\varphi_{i1} < \Phi_1 \wedge \Delta s_i < 0 \wedge \Delta\varphi_i > 0$) and it is negative for all complementary cases.

$$= \underbrace{s_{i2}\left(\frac{\varphi_{i2}-\Pi_1}{\Pi_1} - (\ln\varphi_{i2} - \ln\Pi_1)\right)}_{\varepsilon_{i,N,appr}} + \underbrace{s_{i2}\left(\frac{\varphi_{i2}-\Pi_1}{\Phi_1} - \frac{\varphi_{i2}-\Pi_1}{\Pi_1}\right)}_{\varepsilon_{i,N,ref}}$$
$$+ \underbrace{s_{i2}\left(\frac{\Pi_1-\Phi_1}{\Phi_1}\right)}_{\varepsilon_{i,N,\triangle mean}} \tag{13}$$

The more distant the entering firm's productivity φ_{i2} from the benchmark productivity Π_1, the greater the log approximation error ($\varepsilon_{i,N,appr}$). In line with Proposition (1) and the fact that s_{i2} can take only positive values, $\varepsilon_{i,N,appr}$ must be non-negative. The distortion caused by the reference deviation ($\varepsilon_{i,N,ref}$) is almost identical to its counterpart in the *BFE*, with the exception of its weights. Assuming $\Phi_1 > \Pi_1$, $\varepsilon_{i,N,ref}$ will be positive if $(\varphi_{i2} - \Pi_1) < 0$ and vice versa. The magnitude of the mean deviation ($\varepsilon_{i,N,\triangle mean}$) depends on input share s_{i2}, which scales the difference between the geometric (Π_1) and the arithmetic (Φ_1) means. As the benchmark productivity for evaluating the contribution of entering firms is smaller for logs than for levels, the distortion is always negative.

In the case of exiting firms, we obtain the following mirror image:

$$\begin{aligned}
\varepsilon_{i,X} &= X_{i,lev} - X_{i,log} \\
&= s_{i1}\left(\frac{\Phi_1-\varphi_{i1}}{\Phi_1} - (\ln\Pi_1 - \ln\varphi_{i1})\right) \\
&= s_{i1}\left((\ln\varphi_{i1} - \ln\Pi_1) - \frac{\varphi_{i1}-\Phi_1}{\Phi_1}\right) \\
&= \underbrace{s_{i1}\left((\ln\varphi_{i1} - \ln\Pi_1) - \frac{\varphi_{i1}-\Pi_1}{\Pi_1}\right)}_{\varepsilon_{i,X,appr}} + \underbrace{s_{i1}\left(\frac{\varphi_{i1}-\Pi_1}{\Pi_1} - \frac{\varphi_{i1}-\Pi_1}{\Phi_1}\right)}_{\varepsilon_{i,X,ref}} \\
&\quad + \underbrace{s_{i1}\left(\frac{\Phi_1-\Pi_1}{\Phi_1}\right)}_{\varepsilon_{i,X,\triangle mean}}
\end{aligned} \tag{14}$$

The log approximation error in the exit component ($\varepsilon_{i,X,appr}$) is always negative for exiting firms. The reference deviation ($\varepsilon_{i,X,ref}$) is positive for $\varphi_{i1} > \Pi_1$ and negative for $\varphi_{i1} < \Pi_1$. The mean deviation, ($\varepsilon_{i,X,\triangle mean}$) is always positive.

Overall, as is the case in the previous productivity components, it is conceivable that the individual log distortions in the entry and exit components balance out in the aggregate or induce an overestimation or underestimation, which essentially depends on firm and industry characteristics.

4 Empirical application

4.1 Data

We use firm-level panel data covering the French manufacturing sector for the 2009–2018 period. The relevant information is derived from annual census data named FARE and covers over 3 million companies per year. The data provides information regarding firms' income statements and balance sheets, from which, in turn, we retrieved the value-added and the number of employees. As we did not observe prices, we used industry-specific value-added deflators provided by the French statistical office INSEE. For labor, we used the industry-specific annual number of hours worked per employee (provided by INSEE) and multiplied it by the number of employees to obtain the total number of hours worked per company.

As an industry classification, we used the intermediate SNA/ISIC aggregation A38, which aggregates similar ISIC two-digit divisions to 13 sectors (Eurostat 2008), as listed in Appendix C. We excluded the industry of coke and refined petroleum products (ISIC 19) in our analysis. We restricted our sample to firms with at least ten employees. Increasing the minimum size of the firm ensures higher data quality – a key element in growth rate computations. To avoid artificial breaks in the series, we did not trim observations with fewer than ten employees on a firm-year basis. Instead, we screened out firms for which the median number of employees was strictly lower than ten over the entire period. We focused on labor productivity as our efficiency measure, defined as the value-added to hours worked ratio. We excluded firms reporting a negative value-added. We further truncated the data by excluding firms with at least one observation in the bottom and top 0.5% of the productivity distribution and by discarding firms that experienced suspicious negative and positive jumps in their efficiency series.[7] The application of such restrictions yielded a sample of approximately 260,000 firm-year observations. Appendix D reports the corresponding summary statistics.

Given our sample of firms in the remaining 12 manufacturing sectors, we performed the FHK decomposition using Eqs. (8) and (9) for each industry and year. During the period between 2010 and 2018, this method yields a sample of $12 \times 9 = 108$ decompositions, once in levels and once in logs, which allows us to recover the aggregate log distortions in aggregate productivity growth ($\varepsilon_A = \sum_i \varepsilon_{i,A} = \hat{\Phi}_{lev} - \hat{\Phi}_{log}$), in the *WFE* ($\varepsilon_W = \sum_i \varepsilon_{i,W}$), in the *BFE* ($\varepsilon_B = \sum_i \varepsilon_{i,B}$), in the *CFE* ($\varepsilon_C = \sum_i \varepsilon_{i,C}$), in the entry ($\varepsilon_N = \sum_i \varepsilon_{i,N}$) and the exit component ($\varepsilon_X = \sum_i \varepsilon_{i,X}$). We further separated these total distortions according to our three propositions (log approximation error, reference deviation, mean deviation) as in $\varepsilon_A = \varepsilon_{A,appr} + \varepsilon_{A,ref} + \varepsilon_{A,\triangle mean}$, for the example of aggregate growth. We followed the same logic for the productivity components.

Subsequently, we created a weighted average industry of the industry-level results, using labor input as weights, averaged over the beginning and ending years of the period in which the respective growth rate was measured.

[7] More precisely, we excluded firms for which we observed a change in labor productivity by a factor of more than 3 from one year to another.

4.2 Decomposition results in levels and logs in the manufacturing sector

Table 1 reports the decomposition results in levels and logs and quantifies the misrepresentation caused by the log distortions.

Aggregate productivity growth shows distortions (ε_A) ranging from −0.64 to 1.36 percentage points. Relative to the productivity contribution in levels, the distortions range from −35% (2010) to 27% (2017). The distortions in aggregate growth are not distributed equally among the FHK productivity components. The *WFE* and the exit and entry components are affected most, whereas the *BFE* and the *CFE* are subject to smaller distortions. Nonetheless, if expressed in relative terms, the distortions in the *BFE* and *CFE* can also be substantial. In all five components, productivity growth contributions are neither systematically overestimated nor underestimated using logs. The distortions in the *WFE* range from −17% in 2010 to 25% in 2015. In the *BFE*, the largest overestimation occurs in 2012 with −19%, and the largest underestimation is observed in 2011 with 18%, while the distortions in the *CFE* range from −14% (2011) to 11% (2018). The relative discrepancies are most pronounced with respect to the entry and exit components. For entries, the discrepancy ranges from −6% (2012) to as much as 766% (2011) and for exits from −239% (2013) to 51% (2018).

The magnitude of log distortions can be substantial, implying a severely distorted image of the productivity growth components. Importantly, log distortions can lead to a sign flip in the productivity components (see, for instance, the sign flip in the entry component in 2011), leading to severely misguided conclusions. In our decomposition of industry-level results documented in Appendix E, we provide further evidence for the presence of sign flips in different productivity components and even in terms of aggregate growth.

Apart from investigating the magnitude of productivity components individually, decomposition methods are frequently used to analyze which components, relative to the other components, have been the driving forces behind aggregate productivity growth over a given time span. As Table 1 highlights, such an analysis may be strongly blurred by logs. This is especially visible in the years 2010 and 2011. Both for levels and for logs, the *WFE* is the most relevant component for aggregate growth. However, while levels clearly point toward the *BFE* as the next most relevant component, the impact of exiting firms significantly exceeds that of the *BFE* when using logs.

Strikingly, there is no systematic overestimation or underestimation either in aggregate growth or its decomposed components. This implies that trends in productivity developments may be judged differently in logs than in levels. For instance, calculated in levels, the aggregate growth in 2017 is more than double the aggregate growth in 2010. On the other hand, in logs, it increases by only about 43%. As a further example, while aggregate growth in levels shows a slight increase between 2010 (1.83%) and 2014 (1.99%), logs suggest a slowdown in productivity growth from 2.47 to 1.83%. With respect to individual productivity components, 2016 and 2017 provide a notable example of the potential misconceptions. While the contributions of entering and exiting firms are quite balanced with logs, the exit component clearly dominates with levels, as it is approximately three times the size of the entry component.

Table 1 Productivity decomposition results in levels and in logs

	WFE_{lev}	WFE_{log}	ε_W	BFE_{lev}	BFE_{log}	ε_B	CFE_{lev}	CFE_{log}	ε_C
2010	1.22	1.42	−0.21	0.83	0.82	0.01	−0.68	−0.65	−0.03
2011	2.29	2.27	0.02	0.92	0.76	0.16	−0.85	−0.74	−0.12
2012	3.07	2.84	0.23	0.66	0.79	−0.13	−0.52	−0.47	−0.05
2013	0.51	0.48	0.03	0.60	0.58	0.02	−0.41	−0.39	−0.03
2014	1.70	1.49	0.21	0.46	0.46	0.00	−0.45	−0.44	−0.01
2015	−1.44	−1.80	0.36	0.30	0.30	−0.01	−0.44	−0.42	−0.02
2016	4.21	3.93	0.28	0.24	0.25	−0.01	−0.79	−0.69	−0.11
2017	4.99	3.91	1.08	0.50	0.44	0.06	−0.63	−0.68	0.05
2018	1.79	1.92	−0.14	0.42	0.48	−0.06	−0.62	−0.69	0.07
Mean	2.04	1.83	0.21	0.55	0.54	0.00	−0.60	−0.57	−0.03
	N_{lev}	N_{log}	ε_N	X_{lev}	X_{log}	ε_X	$\hat{\Phi}_{lev}$	$\hat{\Phi}_{log}$	ε_A
2010	−0.09	−0.25	0.16	0.55	1.12	−0.57	1.83	2.47	−0.64
2011	0.02	−0.15	0.18	0.65	1.01	−0.36	3.03	3.15	−0.12
2012	−0.67	−0.63	−0.04	−0.24	−0.19	−0.04	2.31	2.33	−0.03
2013	−0.27	−0.65	0.39	0.12	0.41	−0.29	0.54	0.43	0.12
2014	−0.23	−0.46	0.22	0.52	0.78	−0.27	1.99	1.83	0.16
2015	−0.07	−0.32	0.26	0.32	0.62	−0.30	−1.33	−1.61	0.28
2016	−0.13	−0.52	0.39	0.38	0.57	−0.18	3.91	3.53	0.37
2017	−0.07	−0.30	0.23	0.29	0.35	−0.06	5.07	3.72	1.36
2018	0.51	0.48	0.03	0.16	0.08	0.08	2.26	2.28	−0.02
Mean	−0.11	−0.31	0.20	0.31	0.53	−0.22	2.18	2.01	0.16

Notes: The panel sets out the decomposition results and log distortions for the average manufacturing industry. The productivity components are in %, the log distortions are in percentage points

4.3 Decomposition of the log distortions according to their sources

We now turn to the decomposition of the log-induced distortions according to the three propositions formulated in Sections 2 and 3. This will help explain the observed discrepancies in the total distortions described in the previous section. Table 2 illustrates the mean and the respective interval of each log distortion. To begin with, the table shows that all three distortions, the log approximation error (Proposition 1), the reference deviation (Proposition 2) and the mean deviation (Proposition 3), decisively contribute to the discrepancy between levels and logs, albeit with variations across the productivity components and aggregate growth.

Subsequently, we investigate all three propositions at the level of the individual productivity components. At the end of this section, we shed light on the contribution of the three propositions to the total log distortion for each of the five FHK productivity components and aggregate growth. The analysis will clarify the drivers of the three log distortions to elucidate when and why a respective distortion is of particular relevance.

Table 2 Decomposition of the log distortions

	Log approximation error $\varepsilon_{\cdot,appr}$	Reference deviation $\varepsilon_{\cdot,ref}$	Mean deviation $\varepsilon_{\cdot,\triangle mean}$	Total distortion $\varepsilon_{\cdot}$
WFE	2.75	− 2.54	–	0.21
	[2.29, 3.62]	[− 3.83, − 1.79]		[− 0.21, 1.08]
BFE	0.14	− 0.06	− 0.07	0.00
	[− 0.23, 0.38]	[− 0.10, − 0.03]	[− 0.26, 0.20]	[− 0.13, 0.16]
CFE	− 0.03	0.00	–	− 0.03
	[− 0.17, 0.05]	[− 0.13, 0.22]		[− 0.12, 0.07]
N	0.54	− 0.02	− 0.31	0.20
	[0.36, 0.75]	[− 0.12, 0.01]	[− 0.64, − 0.13]	[− 0.04, 0.39]
X	− 0.61	0.01	0.38	− 0.22
	[− 1.06, − 0.35]	[− 0.03, 0.06]	[0.23, 0.74]	[− 0.57, 0.08]
$\hat{\Phi}$	2.78	− 2.61	0.00	0.16
	[2.36, 3.34]	[− 3.97, − 1.85]	[0.00, 0.00]	[− 0.64, 1.36]

Notes: The table sets out the decomposed log distortions for the average manufacturing industry according to the three propositions stated in Sections 2 and 3. For each productivity component, the first row represents the mean while the second row depicts the interval, that is, the minimum and maximum, of the annual values for the respective distortion during the 2009–2018 period. Note that "·" is a placeholder for the notation of the log distortions for each productivity component, as defined in Section 4.1. All log distortions are reported in percentage points

However, it will become evident that, owing to the variety of influencing factors, a high degree of uncertainty is prevalent with respect to the final impact of the propositions on productivity growth.

4.3.1 Proposition 1: Log approximation error

In the aggregate, the log approximation error (Proposition 1) is consistently positive with a mean of 2.78. With respect to its decomposition, Table 2 reveals that the main driver of the distortion is the *WFE*, where the error term is consistently positive due to the concave logarithmic function. It is particularly large in years with high firm-level growth rates in absolute terms (see Section 3.1). The fact that its mean of 2.75 strongly exceeds the log approximation errors of the other productivity components is unsurprising, given that the approximation error inherent in the *WFE* is weighted with the input shares of all incumbent firms.

The approximation errors in the *BFE* and *CFE* are weighted by changes in input shares, $\triangle s_i$, which induce either positive or negative distortions at the firm level, reducing the aggregate impact of the approximation error (see Sections 3.2 and 3.3). This also implies that there is no discernible pattern in the distortions of the two components.

With respect to entering and exiting firms, one can generally assume that their input shares are small when compared to incumbents, which also explains their comparatively small aggregate approximation errors. Indeed, the investigation of the log approximation error on an annual basis in our dataset reveals a close relationship between the shares of entering and exiting firms, respectively, and their aggregate log approximation error. The annual changes in the share of entry and exit also explain the fluctuations observed in the log approximation errors of entry and exit, as reflected in the intervals (see Table 2). Despite these fluctuations, the two distortions are consistently positive for entering and negative for exiting firms. Once again, this is a consequence of the concavity of the logarithm (see Section 3.4).

In sum, the log approximation error in aggregate growth $\hat{\Phi}$ can generally be assumed to be positive. The log approximation errors of entry and exit can be expected to compensate each other to a certain extent, leaving the always positive and large log approximation error in the *WFE* to dominate the aggregate. This theoretical derivation is also confirmed by our empirical results, which do not report a single industry-year combination with a negative log approximation error in aggregate growth. Still, the possibility of a negative approximation error is theoretically possible, if an industry experiences an enormous wave of exiting firms, combined with low entry activity and an almost stagnant productivity development of incumbents. However, such a scenario is highly unlikely, given that a wave of market withdrawals is likely to be caused by strongly decreasing productivity levels of incumbents. This would induce a strongly positive log approximation error in the *WFE*, which, in turn, would lead to a positive approximation error in aggregate growth.

4.3.2 Proposition 2: Reference deviation

In our sample, the reference deviation (Proposition 2) in aggregate productivity growth is consistently negative with a mean of -2.61. With respect to the composition of the reference deviation across the productivity components, the dominance of the *WFE* is even more pronounced than for the first proposition (see Table 2). A major reason behind this observation is again the previously mentioned high aggregate input share of incumbents (see Section 4.3.1). Another reason can be found in the mechanism causing the reference deviation in the first place. As explained in Section 3.1, the reference deviation in the *WFE* emerges because logs use φ_{i1} instead of Φ_1 as reference. Due to a typically high degree in firm-level productivity dispersion, even within narrowly defined industries (Bartelsman et al. 2013), the discrepancy between φ_{i1} and Φ_1 can be substantial. Interestingly, the reference deviation in the *WFE* was consistently negative. This is an artifact, as we cannot conclude the sign of the aggregate reference deviation from our theoretical analysis in Section 3.1. In fact, only three out of the 108 industry-year combinations show a positive reference deviation. A possible explanation is that firms with below-average productivity ($\varphi_{i1} < \Phi_1$) tended to increase their productivity, whereas firms with above-average productivity ($\varphi_{i1} > \Phi_1$) tended to decrease their productivity.[8] Overall, this contributes to a negative reference deviation in the *WFE*. Despite being a plausible explanation, note that it should be considered only an indication because the exact reference deviation is ultimately determined by the difference between the firm-level and the reference productivity as well as by

[8] Between 2009 and 2018, 57% of firms with below-average productivity increased productivity, and 58% of above-average firms decreased productivity (averages over all industries and years).

the magnitude of each firm's productivity growth and input share (see Section 3.1). This variety of influencing factors also helps explain the large interval in the *WFE*'s reference deviation.

We now turn to the reference deviation in the *BFE*, which is almost negligible compared to the *WFE*. Recall from Section 3.2 that the reference deviation appears in the *BFE* because the log difference entails the geometric mean Π_1 instead of the arithmetic mean Φ_1 as reference. Evidently, the gap between the two means will generally be smaller than between φ_{i1} and Φ_1, as was the case for the *WFE*. Despite showing only small annual deviations, the reference deviation in the *BFE* is consistently negative. This confirms our theoretical analysis in Section 3.2, where we predict that a positive *BFE* (see Table 1) will be subject to a negative reference deviation.

Compared to the *WFE*, the reference deviation in the *CFE* is substantially smaller. This is because in the *CFE* the difference in the reference productivity is multiplied by each firm's change in input share $\triangle s_i$, instead of s_{i1}. Because $\triangle s_i$ can be both positive and negative, we observe a certain balancing effect when aggregating the firm-level reference deviations to the industry level.

With respect to entry and exit, the reasoning behind the small reference deviations is similar to the *BFE*. They appear because logs use Π_1 instead of Φ_1 as reference. Due to the high correlation between the two means combined with their comparatively small input shares, the reference deviation does not yield large distortions in the entry and exit component.

In sum, the reference deviation in aggregate productivity growth $\hat{\Phi}$ is almost exclusively determined by the reference deviation in the *WFE*. Owing to a persistently negative reference deviation in the *WFE* in our sample, it also persists in aggregate productivity growth. It should be emphasized, however, that the sign of the reference deviation in the *WFE* remains undetermined from a theoretical point of view, as we argue in Section 3.1. Moreover, in our decomposition results in Appendix E, we can report positive values for the reference deviation in the *WFE* and the aggregate.

4.3.3 Proposition 3: Mean deviation

By definition, the impact of the mean deviation (Proposition 3) on aggregate productivity growth $\hat{\Phi}$ is zero. Despite its irrelevance to aggregate productivity growth, it is worth investigating the mean deviation because it makes a difference at the level of the productivity components. In contrast to the first two propositions, the mean deviation does not emerge in each component of the FHK decomposition, but only in the *BFE* and the components of entering and exiting firms. As explained in Section 3, the productivity contributions of these three components are measured against a benchmark productivity, Φ_1 in case of levels and Π_1 in case of logs.

We start with the mean deviation in the *BFE*. While the average mean deviation is rather small, the interval at which the distortions range is comparatively large. Recalling Section 3.2 that the difference between Φ_1 and Π_1 is always positive, the actual sign and magnitude of the mean deviation eventually depend on the change of incumbent firms' input shares relative to entering and exiting firms. In our sample, the majority of mean deviations are negative across periods. Hence, the share of entering firms exceeds that of exiting firms in most periods.

The mean deviation in the entry component is consistently negative, and it is consistently positive in the exit component. This finding is in line with our analysis in Section 3.4. However, we see considerable fluctuations in both components' mean deviations, as reported in Table 2. The reasons behind these variations are two-fold: first, high variations in input shares of entering and exiting firms, and second, annual fluctuations in the difference between the arithmetic mean Φ_1 and the geometric mean Π_1.

In sum, the difference between the positive and negative mean deviations of exiting and entering firms will always be compensated by the either positive or negative distortion in the *BFE*, so that the mean deviation in aggregate productivity growth $\hat{\Phi}$ is consistently zero.

4.3.4 Interaction of Propositions 1–3: Total log distortion

In this section, we investigate the extent to which we can derive general statements about the interaction of the three log distortions with respect to the total distortions in the five FHK productivity components and aggregate productivity growth.

As the right column of Table 2 illustrates, the log distortions evoke a positive tendency in the total distortions in the *WFE* (ε_W) and the entry component (ε_N) as well as a negative tendency in the distortion of the exit component (ε_X). The distortions for the *BFE* (ε_B) and *CFE* (ε_C) paint a rather balanced picture. Overall, this leads to a positive tendency in the aggregate log distortion (ε_A). Nonetheless, looking at the intervals of total distortions, we see that both positive and negative values are possible in each productivity component as well as in the aggregate.

When comparing the impact of the three log distortions on the FHK components and aggregate growth, our sample may lead to the conclusion that the log approximation error is the most relevant one, as it is the log distortion with the highest mean in each component in Table 2. However, as shown in Section 4.2, severe misinterpretations in decomposition studies not only occur due to the mismeasurement of a specific observation but also due to flawed comparisons across time and industries. Such comparisons are particularly aggravated if the log distortion is subject to large fluctuations. Hence, if one defines the relevance of log distortions not by their average magnitude but by their variation, the most relevant distortion for the *WFE* and for aggregate productivity growth, for instance, would be the reference deviation due to its large interval, whereas it would be the mean deviation for entering firms. In sum, in a certain sample, determining the most relevant log distortion depends on the definition of relevance, which, in turn, depends on the research purpose.

Overall, we emphasize that determining the most relevant log distortion essentially depends on the sample used for conducting a decomposition exercise. Another sample with a different development of firm-level productivity and input shares will most likely generate different distortions. As there is no deterministic development of these firm-specific values, it is impossible to make a general statement regarding the log distortions. This stresses the gravity of our contribution to a greater extent, as a researcher cannot know the magnitude and direction of the log distortions in productivity decomposition in advance.

4.4 Practical implications for productivity decompositions

The empirical findings of the previous sections have three major implications with respect to productivity decomposition studies that measure firm-level productivity in logs. First, the considerable log distortions and their unpredictable nature imply that the magnitude and even the sign of aggregate productivity growth and productivity components in log-based decomposition studies are unreliable. Second, the unsystematic pattern of the log distortions implies that comparisons between log-based results over time and across industries may be severely flawed. Third, policy implications based on decomposition studies using logs may be subject to pitfalls. In other words, policies that are or were proposed based on inaccurate evidence are likely to be inappropriate.

The solution we propose is straightforward: productivity decomposition should be performed in levels instead of logs. Although there may be several good reasons behind using the logarithm, as mentioned in the introduction of this paper, we claim that the potential pitfalls outweigh the advantages of using log linearization in productivity decomposition exercises.

5 Fallacy in productivity decomposition

The previous section showed that log-based decompositions embody the fallacy in decomposition – using logs may lead to either an inaccurate aggregate growth rate, an inaccurate description of the contribution of the micro sources, or both. In this section, we quantify this fallacy to provide a general idea of the accuracy of log-based decompositions and the scope of the log-induced misinterpretations.

To simplify our investigation, we define three micro sources of economic growth – firm learning, defined as the within-firm effect holding the input share constant and allowing firm efficiency to vary (WFE in Eqs. 8 and 9); the resource reallocation effect, resulting from changes in the input shares of firms ($BC = BFE + CFE$); and industry churning, defined as the effect of entry into and exit from the market ($NX = N + X$).

To quantify the fallacy in decomposition, we perform the FHK decomposition using Eqs. (8) and (9), respectively, at the four-digit industry level for each year. We then measure the overall log distortion ε_A and the log distortions in the individual productivity components, that is, in the within component ε_W, the reallocation component ε_{BC} ($\varepsilon_{BC} = \sum_i \varepsilon_{i,B} + \sum_i \varepsilon_{i,C}$), and the churning component ε_{NX} ($\varepsilon_{NX} = \sum_i \varepsilon_{i,N} + \sum_i \varepsilon_{i,X}$). Trimming the bottom and the top 1% of each of these four log distortions yields 2805 observations.

We, in turn, infer the fallacy in decomposition by simply counting the frequency of 'inaccurate' measurements, which we define as follows: first, with respect to aggregate growth, we arbitrarily qualify a measurement as 'accurate' if the aggregate growth in logs does not differ from the aggregate growth in levels by more than $\pm\alpha\%$, where α represents the tolerance level below which the log measure is considered accurate: $|\varepsilon_A/\hat{\Phi}_{lev}| \leq \alpha\%$. Second, we define the measurement of the contribution of components as accurate if no log components' contribution to aggregate growth in logs deviates by more than $\pm\alpha$ percentage points from the respective counterpart in levels. That is: $\triangle\theta_{WFE} \wedge \triangle\theta_{BC} \wedge \triangle\theta_{NX} \leq \alpha/100$, where $\triangle\theta_Z = |Z_{lev}/\hat{\Phi}_{lev} - Z_{log}/\hat{\Phi}_{log}|$

Table 3 The fallacy in decomposition – four-digit industry level

Aggregate growth	Contribution of components: Accurate	Contribution of components: Inaccurate
$\alpha = 5$		
Accurate	7.9	6.5
Inaccurate	23.7	61.9
$\alpha = 10$		
Accurate	21.6	6.6
Inaccurate	28.2	43.6
$\alpha = 20$		
Accurate	43.3	5.9
Inaccurate	24.8	26.0

Notes: $N = 2805$ Parameter α denotes the tolerance level that determines whether a measure is accurate or inaccurate. As for aggregate growth, it represents the magnitude of the aggregate distortion (ε_A) relative to the true growth rate ($\hat{\Phi}_{lev}$). A log-based aggregate growth rate is considered accurate when $|\varepsilon_A/\hat{\Phi}_{lev}| \leq \alpha\%$. The contribution of components is considered accurate if no log components' contribution to aggregate growth in logs deviates by more than α percentage points from the respective level counterpart. That is: $\triangle\theta_{WFE} \wedge \triangle\theta_{BC} \wedge \triangle\theta_{NX} \leq \alpha/100$, where $\triangle\theta_Z = |Z_{lev}/\hat{\Phi}_{lev} - Z_{log}/\hat{\Phi}_{log}|$ and $Z = \{WFE; BC; NX\}$. The χ^2 test reveals that the two events 'accuracy in aggregate growth using logs' and 'accuracy of contributions using logs' are related at 1% significance level, irrespective of the tolerance level α. Numbers in the table are in % of industry-year combinations, that is, of $N = 2805$

and $Z = \{WFE; BC; NX\}$. We counted the frequency of accurate and inaccurate measurements of both aggregate growth and the contribution of the individual productivity components and built a 2×2 table that presents the frequencies of the four possible cases.[9] Table 3 documents the results for three tolerance levels, namely $\alpha = 5$, $\alpha = 10$, and $\alpha = 20$.

Starting with a low tolerance level, where $\alpha = 5$, we observe that the log-based decomposition exercise proves accurate for only 8% of the decomposition exercises. In the majority of cases (62%), decomposition using logs yields an inaccurate aggregate growth rate and inaccurate contributions of the three components. In almost one out of four cases, log-based decomposition yields inaccurate aggregate growth rates without affecting individual contributions. This result implies that, in these cases, the overall mismeasurement in aggregate growth stems from a roughly equal mismeasurement in all growth components. Only 6.5% of the decomposition exercises yield inaccurate contributions with accurate aggregate growth rates. Altogether, at $\alpha = 5$, decompositions are inaccurate in more than nine out of ten cases.

Increasing the tolerance level α to 10 and 20 mechanically increases the number of accurate decomposition exercises to 22 and 43%, respectively. However, this hardly

[9] In Appendix F, we provide a more detailed explanation of the four possible cases.

affects the 'inaccurate-accurate' cases, vice versa, with still approximately three cases in ten. Hence, increasing the tolerance level barely affects the fact that 30% of such log-based decompositions produce inaccurate results. The second observation is that even though an increase in the tolerance level increases the frequency of accurate results, a substantial number of decompositions remains inaccurate – eight decompositions out of ten for $\alpha = 10$ and six out of ten for $\alpha = 20$. Overall, the message from Table 3 is clear – log-based decompositions generally yield inaccurate results with accurate decompositions as the exception, and not the rule.

6 Log distortions and industry characteristics

This section investigates the extent to which distortions caused by log-based productivity growth rates correlate with industry characteristics. We thereby provide evidence regarding whether some industries are more exposed to log distortions than others.

We deployed industry characteristics often used in the literature as a candidate explanation for the observed aggregate productivity growth. These comprehend export intensity (ExpInt: industry sum of export divided by the industry sum of sales), profit rate (PrRate: industry sum of profit divided by the industry sum of value-added), investment rate (InvRate: industry sum of investment divided by the industry sum of value-added), the number of firms in the four-digit industry (firm count FC, in logs), mean firm size (MFS, in logs: industry sum of working hours divided by industry number of firms), and industry concentration as measured by the Herfindahl–Hirschman Index (HHI) for market shares in sales. We do not have any particular prior on whether and how these industry characteristics are associated with log distortions, and by no means do we intend to depict causal relationships running from industry characteristics to log distortions. This advocates the use of an ordinary least squares estimator and the following model specification:

$$\mathbf{Y}_{st} = \alpha + \mathbf{B}'\mathbf{X}_{st} + \epsilon_{st} \tag{15}$$

where $\mathbf{Y} = \{\varepsilon_A, \varepsilon_W, \varepsilon_{BC}, \varepsilon_{NX}\}$ and $\mathbf{X}$ includes the six industry characteristics mentioned above. Subscripts s and t stand for four-digit sector s at time t. Column vector $\mathbf{B}$ represents the parameter estimates, which, in this case, should be interpreted as mere partial correlation coefficients.

The process of documenting how industry characteristics are associated with log distortions is not as straightforward as it may initially seem. As log distortions can be positive or negative, the sign of the parameter estimates in model (15) cannot simply be interpreted as increasing or decreasing the distortion. To illustrate this point, imagine that the distortion is positive ($\mathbf{Y} > 0$), that is, that logs underestimate the productivity component. Subsequently, a positive parameter estimate suggests that the given industry characteristic is positively associated with log distortions. Instead, imagine that the average distortion is negative. Then, a positive parameter implies that the given industry characteristic moderates log distortions. To resolve this ambiguity, one could use the absolute value of distortions as the LHS variable. This, however, excludes the possibility of an asymmetrical correlation, that is, that a given industry characteristic only underestimates but does not overestimate aggregate growth and vice versa. Therefore, we allowed the partial correlations to differ between a positive ($\mathbf{Y} > 0$) and a negative log distortion ($\mathbf{Y} < 0$) and ran model (15) on the two respective subsamples.

Furthermore, we built our two subsamples exclusively on whether ε_A is positive or negative. In our data, the overall number of observations is 2805, of which 1619 pertain to an underestimation of aggregate productivity growth ($\varepsilon_A > 0$), and 1186 pertain to an overestimation of aggregate productivity growth ($\varepsilon_A < 0$). We then regressed ε_A, ε_W, ε_{BC} and ε_{NX}, as defined in Section 5, sequentially on the vector of explanatory variables $\mathbf{X}$. As $\varepsilon_A = \varepsilon_W + \varepsilon_{BC} + \varepsilon_{NX}$, the reported parameter estimates pertaining to the dependent variables ε_W, ε_{BC} and ε_{NX} all sum to the estimate pertaining to ε_A: $\hat{\beta}_{\varepsilon_A} = \hat{\beta}_{\varepsilon_W} + \hat{\beta}_{\varepsilon_{BC}} + \hat{\beta}_{\varepsilon_{NX}}$. This method allows us to depict where the sources of the overall log distortion stem from and whether this affects the respective contributions.[10]

The left (right) panel of Table 4 displays the results for underestimated (overestimated) aggregate growth rates. Focusing first on the left panel and starting with export intensity, we observe that sectors more committed to international trade are associated with larger log-induced underestimations ($\hat{\beta}^{\varepsilon_A>0}_{\varepsilon_A,ExpInt} = 0.615$). This result mainly stems from a significant underestimation of the within component ($\hat{\beta}^{\varepsilon_A>0}_{\varepsilon_W,ExpInt} = 0.872$), though partially compensated by a moderating churning coefficient ($\hat{\beta}^{\varepsilon_A>0}_{\varepsilon_{NX},ExpInt} = -0.310$). This result implies that the contribution of firm learning is systematically underestimated in more open industries. Looking at the right panel, we find no significant overestimation issue for more open industries, except for the within component ($\hat{\beta}^{\varepsilon_A<0}_{\varepsilon_W,ExpInt} = 0.395$). Altogether, industry openness is associated with a systematic underestimation of aggregate growth and affects the contributions of the micro sources of growth.

Turning to the profit rate, we observe a similar pattern – industries with higher profit rates are associated with a larger log-induced underestimation ($\hat{\beta}^{\varepsilon_A>0}_{\varepsilon_A,PrRate} = 0.912$), whereas there is no significant association with an overall log-induced overestimation. In the former case, the distortions essentially stem from an underestimation of the reallocation component ($\hat{\beta}^{\varepsilon_A>0}_{\varepsilon_{BC},PrRate} = 0.500$). In the latter case, where no significant association between profit rate and a log-induced overestimation can be identified, the distortion is caused by a positive association of the within component ($\hat{\beta}^{\varepsilon_A<0}_{\varepsilon_{BC},PrRate} = 0.651$), which is compensated by the negative association of the churning coefficient ($\hat{\beta}^{\varepsilon_A<0}_{\varepsilon_{NX},PrRate} = -0.726$).

Industry concentration is a characteristic that always exacerbates log distortions by increasing both underestimations and overestimations ($\hat{\beta}^{\varepsilon_A>0}_{\varepsilon_A,HHI} = 3.306$ and $\hat{\beta}^{\varepsilon_A<0}_{\varepsilon_A,HHI} = 1.785$). Such distortions spread across all components, except for the reallocation and the net entry distortion component in the case of overestimation. This

[10] This also implies that the interpretation of whether the respective industry characteristic exacerbates or reduces log distortions applies to ε_A exclusively.

Table 4 Industry characteristics and the magnitude of log distortions

	Log-induced underestimation ($\varepsilon_A > 0$)				Log-induced overestimation ($\varepsilon_A < 0$)			
	ε_A	ε_W	ε_{BC}	ε_{NX}	ε_A	ε_W	ε_{BC}	ε_{NX}
ExpInt	0.615**	0.872***	0.053	−0.310*	0.114	0.395*	−0.091	−0.190
	(0.262)	(0.249)	(0.078)	(0.175)	(0.254)	(0.228)	(0.082)	(0.211)
PrRate	0.912**	0.279	0.500***	0.134	−0.041	0.651**	0.033	−0.726**
	(0.379)	(0.361)	(0.113)	(0.253)	(0.366)	(0.328)	(0.118)	(0.305)
HHI	3.306***	1.828***	0.308*	1.171***	1.785***	1.786***	0.039	−0.041
	(0.608)	(0.580)	(0.181)	(0.406)	(0.554)	(0.497)	(0.179)	(0.461)
InvRate	0.005	−0.003	0.019	−0.011	−0.110*	−0.095*	0.003	−0.018
	(0.060)	(0.058)	(0.018)	(0.040)	(0.062)	(0.056)	(0.020)	(0.052)
FC	−0.271***	−0.250***	0.026*	−0.046	−0.307***	−0.143***	−0.007	−0.158***
	(0.047)	(0.044)	(0.014)	(0.031)	(0.043)	(0.039)	(0.014)	(0.036)
MFS	−0.397***	−0.257***	−0.014	−0.126**	−0.193***	−0.036	−0.018	−0.139**
	(0.073)	(0.070)	(0.022)	(0.049)	(0.066)	(0.059)	(0.021)	(0.055)
Observations	1,619	1,619	1,619	1,619	1,186	1,186	1,186	1,186
R-squared	0.110	0.083	0.017	0.014	0.117	0.093	0.002	0.028

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Constant not reported. The left panel shows log-induced underestimations ($\varepsilon_A > 0$), and the right panel overestimations ($\varepsilon_A < 0$). The dependent variable is the absolute value of the difference between levels and logs for the individual components: Aggregate (ε_A); Within (ε_W); Reallocation (ε_{BC}); and Churning (ε_{NX}). A positive (negative) sign for the parameter estimate indicates that a given industry characteristics is associated with a larger (smaller) underestimation or overestimation using logs is a clear indication that highly concentrated industries are prone to distortions when decomposing aggregate productivity growth based on log-transformed measures of efficiency. This not only affects the estimated productivity growth but also casts doubt on the relevance of the contribution of each component.

With respect to the investment rate, industries with a high investment rate appear not to correlate with log distortions (apart from a negligible but still significant, distortion-reducing firm learning effect in the case of overestimation), neither with regard to the aggregate growth rate nor with any of the respective contributions of the micro sources of growth.

The two key variables that significantly reduce log distortions are the number of firms and the average size of firms. Regarding firm count, the within (ε_{NX}) and the churning (ε_W) components reduce distortions. For the reallocation component, a high number of firms is associated with a larger underestimation ($\hat{\beta}^{\varepsilon_A>0}_{\varepsilon_{BC},FC} = 0.026$). If competition becomes more intense with an increasing number of firms in an industry, then profit rates, market shares, and price–cost margins should decline. This, in turn, is likely to translate into a less right-skewed distribution of sales and size, weakening the position of dominant firms. With respect to average firm size, the churning component decreases distortions, supported by a distortion-reducing firm-learning component. Our interpretation is that a higher mean firm size is a proxy for entry barriers. In turn, fewer movements in firm entry and exit reduce distortions due to industry churning.

Overall, the results unambiguously show that industries with a low degree of competition, as measured by industry concentration or profit rate, as well as industries with a high openness to international trade, proxied by export intensity, are associated with higher log distortions. Accordingly, more competitive industries are associated with lower log distortions. With this in mind, the validity of decomposition studies with log-based productivity growth rates, which examine the role of industry characteristics in productivity growth, must be put into perspective as the use of log-transformed productivity components will inevitably induce severe endogeneity problems in inferential regression analyses.

7 Conclusion

The use of logs in productivity decomposition induces fallacious conclusions – using logs may lead to either inaccurate aggregate productivity growth, an inaccurate description of the contribution of the productivity components, or both. As we show, this fallacy is caused due to three log distortions: (i) the log approximation error, as a consequence of the logarithm's concavity; (ii) the reference deviation, arising from a different reference assumption implicit in log differences; and (iii), the mean deviation, caused by the difference in the deployed benchmark productivity.

Leveraging the FHK decomposition method, we calculated the respective distortions analytically and showed their magnitude empirically using firm-level data of the French manufacturing sector during 2009–2018. The results suggest that the use of logs can lead to substantial misconceptions regarding productivity developments.

Log-induced distortions appear to be unsystematic, which implies that each productivity component as well as aggregate productivity growth may be either overestimated or underestimated. This impairs the comparison between log and level results as well as the comparison between log results themselves. Overall, our empirical exercise suggests that logs tend to underestimate the growth contribution of the *WFE* and the entry component, while overestimating the contribution of the exit component. Conversely, the *BFE* and *CFE* appear to be less affected by the use of logs. In sum, these tendencies result in a log-induced underestimation of aggregate productivity growth.

Performing decompositions at a fine-grained industry level has allowed us to quantify this fallacy in log-based decompositions. As the results show, even with reasonably high levels of tolerance, the odds that a log-based decomposition will yield misleading results are high. With a simple study on the association of industry characteristics with log distortions, we further show that the magnitude of log distortions is substantial for inferential productivity analyses. The results unambiguously show that industries with a low level of competition are associated with higher log distortions. We, therefore, conclude that the use of log-based growth rates in productivity decomposition studies should be avoided.

Acknowledgements We would like to thank the participants of the 18th ISS Conference for valuable remarks on an earlier draft of this paper. We also thank two anonymous referees and the Editor of the Journal of Evolutionary Economics for their comments on previous versions of the paper.

Funding Open Access funding enabled and organized by Projekt DEAL.

Data Availability The dataset used in this study is not publicly available as it contains proprietary information that the authors acquired through a license. Information on how to obtain it and reproduce the analysis is available from the authors on request.

Declarations

Conflict of interest The authors have no competing interests to declare that are relevant to the content of this article.

Appendix A: Literature overview

Table 5 Application of levels and logs as a labor productivity measure in past decomposition studies

Contribution	Method	Data	Levels/Logs
Griliches and Regev (1995)	GR	Israel Central Bureau of Statistics (CBS) - Industrial Surveys	Levels
Baily et al. (1996)	BHC (modified)	US Census - Manufacturing Sector	Levels
Davis and Haltiwanger (1999)	FHK	US Census - Manufacturing Sector	Logs
Baily et al. (2001)	GR (modified)	US Census - Annual Survey of Manufactures of the Longitudinal Research Database	Levels
Foster et al. (2001)	FHK	US Census - Manufacturing and Services Sector	Logs
Scarpetta et al. (2002)	GR, FHK	Firm-level data from ten OECD countries: United States, Germany, France, Italy, United Kingdom, Canada, Denmark, Finland, Netherlands, and Portugal	Logs
Bernard et al. (2003)	FHK	Simulated data - Based on parameters from US Manufacturing	Levels
Disney et al. (2003)	BHC, GR, FHK	UK Census of Production - Annual Census of Production Respondents Database	Logs
Van Biesebroeck (2003)	BHC (modified)	US - Automobile Assembly Plants and Longitudinal Research Database	Logs
Bartelsman et al. (2004)	GR, FHK	Firm-level data from 24 countries	Logs
Van Biesebroeck (2005)	BHC (modified)	Firm-level data from nine African countries: Based on surveys in the Manufacturing Sector	Logs
Foster et al. (2006)	FHK	US Census - Census of Retail Trade	Logs
Hakkala (2006)	GR, FHK	Statistics Sweden - Sample Manufacturing Sector	Levels
Lentz and Mortensen (2008)	FHK	Danish Business Statistics Register - Annual panel of privately owned firms	Levels
Bartelsman et al. (2009)	GR, FHK	Firm-level data from 14 countries: Estonia, Hungary, Latvia, Romania, Slovenia, Argentina, Brazil, Chile, Colombia, Mexico, Venezuela, Indonesia, South Korea, and Taiwan [China]	Logs
Haskel and Sadun (2009)	FHK	UK Annual Respondents Database (ARD) - Retail Sector	Logs
Maliranta and Määttänen (2015)	OP (augmented)	Statistics Finland - Structural Business Statistics Data	Logs

Table 5 continued

Contribution	Method	Data	Levels/Logs
Melitz and Polanec (2015)	DOPD	Slovenian AJPES - Slovenian Manufacturing Sector	Logs, (Levels)
Decker et al. (2017)	DOPD, FHK	US Census - Revenue-enhanced Longitudinal Business Database (ReLBD)	Logs
Acemoglu et al. (2018)	FHK	US Census - Manufacturing Sector	Levels
Alon et al. (2018)	DOPD	US Census - Revenue-enhanced Longitudinal Business Database (ReLBD)	Logs
Brown et al. (2018)	DOPD	Mexico - Annual Industrial Survey (EIA); Columbia - Manufacturing Survey (EAM); Chile - National Annual Manufacturing Survey (ENIA); Peru - Annual Economic Survey (EEA)	Levels, (Logs)
Dias and Marques (2021b)	DOPD/FHK (modified)	Statistics Portugal - Portuguese nonfinancial firms	Logs

Notes: This table provides an overview of recent decomposition literature and documents the measure deployed for representing firm-level productivity (levels and/or logs). BHC: Baily, Hulten and Campbell (1992), GR: Griliches and Regev (1995), FHK: Foster, Haltiwanger and Krizan (2001), DOPD: 'Dynamic Olley-Pakes Decomposition' by Melitz and Polanec (2015); 'Levels' and 'Logs' in parentheses means that some results were reported in these measures as a supplement to the mainly applied measure

Appendix B: Generalization of log distortions

Here, we show the validity of our propositions regarding the distortions induced by logs when computing aggregate productivity growth rates in a more generalized form. In the main text, we focused on the FHK decomposition method and showed how the use of logs induces three types of distortions which we formalized in three propositions. Proposition (1) depicts the log approximation error as a consequence of the logarithm's concavity. In Proposition (2), we explain that logs induce a reference deviation, arising from the difference in the reference productivity deployed by levels and logs, respectively. Proposition (3) captures the mean deviation, caused by the discrepancy in the deployed benchmark productivity.

In the following, we do not decompose productivity growth and abstract from within-, between- or cross-firm effects as well as from entry and exit components. Consequently, the findings are independent of a specific decomposition method and can be applied more generally to productivity growth computations.

To start with, this more general approach entails an important change regarding the separation of log-induced distortions according to the aforementioned propositions. As we are not decomposing growth contributions, the requirement for a benchmark productivity no longer applies. This implies that the mean deviation (Proposition 3) will have no impact on this aggregate perspective – a finding which is also in line with our statements in Section 3, where we introduce the mean deviation as our third proposition. Moreover, dropping the benchmark productivity will also affect the impact of the two distortions depicted by Propositions (1) and (2), because their manifestation in the FHK components takes the existence of the two different means for levels and logs, respectively, into account.

To reveal the manifestation of the two remaining log distortions of Propositions (1) and (2) in a more general manner, we concentrate on the contribution of a single firm i to the aggregate productivity growth rate, as we have done in the main text:

$$C_{i,lev} = (s_{i2}\varphi_{i2} - s_{i1}\varphi_{i1}) \cdot \Phi_1^{-1} \tag{B1}$$

$$C_{i,log} = s_{i2} \ln \varphi_{i2} - s_{i1} \ln \varphi_{i1} \tag{B2}$$

The difference in the contribution between levels (Eq. B1) and logs (Eq. B2) will only be zero, if the log distortions, that is, the log approximation error and the reference deviation, are simultaneously nil. Obviously, $\Delta C_i \neq 0$ will be the most common case. Calculating this difference yields:

$$\Delta C_i = (s_{i2}\varphi_{i2} - s_{i1}\varphi_{i1}) \cdot \Phi_1^{-1} - (s_{i2} \ln \varphi_{i2} - s_{i1} \ln \varphi_{i1}) \tag{B3}$$

As the use of logs only affects the productivity measure but not input shares, we rewrite Eq. (B3) by separating the productivity contribution caused by changes in shares and changes in productivity:

$$\Delta C_i = s_{i1}\left(\frac{\Delta\varphi_i}{\Phi_1} - \Delta \ln \varphi_i\right) + \Delta s_i\left(\frac{\Delta\varphi_i}{\Phi_1} - \Delta \ln \varphi_i\right) + \Delta s_i\left(\frac{\varphi_{i1}}{\Phi_1} - \ln \varphi_{i1}\right) \tag{B4}$$

Using Eq. (B4), we can now separate the total difference according to the two distortions depicted by Propositions (1) and (2) in the main text, namely the approximation error and the reference deviation, as follows:

$$\begin{aligned} \Delta C_i = {} & \underbrace{(s_{i1} + \Delta s_i)\left(\overbrace{\frac{\Delta\varphi_i}{\varphi_{i1}} - \Delta \ln \varphi_i}^{\text{T1}}\right) + \Delta s_i\,(\overbrace{1 - \ln \varphi_{i1}}^{\text{T2}})}_{\varepsilon_{i,appr}} \\ & + \underbrace{(s_{i1}\Delta\varphi_i + \Delta s_i \Delta\varphi_i + \varphi_{i1}\Delta s_i)\left(\Phi_1^{-1} - \varphi_{i1}^{-1}\right)}_{\varepsilon_{i,ref}} \end{aligned} \tag{B5}$$

The log approximation error ($\varepsilon_{i,appr}$) contains the difference between values in levels and their logged counterparts. The reference deviation ($\varepsilon_{i,ref}$) contains the distortion caused by the difference in the respective reference productivity, which is φ_{i1} as the implicit reference productivity of a log difference, and Φ_1, as the actual reference productivity of levels.

Note that the log approximation error arises from two terms. The first term (T1) captures the log approximation of the contribution of a firm-level productivity increase to aggregate productivity growth. The second term (T2) depicts the fact that, even if firm-level productivity is constant, logs approximate the contribution of a change in firm-level input shares to aggregate productivity growth. To illustrate the derivation of this second term, we compare Eqs. (B1) and (B2) under the assumption of a constant productivity level over time ($\varphi_{i1} = \varphi_{i2}$). Moreover, we take the reference deviation substantiated in Proposition (2) into account, that is, we assume that levels had the same implicit reference φ_{i1} instead of Φ_1. Then, a change in share weight Δs_i will only be equal in levels ($(s_{i2} \cdot \varphi_{i1} - s_{i1} \cdot \varphi_{i1}) \cdot \varphi_{i1}^{-1}$) and logs ($s_{i2} \cdot \ln \varphi_{i1} - s_{i1} \cdot \ln \varphi_{i1}$) if $\ln \varphi_{i1} = \varphi_{i1}/\varphi_{i1} = 1$. From this, it follows that any deviation of φ_{i1} from the logarithm's base will create a distortion induced by the log approximation error if the respective firm changes its input share.

Conclusively, Eq. (B5) comprises the two log distortions pointed out by Propositions (1) and (2), and thereby accounts for the total discrepancy between levels and logs.

Appendix C: Industry classification

Table 6 A38 and ISIC industry classification in manufacturing

A38	ISIC	Description
CA	10-12	Manufacture of food products, beverages, and tobacco products
CB	13-15	Manufacture of textiles, wearing apparel, leather, and related products
CC	16-18	Manufacture of wood and paper products; printing and reproduction of recorded media
CD	19	Manufacture of coke and refined petroleum products
CE	20	Manufacture of chemicals and chemical products
CF	21	Manufacture of basic pharmaceutical products and pharmaceutical preparations
CG	22-23	Manufacture of rubber and plastics products, and other non-metallic mineral products
CH	24-25	Manufacture of basic metals and fabricated metal products, except machinery and equipment
CI	26	Manufacture of computer, electronic, and optical products
CJ	27	Manufacture of electrical equipment
CK	28	Manufacture of machinery and equipment n.e.c
CL	29-30	Manufacture of transport equipment
CM	31-33	Other manufacturing; repair and installation of machinery and equipment

Notes: The table sets out the intermediate SNA/ISIC aggregation A38, which aggregates similar ISIC two-digit divisions to 13 different categories. It is the industry classification deployed in the main text, excluding the industry of coke and refined petroleum products (ISIC 19)

Appendix D: Summary statistics

Table 7 Summary statistics for French manufacturing 2009-2018

	Obs	Mean	Sd	Min	Med	Max
All manufacturing						
Value-Added	260,674	5,951,312	$4,81 \cdot 10^7$	1,405.84	1,309,477	$6,04 \cdot 10^9$
Employees (FTE)	260,674	77.14	519.53	5	23	68326.5
Working Hours	260,674	117,363	791,967.40	7,177.97	35,211	$1,04 \cdot 10^8$
Value-Added/Working Hours	260,674	40.64	20.44	0.09	36.30	724.62
Manufacturing of chemicals and chemical products (ISIC 20)						
Value-Added	8,252	$1,47 \cdot 10^7$	$4,42 \cdot 10^7$	65,269.38	3,302,234	$8,96 \cdot 10^8$
Employees (FTE)	8,252	139.16	422.65	5	40	8447.5
Working Hours	8,252	208,831.10	634,371	7,455.88	59,790.08	$1,26 \cdot 10^7$
Value-Added/Working Hours	8,252	61.01	38.49	2.89	52.35	486.37

Notes: The numbers for 'All manufacturing' include firms of all manufacturing industries with the exception of the coke and refined petroleum products industry (ISIC 19). The number of employees is documented in the form of full-time equivalents (FTE). The statistics are reported for the cleaned sample. Value-Added is reported in deflated €

Appendix E: Log distortions in individual industries

In this appendix, we report log distortions within individual industries, as opposed to the average industry values that we investigated in the main text. To this end, we applied Eqs. (8) and (9) to each of the 12 industries listed in Table 6. We subsequently calculated the log-induced discrepancy, resulting in 108 industry-year observations. We treated the large quantity of results by reporting percentiles in Table 8. Subsequently, we present detailed results for manufacturers of chemicals and chemical products (ISIC 20). Over the 2009–2018 period, this industry shows a high frequency of log-induced sign flips in aggregate growth and its components, which highlights the potential misconceptions induced by logs in productivity decomposition.

Table 8 Log distortions in individual industries

	Min	p_{10}	p_{25}	Med	p_{75}	p_{90}	Max
ε_W	– 1.84	– 0.78	– 0.29	0.24	0.66	1.46	3.80
ε_B	– 0.56	– 0.17	– 0.07	– 0.01	0.08	0.15	0.77
ε_C	– 0.85	– 0.23	– 0.09	– 0.03	0.03	0.10	0.49
ε_N	– 1.30	0.02	0.08	0.16	0.32	0.52	1.22
ε_X	– 2.94	– 0.73	– 0.36	– 0.20	– 0.10	– 0.04	1.44
ε_A	– 3.36	– 1.15	– 0.43	0.09	0.72	1.34	4.56

Notes: $N = 108$. The table sets out the distribution of the annual log distortions occurring in the decomposition exercises conducted for the 12 industries in our sample in the period 2009-2018. For each component, the number of industry-year combinations is $N = 108$. The reported values for the log distortions are in percentage points; $p_{(\cdot)}$ reflect percentiles

As shown in Table 8, most distortions are restricted within the range of approximately ±1 percentage point, even though it is possible that log distortions reach or even exceed values of 3 percentage points. Analogous to our previous findings, the *BFE* and *CFE* are, in absolute terms, less affected by log distortions. Nonetheless, even these two components can be subject to considerable distortions, exceeding our observations for the average industry. Moreover, for the 108 industry-year combinations within our sample, we identified 25 combinations with a sign flip either in at least one of the five components or in aggregate productivity growth. This result implies that almost one out of four industry-year combinations is affected by a sign flip, which underlines the potential impact of log distortions on individual industries.

Comparing the distortions between different industries, we detect that some industries are strongly affected by logs, whereas others are less affected. This means that when performing a decomposition exercise for an individual industry, the results are not necessarily strongly distorted by logs. The observed differences between industries raise the question of whether there is a systematic pattern or certain characteristics that make an industry more or less prone to log distortions. We provided evidence regarding such relationships in Section 6.

We now turn to the decomposition results for manufacturers of chemicals and chemical products (see Table 9). The summary statistics for the chemicals industry are reported in Appendix D in Table 7. Overall, the results are in line with our findings regarding the average industry. However, the magnitude and fluctuations of the distortions exceed those for the average industry. The distortions in aggregate productivity growth range from −2.07 to 3.96 percentage points. In relative terms, the distortion varies between −69% (2012) and 143% (2017). These distortions are mostly driven by the large deviations in the *WFE,* which range from −47% in 2012 to 191% in 2017. In the *BFE*, the span of distortions reaches from −94% in 2017 to 71% in 2011. For the *CFE,* the largest overestimation amounts to −81% in 2010, whereas the underestimation is the strongest in 2017, with 28%. Once again, the relative distortions are most pronounced in the entry and exit components. In the case of the entry component, the discrepancy is always positive, ranging from 21% in 2011 to 434% in 2018. The distortion in the exit component is consistently negative, ranging from −1122% in 2017 to −8% in 2011.

It is obvious that the combination of the magnitude and volatility of these log distortions can lead to severe misconceptions concerning productivity growth. This is especially evident in the prevalence of sign flips in aggregate productivity growth (2011 and 2017), the *WFE* (2017) and the exit component (2013 and 2017), as reported in Table 9. A look at the *BFE* reveals a further example of a misconception. Considering the development of the *BFE* between 2011 and 2012, logs create the impression that the *BFE* has almost doubled between 2011 and 2012, whereas the results in levels show that it has actually decreased.

The described high fluctuations are also reflected in the large ranges of the log distortions in all components, especially in the *WFE, BFE, CFE,* and aggregate productivity growth (see Table 10). Despite the volatility in distortions, we again detect an average positive distortion in the *WFE* (ε_W), the entry component (ε_N), and aggregate growth (ε_A), whereas the average distortion in the exit component (ε_X) is negative.

Table 9 Productivity decomposition results in levels and in logs in the chemical industry

	WFE_{lev}	WFE_{log}	ε_W	BFE_{lev}	BFE_{log}	ε_B	CFE_{lev}	CFE_{log}	ε_C
2010	−4.16	−3.53	−0.63	−0.09	−0.13	0.04	−0.36	−0.07	−0.29
2011	−1.76	−1.97	0.21	0.69	0.20	0.49	−0.89	−0.67	−0.22
2012	0.77	1.14	−0.36	0.47	0.37	0.11	−0.43	−0.27	−0.16
2013	7.94	8.62	−0.68	0.80	0.69	0.12	−0.84	−0.63	−0.21
2014	−2.53	−3.00	0.47	0.35	0.46	−0.11	−0.80	−1.01	0.21
2015	−5.10	−8.91	3.80	1.12	1.24	−0.12	−1.59	−1.73	0.14
2016	15.05	16.89	−1.84	1.45	1.48	−0.03	−2.21	−2.27	0.06
2017	1.38	−1.26	2.63	−0.23	−0.01	−0.21	−0.12	−0.16	0.03
2018	5.03	5.84	−0.81	0.51	0.48	0.03	−0.76	−0.71	−0.05
Mean	1.85	1.54	0.31	0.57	0.53	0.04	−0.89	−0.84	−0.05
	N_{lev}	N_{log}	ε_N	X_{lev}	X_{log}	ε_X	$\hat{\Phi}_{lev}$	$\hat{\Phi}_{log}$	ε_A
2010	1.41	1.03	0.38	−1.00	−0.26	−0.73	−4.20	−2.97	−1.23
2011	2.29	1.81	0.48	0.38	0.41	−0.03	0.71	−0.22	0.94
2012	−0.29	−0.39	0.10	0.52	0.93	−0.41	1.05	1.77	−0.72
2013	−0.12	−0.29	0.17	−0.08	0.05	−0.13	7.71	8.44	−0.74
2014	−0.14	−0.33	0.19	−0.37	−0.20	−0.16	−3.49	−4.08	0.59
2015	−0.42	−0.65	0.23	0.25	0.34	−0.09	−5.75	−9.70	3.96
2016	−0.16	−0.24	0.08	−0.54	−0.20	−0.34	13.59	15.66	−2.07
2017	0.88	0.42	0.46	−0.02	0.20	−0.22	1.89	−0.81	2.70
2018	−0.09	−0.50	0.41	−0.49	−0.31	−0.18	4.20	4.80	−0.60
Mean	0.37	0.09	0.28	−0.15	0.11	−0.26	1.75	1.43	0.31

Notes: The panel sets out the decomposition results and log distortions for the industry of 'Chemicals and chemical products' (ISIC 20). The productivity components are in %, the log distortions are in percentage points

The occurrence of positive values in the reference deviation of the *WFE* ($\varepsilon_{W,ref}$) in the results for the chemicals industry in Table 10 is most striking. Recall that the reference deviation exhibits a strong negative tendency in our sample, for which we offered the opposite direction of the development of below and above-average productivity firms as a possible explanation. In the chemicals industry, we detect positive values for the reference deviation in the years 2015 (0.76) and 2017 (0.064). Again, the development of firms with below and above-average productivity may provide one possible explanation for the reference deviation in those two years. As both below and above-average firms mostly decrease their productivity in 2015 and 2017, the negative reference deviation of the first group seems to be compensated by the positive reference deviation of the second group, yielding an overall positive reference deviation.[11]

[11] Of firms with an initial productivity above the mean ($\varphi_{i1} > \Phi_1$), 64% (2015) and 61% (2017) decrease their productivity. Of below-average firms ($\varphi_{i1} < \Phi_1$), 61% and 55%, respectively, decrease their productivity.

Table 10 Decomposition of the log distortions in the chemical industry

	Log approximation error $\varepsilon_{\cdot,appr}$	Reference deviation $\varepsilon_{\cdot,ref}$	Mean deviation $\varepsilon_{\cdot,\Delta mean}$	Total distortion $\varepsilon_{\cdot}$
WFE	3.41	− 3.10	–	0.31
	[2.11, 6.05]	[− 7.89, 0.76]		[− 1.84, 3.80]
BFE	0.25	− 0.08	− 0.14	0.04
	[− 0.13, 1.08]	[− 0.23, 0.01]	[− 0.47, 0.09]	[− 0.21, 0.49]
CFE	− 0.08	0.03	–	− 0.05
	[− 1.25, 0.29]	[− 0.40, 1.31]		[− 0.29, 0.21]
N	0.61	− 0.07	− 0.26	0.28
	[0.25, 1.16]	[− 0.28, 0.02]	[− 0.49, − 0.08]	[0.08, 0.48]
X	− 0.72	0.07	0.39	− 0.26
	[− 1.50, − 0.18]	[− 0.01, 0.19]	[0.10, 0.86]	[− 0.73, − 0.03]
$\hat{\Phi}$	3.47	− 3.16	0.00	0.31
	[2.69, 4.61]	[− 6.69, 0.52]	[0.00, 0.00]	[− 2.07, 3.96]

Notes: The table sets out the decomposed log distortions for the industry of 'Chemicals and chemical products' (ISIC 20) according to the three propositions stated in Sections 2 and 3. For each productivity component, the first row represents the mean while the second row depicts the interval of the annual values for the respective distortion during the 2009-2018 period. Note that "·" is a placeholder for the notation of the log distortions for each productivity component, as defined in Section 4.1. All log distortions are reported in percentage points

Hence, by revealing how log distortions are driven by the idiosyncratic development of firms and industries, the chemicals industry offers an instructive example of the difficulty in predicting the impact of log distortions.

Appendix F: Decomposition fallacy: Four hypothetical cases

In Section 5, we quantified the fallacy in productivity decomposition, which we infer by counting the frequency of the different types of log-induced mismeasurements. Table 11 illustrates these types of mismeasurement. It is based on the assumption that the correct decomposition exercise is performed using levels according to Eq. (8), as opposed to using logs according to Eq. (9).

In Table 11, case 1 describes the situation when the log-based decomposition produces the same results as those obtained from the level-based decomposition. In case 2, using logs leads to an overestimation of aggregate growth (6 versus 4%), while leaving the relative contribution of each component unaffected. In case 3, log-based decomposition generates inaccurate relative contributions (for example, 25 versus 50% for the *WFE* component) while leaving aggregate growth unaffected. Case 4 is the worst-case scenario when both types of mismeasurement apply. Cases 2 to 4 all represent the fallacy in decomposition when using logs. All imply an incorrect representation of the sources of aggregate productivity growth.

Table 11 Decomposition fallacy: Four hypothetical cases

	$\hat{\Phi}$	WFE	BC	NX
	Results using level-based decomposition			
Correct values	4	2	1.5	0.5
	100	*50*	*37.5*	*12.5*
	Possible cases using log-based decomposition			
Case 1: No error	4	2	1.5	0.5
	100	*50*	*37.5*	*12.5*
Case 2: Error in aggregate growth	6	3	2.25	0.75
	100	*50*	*37.5*	*12.5*
Case 3: Error in contributions	4	1	2	1
	100	*25*	*50*	*25*
Case 4: Both types of errors	6	1.5	3	1.5
	100	*25*	*50*	*25*

Notes: Figures in italics represent the respective contributions of the within-firm effect (*WFE*), the reallocation effect (*BC*), and the net-entry effect (*NX*) in % of aggregate growth ($\hat{\Phi}$) as defined in Section 5

Appendix G: Log distortions in related decomposition methods

The aim of this section is to show that the general patterns we have identified for the FHK decomposition also hold true for the related methods proposed by Griliches and Regev (1995) (GR) and Melitz and Polanec (2015) (DOPD: 'Dynamic Olley-Pakes Decomposition'), which represent two commonly used alternatives. In addition to the similarities, we will note important differences in the DOPD method.

Like the FHK method, the method proposed by Griliches and Regev (1995) is a longitudinal approach. The GR method decomposes productivity using average weights. For simplicity, we express the decomposition in a somewhat 'neutral' form, not differentiating between levels and logs in the denotation, as done, for instance, by Baily et al. (2001).

$$\hat{\Phi}_{GR} = \sum_{i \in S} \underbrace{\overline{s_i} \cdot \triangle\varphi_i}_{WFE_i} + \sum_{i \in S} \underbrace{\triangle s_i \cdot (\overline{\varphi_i} - \overline{\Phi})}_{BFE_i} + \sum_{i \in N} \underbrace{s_{i2} \cdot (\varphi_{i2} - \overline{\Phi})}_{N_i} + \sum_{i \in X} \underbrace{s_{i1} \cdot (\overline{\Phi} - \varphi_{i1})}_{X_i} \tag{G1}$$

As with the FHK method, when using levels, the decomposition formula above would require a reference productivity for calculating growth rates. As Griliches and Regev use average weights, the choice of $\overline{\Phi}$ may be the most intuitive one in this case (Van Biesebroeck, 2008). However, in line with the approach of Baily et al. (2001), who deployed a modified version of the GR method, we use Φ_1 as a reference productivity. This method also facilitates a comparison between our results for the GR and the FHK method. By using the averages of firm-level productivity $\overline{\varphi_i}$ and input shares

$\overline{s_i}$, there is no interaction term or cross-firm effect, as in the FHK method. A further important difference from the FHK method is the choice of benchmark productivity, which measures the impact of input share reallocations and entering and exiting firms. Instead of the initial aggregate productivity Φ_1, the GR method deploys $\overline{\Phi}$, that is, the average between the aggregates in the starting and ending period.

The log distortions for the weighted average industry for the 2009–2018 period are shown in Table 12. The results are similar to those presented for the FHK method. The distortions in aggregate productivity growth range from −0.64 to 1.36 percentage points and are mostly driven by the distortions in the *WFE*. The exit and entry components also show considerable absolute distortions, whereas the *BFE* is less affected. Analogous to the results for the FHK method, on average, logs underestimate the *WFE* and the entry component, but they overestimate the exit component. The *BFE* shows a minor negative tendency. Taken together, this induces, on average, a positive log distortion in aggregate productivity growth.

The decomposition method proposed by Melitz and Polanec (2015) is based on the cross-sectional approach proposed by Olley and Pakes (1996). Instead of tracking individual firms over time, the DOPD method decomposes aggregate productivity in two different periods and, in turn, contrasts the individual components. Apart from the entry and exit components, they decompose the contribution of incumbents into a

Table 12 Decomposition of the log distortions in the GR decomposition

	ε_W	ε_B	ε_X	ε_N	ε_A
2010	− 0.22	0.00	− 0.59	0.17	− 0.64
2011	− 0.04	0.11	− 0.36	0.17	− 0.12
2012	0.21	− 0.15	− 0.03	− 0.06	− 0.03
2013	0.02	0.01	− 0.29	0.38	0.12
2014	0.21	− 0.01	− 0.26	0.22	0.16
2015	0.35	− 0.01	− 0.30	0.25	0.28
2016	0.23	− 0.06	− 0.18	0.38	0.37
2017	1.10	0.01	0.02	0.22	1.36
2018	− 0.10	− 0.05	0.07	0.06	− 0.02
Mean	0.19	− 0.02	− 0.21	0.20	0.16
SD	0.39	0.07	0.21	0.14	0.53
Median	0.21	− 0.01	− 0.26	0.22	0.12

Notes: The table sets out the annual decomposed log distortions during the 2009–2018 period for the entire manufacturing sector, using the decomposition method by Griliches and Regev (1995) (GR). The results are based on the annual averages of the industry-level results for the 12 industries in our sample. As industry weights, we used labor input in the form of hours worked, averaged over the beginning and ending years of the period in which the respective growth rate was measured. All reported values for the log distortions are in percentage points

Table 13 Decomposition of the log distortions in the DOPD decomposition

	ε_W	ε_B	ε_X	ε_N	ε_A
2010	− 0.42	− 0.03	− 0.62	0.20	− 0.87
2011	− 0.22	0.10	− 0.40	0.19	− 0.33
2012	− 0.02	− 0.05	0.01	− 0.19	− 0.25
2013	0.08	− 0.15	− 0.29	0.39	0.02
2014	− 0.01	0.08	− 0.29	0.23	0.01
2015	0.27	0.03	− 0.32	0.25	0.22
2016	0.63	− 0.62	− 0.19	0.39	0.21
2017	0.73	0.23	− 0.05	0.22	1.14
2018	0.33	− 0.62	0.11	0.04	− 0.14
Mean	0.15	− 0.11	− 0.23	0.19	0.00
SD	0.38	0.31	0.22	0.18	0.54
Median	0.08	− 0.03	− 0.29	0.22	0.01

Notes: The table sets out the annual decomposed log distortions during the 2009–2018 period for the entire manufacturing sector, using the decomposition method by Melitz and Polanec (2015) (DOPD). The results are based on the annual averages of the industry-level results for the 12 industries in our sample. As industry weights, we used labor input in the form of hours worked, averaged over the beginning and ending years of the period in which the respective growth rate was measured. All reported values for the log distortions are in percentage points

within-firm and a between-firm effect.

$$\hat{\Phi}_{DOPD} = \underbrace{\triangle\overline{\varphi}_S}_{WFE} + \underbrace{\triangle cov_S(\varphi_{it}, s_{it})}_{BFE} + \underbrace{s_{N2}\cdot(\Phi_{N2} - \Phi_{S2})}_{N} + \underbrace{s_{X1}\cdot(\Phi_{S1} - \Phi_{X1})}_{X} \tag{G2}$$

In the DOPD decomposition, the *WFE* is represented by the development of the unweighted average of firm-level productivity in surviving firms. The *BFE* is expressed by the change in the covariance between the firm-level productivity of incumbents and their input shares. The last two terms represent the contribution of entering and exiting firms relative to the aggregate productivity of surviving firms at a certain point in time.

When representing firm-level productivity in logs, Eq. (G2) can simply be used in the above form. For levels, however, it requires a slight modification to ensure scale invariance in the covariance term. Melitz and Polanec (2015) provide a level representation of the decomposition method in the appendix of their paper. Note that they deployed $\overline{\Phi}$ as a reference productivity. We followed their suggested approach for the results presented in Table 13.

The distortions in aggregate productivity growth range from −0.87 to 1.14 percentage points. What stands out as a striking difference between the DOPD method and the FHK and GR methods, is that the *BFE* is subject to significantly stronger distortions, ranging from −0.62 to 0.23 percentage points with a negative mean. This result implies that, in contrast to the other two methods, whose reallocation component is (in absolute terms) affected only to a limited extent, each of the four productivity components in the DOPD decomposition may be considerably distorted by using logs. Moreover, the log distortions in the *BFE* show a clearly negative tendency on average. This implies that logs tend to overestimate the *BFE* in the DOPD method. This negative tendency also appears to balance out the distortion in aggregate growth, resulting in an average distortion of approximately zero. Hence, for the DOPD method, our sample shows that calculations based on logs are, on average, on spot with respect to aggregate growth.

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