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# METHODICAL PROCEDURE FOR A SURROGATE MODEL BASED FATIGUE CALCULATION TO SUPPORT THE DESIGN PROCESS OF EBIKE DRIVE UNITS

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## ABSTRACT

In this paper, a method is developed to consider multiaxial load spectra and their variation in a computationally efficient local fatigue calculation procedure. This method is based on an FE data-based surrogate model and is intended to support the simulation-based product design process. To demonstrate their application and necessity, a case study on the design of eBike drive units is presented. For this purpose, the general requirements for the design of eBike drive units as well as the fundamentals of multiaxial fatigue analysis and surrogate modeling are outlined. In addition, a validation process of the surrogate model and its use for fatigue calculation is presented and discussed.

## 1. INTRODUCTION

Many structural components of real products are exposed to highly variable and versatile load spectra during their use, resulting in different and geometry dependent multiaxial stresses. In many cases, these loads can also be considered as non-proportional, causing different principal stress axis or a rotation of the principal stress directions, resulting in a challenging fatigue calculation. Relevant examples of multiaxial loads can usually be found at components in aerospace, railway, naval or automotive industry [1]. An emerging product that also falls into this category is the eBike drive unit (DU), which is affected by highly variable user and situation-dependent load spectra coupled with internal loads of the engine [2].
For time and cost reasons, real component durability tests are often not feasible and are therefore (especially in early stages of the product design) predominantly replaced by simulation-based approaches. To enable simulative frontloading in product design, suitable methods for the evaluation of these variable multiaxial loads have to be investigated in order to dimension the products appropriately from the beginning. In addition, this frontloading provides the opportunity to minimize the uncertainties regarding structural behavior that emerge between the loads in the real application and the test loads that are usually simplified due to time and cost constraints. Unfortunately, in practice, the occurrence of multiaxial and non-proportional loads cannot be avoided for the structural design in the case of complex geometries and for certain applications [3, 4]. Thus, their effects must be incorporated in the simulation-based product design process.
For the evaluation of non-proportional multiaxial loads, it is necessary to determine the real load-time curves for the stresses and strains at all critical points of the component. Subsequently, these have to be evaluated by suitable fatigue or damage criteria based on e.g. a critical plane model to condense these variable loading histories to a cyclic load collective of uniaxial equivalent stress or strain amplitudes. This equivalent amplitude can then be used as a comparable value to the usually uniaxially determined material properties and SN-curves for

fatigue life estimation. Although the definition of these fatigue damage criteria is currently still a part of research, several mostly validated (depending on material characteristics and the load combinations) models for different materials and load combinations already exist. [5-8]

However, simplifications are often adopted in product design to minimise the computational effort for calculating loads or damage. The main reasons for this are either the numerically complex simulation of the real load profile and its evaluation, or the initial lack of information about the potential load spectrum and its alteration. In terms of fatigue calculations, non-proportional loads histories are often replaced by quasi static load cases that are assumed to be proportional and are therefore calculated by the well-known strength hypothesis like von Mises and Tresca, even though considerable errors may occur. To minimize the computational effort in the FE simulation, entire load spectra are decomposed into individual loads or partial loads, which are then scaled or superimposed. Again, tolerating an unknown error in the calculation. [9-10]

Although the motivation behind these simplifications is evident, it conflicts with the demand for simulative frontloading in modern product design to reduce development time, uncertainties, and especially expensive real component testing. Particularly since it is desirable in the context of product development that the critical product properties are already addressed at the very beginning of the synthesis and analysis cycles of the design process [11]. Hence an efficient method should be developed to evaluate one of the most important properties, the durability of the product, as early and as accurately as possible. Thereby, the main focus should be on the uncertainty and risk minimization regarding the load assumption and the fatigue calculation methodology, in order to avoid further iteration loops later in the design process. This is particularly important since several studies have already identified significant differences in the service life of uniaxially or multiaxially proportionally and non-proportionally loaded components. [12,13]

Based on the needs described, this work proposes a method based on surrogate modelling, which makes it possible to consider all the multiaxial and non-proportional stress states of a versatile and real load collective efficiently. Furthermore, this approach provides the option for design optimization during the product design cycle by also including effects of changes in design or system parameters. To illustrate this with a practical example, the housing of the aforementioned eBike drive unite is used as a case study.

## 2. STATE OF THE ART

First of all, this section describes the state of the art in the calculation of multiaxial strength hypotheses and the load condition on the eBike DU. Additionally, the general as well as lifetime calculation related benefits of machine learning and deep leaning methods and their application in recent studies are presented.

### 2.1 eBike drive units

Since eBike drive units are typically considered as standard components and are therefore installed in all bicycle categories such as cargo, mountain and trekking bikes, multiple influencing parameters result from the entire bicycle system. In addition to these bike setup-dependent parameters, previous investigations and measurements of the riding behaviour revealed a wide range of loads acting on the drive unit due to the different riders, bicycles and riding situations. This causes complex, highly variable multiaxial and non-proportional loads on several parts of the engine [2].

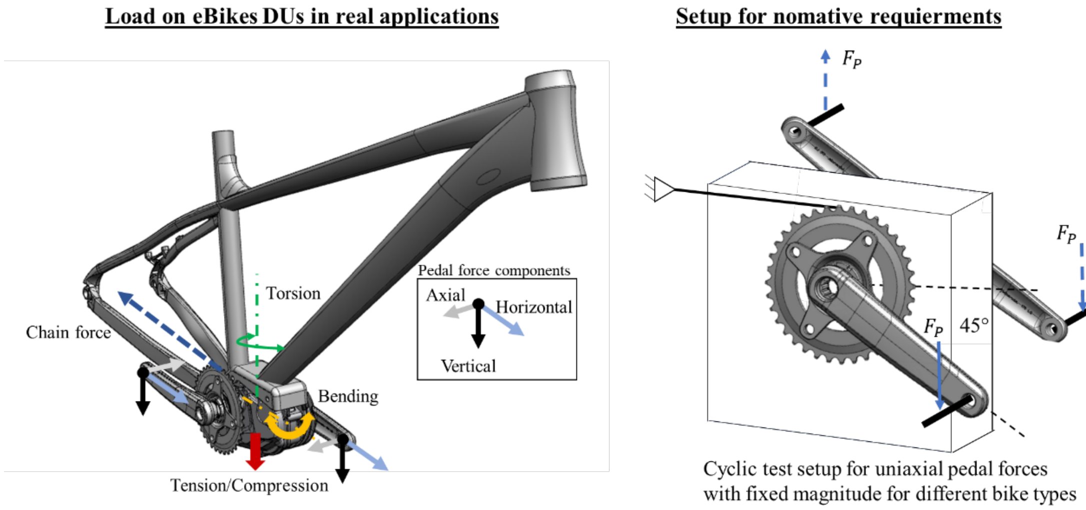


*Figure1: Schematic representation of the real load situation of an eBike DU compared to the normative [2]*

Furthermore, usual manufacturing tolerances of the product-specific design parameters resulting from the manufacturing process and the related static loads must be considered. Additionally internal loads due to the gear and motor forces as well as thermal loads must be incorporated.

Current designs are based on normative specifications that have been adopted from conventional, purely mechanical bicycles and are considering only uniaxial cyclic loadings on the pedals. Real pedal forces, on the other hand, show relevant amounts of force in all three cartesian coordinate directions, resulting in a completely different combination of bending, torsion, and tension or compression loading compared to the norm test (see Figure 1). Moreover, the impact of the bicycle frame, its stiffness and the defined boundary conditions of the bicycle system are neglected in the normative requirements. [2,14]

It is still unknown whether the real loads can be reduced to these one-dimensional partial loads of the normative requirement without significant errors. As an important step to minimize the uncertainties in the product design of eBike drive units, the fatigue caused by real eBike rides should therefore be characterised. In this context, the focus is on the different driver characteristics and, in particular, the combination of different driving situations. Additionally, the calculation has to include the variety of product parameters such as the frame geometry and stiffness, geometries of the crank setup and the drive unit as well as static loading and residual stresses due to manufacturing processes. All these factors are important since they are superimposed with the multiaxial user-dependent pedal loads to calculate the potential load situations on different eBikes.

In order to convert this high variance into a representative fatigue calculation, every possible driving load must be calculated not as an individual load, but over either a measured or synthetic and realistic load time history to evaluate a fatigue life prediction for real riding situations. Based on these results, the uncertainty regarding the differences between the normative uniaxial loads rated as the "worst case" and the totality of potential eBike loads can then be removed.

Overall, it can be stated that, in addition to the dynamic loads of the rider and the corresponding motor and gear loads, static loads due to the assembly and manufacturing processes as well as thermal loads must be factored in. To incorporate the variety of bicycle applications, other influencing variables such as frame stiffness or the geometry of the crank setup must also be taken into account for the fatigue calculation of the drive unit.

 

## 2.2 Multiaxial Fatigue calculation

In general, the fatigue strength under multiaxial loading can be determined by certain strength hypotheses, which are based on the idea of assigning a damage equivalent uniaxial load to each multiaxial stress state. On one hand this equivalent stress drastically helps to reduce the complexity of the three-dimensional stress state, on the other hand it enables the calculation of the fatigue life, based on the reference to and fatigue limits for uniaxial test loading. In this case the equivalent stress is then usually compared to a typical SN-Curve of the material to define the damage of the investigated load [10].

When calculating multiaxial loading, the separation into proportionally and non-proportionally changing load components is of great relevance, since for non-proportional changes a rotation of the principal stress system occurs, which of course has to be investigated differently. In this case, the application of common equivalent stresses hypotheses like von Mises or Tresca is not appropriate [15].

Consequently, for the calculation of non-proportional loads, the entire load time sequences and their effects must be considered with different kinds of strength hypotheses. Likewise, the straightforward scaling of the amplitude in reference to the individual load signal, which is often used for proportional variable amplitude loading, is also not recommended. Further information on that can for example be found in [13,15].

In general, there are two categories of fatigue calculation, that can be distinguished in the frequency and time domains. Typically, the application of these two categories depends on the variability of the loading situation, where random loads are usually considered in a statistical and spectral calculation in the frequency domain. Deterministic loads with constant or variable amplitude are calculated in the time domain [1].

Although frequency domain analysis has been established for uniaxial variable loads, the application of a frequency-based method for multiaxial non-proportional loads is still a point of current research. Therefore, time-based analyses are predominantly used, although they require an enormous computational effort for the determination of the time-based and local loads of a component geometry [16,17]. For this reason, only the time domain calculation methods of multiaxial loads will be discussed in the following.

In recent years, numerous hypotheses have been defined for the determination of an equivalent stress for certain time-dependent cycles on multiaxial non-proportional loads. Comparable to the more common hypotheses for proportional loads, these are also based on the physical quantities that are considered to be decisive for the initiation of fractures and cracks. These are divided into stress, strain and energy-related considerations, which are focused on the application in the typical low cycle, high cycle or infinite life range of the SN-curve. Regarding the intended application in the high cycle fatigue area, only the relevant stress-based calculation will be described in the following. In contrast, for the low cycle fatigue range usually strain or energy-based models are considered to contain the effects of plasticity. Further subdivisions can be found for ductile or brittle materials according to the material behaviour, which generally strongly affects the failure mechanism.[7].

In general, the primary factor for fatigue crack initiation is a combination of (cyclic) principal shear stress and the associated normal stress. Here the underlying assumption is that the shear stress or strain is responsible for the initiation of microstructural cracks which are further opened and enlarged by the normal stress component acting on the crack plane.

Based on this fundamental assumption, that fatigue cracks develop in a certain cracking plane, the most commonly used hypotheses for the investigation of non-proportional loads are the so-called critical plane approaches. This method is intended to calculate either the most

critical plane or the sum of damage by all planes via a damage criterion derived from the normal and shear stress histories at each plane. For this purpose, the stress state at a specific location must be subdivided and evaluated for an arbitrarily fine discretization of possible planes [7,14].
To determine the relevant shear and normal stress components in the plane, first the stress tensor is projected onto a plane defined by the angles $\varphi$ and $\theta$ and the vectors $n_1$, $n_2$, $n_3$ (Formular 1.1-1.3) as shown in Figure 2. Then the projection of the time dependent stress tensor on the given plane delivers a clearly defined normal stress component $\sigma_{\theta\varphi}(t)$ and two shear stress components $\tau_{\theta\varphi,1}(t)$ and $\tau_{\theta\varphi,2}(t)$, which define a closed path curve that can be projected onto the respective plane (see Figure 2). At this point, the non-proportionality of the load case can be identified, as for proportional load cases, a path curve in the form of a straight line is determined, while for non-proportional loads, a curve path with several turns into different direction can be obtained. [6,18]

$$n_1 = \begin{pmatrix} \sin(\varphi)\cos(\theta) \\ \sin(\varphi)\sin(\theta) \\ \cos(\varphi) \end{pmatrix} \quad (1.1) \qquad n_2 = \begin{pmatrix} -\sin(\theta) \\ \cos(\theta) \\ 0 \end{pmatrix} \quad (1.2) \qquad n_3 = \begin{pmatrix} -\cos(\varphi)\cos(\theta) \\ -\cos(\varphi)\sin(\theta) \\ \sin(\varphi) \end{pmatrix} \quad (1.3)$$

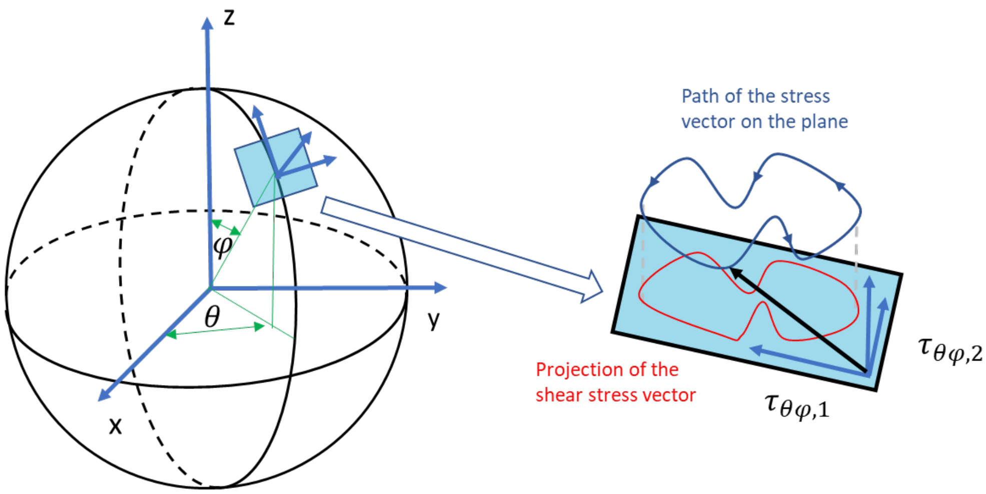


*Figure 2: Spherical coordinate system defining the projection of the stress tensor onto the cutting plane and the subsequential projection of the resulting shear stress curve (according to [18,19])*

$$\sigma_{\theta\varphi}(t) = n_1 \begin{bmatrix} \sigma_x(t) & \tau_{xy}(t) & \tau_{xz}(t) \\ \tau_{yx}(t) & \sigma_y(t) & \tau_{zy}(t) \\ \tau_{xz}(t) & \tau_{yz}(t) & \sigma_z(t) \end{bmatrix} * {n_1}^T \quad (2.1)$$

$$\tau_{\theta\varphi,1}(t) = n_1 \begin{bmatrix} \sigma_x(t) & \tau_{xy}(t) & \tau_{xz}(t) \\ \tau_{yx}(t) & \sigma_y(t) & \tau_{zy}(t) \\ \tau_{xz}(t) & \tau_{yz}(t) & \sigma_z(t) \end{bmatrix} * {n_2}^T \quad (2.2)$$

$$\tau_{\theta\varphi,2}(t) = n_1 \begin{bmatrix} \sigma_x(t) & \tau_{xy}(t) & \tau_{xz}(t) \\ \tau_{yx}(t) & \sigma_y(t) & \tau_{zy}(t) \\ \tau_{xz}(t) & \tau_{yz}(t) & \sigma_z(t) \end{bmatrix} * {n_3}^T \quad (2.3)$$

For the normal stress, calculated by formula 2.1, the mean stress and amplitude which are relevant for the calculation of the strength hypothesis can be determined directly. [6, 18, 19]
For the determination of mean and amplitude of the 2D shear stress derived from formula 2.2 and 2.3 a further mathematical step is required. Their calculation can either be performed by searching for the longest chord, or by the method of the Minimum Circumscribing Circle (MCC) [20]. Both of these methods try to define proper amplitude and mean stress values to reassemble the two-dimensional shear stress curve into a one-dimensional signal. With both methods being solved iteratively (see figure 3).

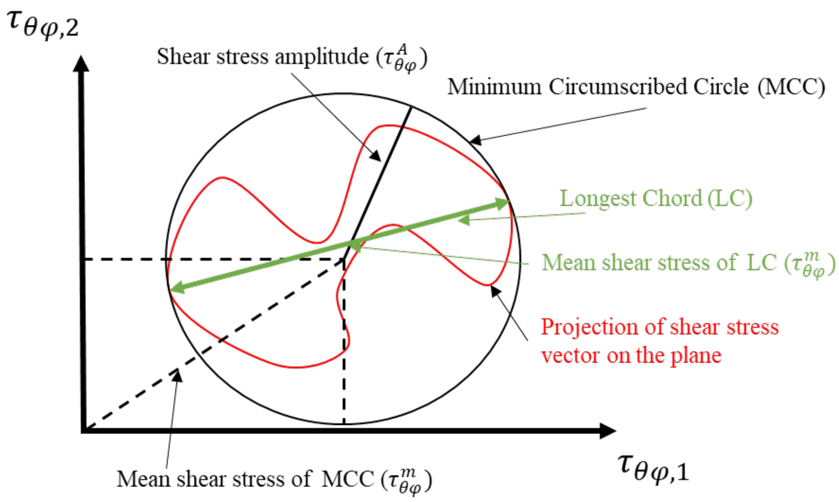


*Figure 3: Projection of the resulting shear stress curve on the critical plane (according to [18,19])*

Finally, an evaluation of these values over a given load-time sequence will provide the time-dependent values for the mean and amplitude stress of the normal and shear stress inside the defined plane relevant for the calculation of multiaxial damage models like the Shear Stress Intensity Hypothesis (SIH) which is listed below and is further explained in [6].

*Shear Stress Intensity Hypothesis [6]:*

$$\sigma_{eq} = \sqrt{\frac{15}{8}\int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi}[a\tau_a{}^2(1+c\tau_m{}^2)+b\sigma_a{}^2(1+d\sigma_m)]\sin(\theta)\,d\varphi d\theta} \quad (3)$$

*With a, b, c, d as material constants, $\tau_a$= Shear stress amplitude, $\tau_m$= Mean shear stress, $\sigma_a$= Noraml stress amplitude, $\sigma_m$= Mean normal stress*

In addition to calculating an equivalent stress for a given load history, it is also crucial to identify a suitable number of cycles, or rather to convert the load-time sequence into different amplitudes and associated numbers of cycles. For cycle definition, similar to uniaxial loads, rainflow-counting is carried out. Rainflow-counting is a procedure in which a load curve is examined for closed hysteresis curves and thus complete load cycles. For its application to multiaxial non-proportional loads, there are different procedures based on one input channel focusing on either the normal, shear or an equivalent stress or a two-channel method representing both the shear and normal stress components. [21, 22]
For the computation of multiaxial loads, it can thus be concluded that a large number of strength hypotheses exist, which need to be distinguished primarily based on the material

 

data, the evaluated target variable and the proportionality or non-proportionality of the load case. The most challenging aspect of their application is the computationally intensive simulation of the load-time curves at the locations of the component under investigation. Furthermore, it should be emphasized that these methods can potentially enable a more precise investigation of the loading situation during the early stages of product design but cannot be used for a final validation without a calibration based on real component tests.

### 2.3 Use of machine leaning and deep learning surrogate models

In recent years, the use of data-based surrogate models in the simulation and calculation of products has become increasingly popular. The primary purpose of using surrogate models is to replicate the computational expensive simulation of numerical models, such as an FEM model, by computationally less expensive analytical and statistical models. These surrogate models are based on a small number of simulation input and output data, which are calculated by only a few of the expensive high-fidelity calculations [23, 24]. Consequently, the formation of metamodels can also be regarded as an advanced type of post-processing [25]. Besides the numerical sources, information from real testing and measurements can also be used for the formation of the surrogate model. Based on this model, the required results for given input parameters can be determined without the need for a new measurement or simulation. Thus, this modelling can be used to efficiently discover optimization potentials, expand the system understanding and minimize uncertainties.

For the formation of surrogate models, a variety of methods with different methodical approaches and levels of complexity have been developed and established in recent years. A general overview of the model types and the general application possibilities of these surrogate models can be found, for example, in [23, 24, 26]. In this contributions different analytical and stochastic regression methods as well as more modern Machine Learning (ML) and Deep Learning (DL) methods based on decision trees and neural networks are discussed and explained.

For the generation of the surrogate models the mostly used strategy is supervised learning, which can be seen as a training with the combined input an output data gathered by the simulation or measurement. This so-called training is an optimization loop that minimizes the error between the surrogate model prediction and the given input data [27].

General examples of such models and their use in combination with FEM simulation can be found in [28-33]

With respect to the fatigue calculation, recent approaches have been presented in which a surrogate model was trained to predict (equivalent) stress values, that are later used to deduce the fatigue information [34] or to predict direct damage [35] values depending on the load situation or geometries. Thereby, the study of [35] considers multiaxial and also non-proportional loads and determines the damage directly based on parameterized and generalized load-time sequences.

Other approaches focused on characterizing the multiaxial damage model or damage criterion using an ML- or DL- model trained for the stress and strain input quantities and the associated measured fatigue behaviour. Thereby, both simulative and experimentally determined data were used for the formation and training of the surrogate model [36, 37].

It can be concluded that surrogate models are frequently used to as a substitution or extension of computational expensive numerical simulations. In that way they show their ability to accurately predict defined output parameters for arbitrary numerical models under varied input parameters, if the input changes are still within the parameter space used for training.

 

Also, several studies already experienced the use of surrogate models with regard to fatigue calculation. The approaches focusing on building a material or component dependent damage criteria can be considered as an interesting extension but not as a direct solution to the problem of this study on the eBike DU. More relevant approaches can be taken from the two studies [34,35] that are intended to predict load or geometry dependent fatigue information. However, even these do not represent a readymade solution, since on the one hand no multiaxiality or non-proportionality is considered and on the other hand only a parameterized generalized load sequence is considered.

# 3. CALCULATION METHOD

## 3.1 Approach

The insights into the state of the art eBike design process clearly show that the variance of the load collectives must be investigated more intensively. Due to all the different dynamic and static loads, clearly multiaxial and non-proportional stress states can be expected, so that easy-to-calculate quasi static equivalent stresses like von Mises or Tresca cannot be applied. For a theoretically correct analysis of these loads, it is therefore necessary to use a calculation method based on critical plane models and a sufficient damage criterion. As already mentioned at the beginning, the biggest limitation hereby is the computational effort.
Overall, a calculation of the local load conditions over the whole geometry would be possible in an FEM simulation. For the computation of the load time histories of all or as many measured sequences of the real load collectives as possible, the calculation is far too costly and time-intensive and thus simply not reasonable. It must also be mentioned that it is not possible to concentrate on one critical point of the product, because the influence of the multiaxial load in combination with the complex geometry does not allow an estimation of a maximum loaded location. Instead, it can even be expected that, depending on the individual load sequences, different points of the product are most critically loaded.
However, the state of the art has shown that the use of ML- or DL-based regressors can be used to generate efficient data-based surrogate models capable of minimizing the overall computational effort and, at the same time, increasing the insight into the observed system behavior. It is obvious that a direct prediction of the damage for the large number of load channels, the different time sequences and the complexity of the critical planes models is difficult to implement and requires an enormous amount of training data. Here the main problem is the dependence on the load history and its variety, which is neither easy to parameterize nor to calculate. Due to the multi-channel load case, the transfer of the calculated damage of one load sequence to another load sequence is also not as trivial as for uniaxial variable amplitude loading and thus again potentially error-prone. Further problems result from the requirement that, apart from load variation, other system and design parameters should also be varied to enable optimization. Because of that, it is presumably more effective to exclude the influence of the load spectrum from the prediction and to not directly aim at a prediction of the damage comparable to [35].

Therefore, the use of the surrogate model must be applied prior to the fatigue calculation. For the design of the eBike engine, the assumptions are made that the loads of the cyclist should be in the range of either infinite life or at worst in the high cycle fatigue range, which is why a stress-based fatigue calculation is the feasible choice. Consequently, a suitable output variable for the prediction of the surrogate model is the local stress tensor, which can be predicted for

an arbitrarily fine discretization of the load time history to provide the required input for the multiaxial fatigue calculation.
Training the regressor for the quasi-static prediction of the local stress tensor on the basis of a given sample data representing the overall parameter space can provide the required stress states needed to assemble any load-time sequences in a computationally efficient way and in real time. As the computational effort is rather low for the fatigue calculation compared with the FEM simulation this will still bring a significant reduction in computational resources. In addition, once the regressor has been trained and the sample data has been calculated, it offers unlimited flexibility in fatigue calculation of various cycles. Thus, the variety of bicycle loads can be calculated, and the associated uncertainty can be minimized.
This initial required parameter space is defined by any individual load and parameter combination from the measured load collectives, the relevant geometry parameters and further influences from the overall eBike system such as the frame stiffness.

Based on the sequence of stress tensor predictions that is representing the load history, the multiaxial damage calculations can then be performed using a critical plane-based damage criterion. Consequently, the initially mentioned simplifications in the FEM calculation as well as in the fatigue calculation can be avoided.
In general, this approach is kind of comparable to the separate calculation of unified load cases, their linear extrapolation and interpolation and later superposition. However, the advantage of the ML- or DL- based surrogate model is the identification of nonlinear relationships between the individual load and influence parameters, as they are interpolating between real samples of the potential parameter space. Because the parameter space is known, it can be ensured that only interpolations are made, which of course increases the accuracy of the predictions compared to extrapolation. In addition, the validation of the model using different splits of training and test, or validation data can determine the model performance using common metrics such as the $R^2$ or the Root Mean Square Error (RMSE) value. Depending on that those metrics the user can decide whether the sample data are delivering good enough models or if the sample data must be increased. By this way also the best performing regressor model or algorithm can be chosen.
If the parameter variation of the eBike system or certain design parameters are also considered when forming the surrogate model, the calculation of the fatigue strength can be used not only to validate the mechanical strength, but also to optimize the design. The decisive advantage here is that the multiaxial and non-proportional loading of the entire collective can be taken into account in order to enable robust optimization of the component for all common field loads.
In order to keep the computational effort for the surrogate model and the fatigue calculation low, it is nevertheless necessary to identify the relevant locations of the component and to focus the calculation on them. For this reason, assumptions can be made that those parts of the product which do not show any relevant stresses in the individual sample FEM calculations do not have to be considered for the calculation of the load-time sequence. Hence, the nodal values of the FEM can be clustered and filtered according to a certain tolerance value. A more detailed derivation of this strategy and the algorithm for the data reduction and clustering which was implemented based on that idea can be found in [38].

All in all, the methodical approach of the two-part process shown in figure 4 can be concluded. This consists of the essential points of surrogate model building based on the considered load and parameter combinations as well as the fatigue calculation fed by the surrogate model.

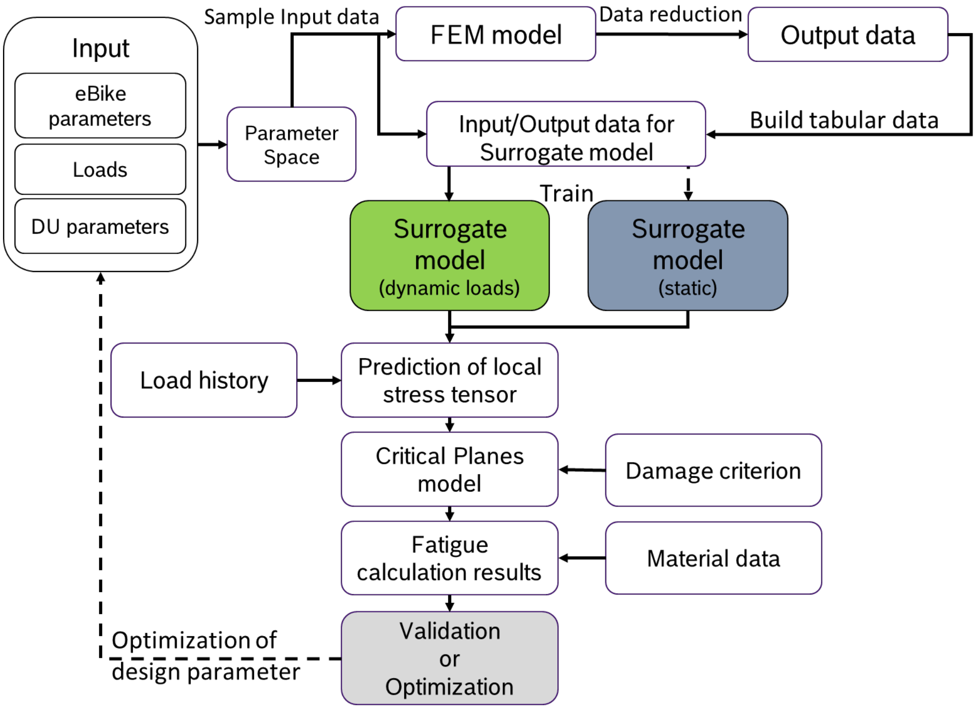


*Figure 4: Methodical approach for the surrogate modelling and subsequent fatigue calculation and optimization*

So far, only dynamic loads (pedal forces, gear forces) of the DU have been considered since they are the obvious cause of the multiaxial non-proportional loading. For their calculation, relevant influencing factors such as the frame stiffness and boundary conditions or the mounting position of the engine are already taken into account, as they are indispensable for the proper modelling of the product.
However, static loads resulting from manufacturing and thermal expansion must also be considered and can have a vital role for the evaluation of the multiaxial fatigue. To keep the dimension of input parameters and the complexity of the surrogate model low, these static stress states are modelled by a separate surrogate model. Thus, the dimension of the input data can be drastically reduced, and the model can be trained with sufficient accuracy with less sample data and therefore less computational effort for the FEM calculations. Due to the assumption that changes in the static load states have no relevant effects on the load situation of the dynamic driving load, a pure superposition of the separately considered stress states can be performed. This assumption is associated with the condition that the static stress states only vary within certain tolerances, at which the functionality of the product is not changed or restricted. Therefore, the indispensable and functionally relevant static loads, due to press-fit and prestressing process, are also calculated for the dynamic model in a preliminary step, although their stress state is subsequently subtracted from the dynamic load states in order to maintain the clear separation of the two kinds of loads.

## 3.2 Validation

The planned surrogate model-based calculation procedure is divided into two parts, the surrogate model creation and its training and the use of the surrogate model for the lifetime calculation. Consequently, both parts must be validated in order to enable an appropriate application of the procedure.

 

As a first step, the general validation of the surrogate model for the single prediction of the respective stress tensor components has to be performed. In addition to the validation of this step, it is particularly important to estimate the size of the sample data set and the correct model approach for the surrogate models generated. This can be performed by the evaluation of the training and test metrics, which can be defined by the offset of the predicted value to some simulated or measured date. In this case, this means that the deviation between the predicted and simulated values of the stress tensor will be analyzed.
To keep the FE simulated sample size feasible, it is necessary to find a compromise between a sufficient model accuracy and the initial simulation effort for the calculation of the sample data. Since deviations between the predictions and the real simulation are expected in any case, the summation of the errors and their effect on the fatigue calculation is particularly relevant for the determination of certain load-time sequences.
Therefore, in a second step, a determination of the accuracy for the fatigue calculation results based on the surrogate data must be made.
To perform this second step, selected load cycles are evaluated by the regressor of the surrogate model and a full FE simulation to obtain the required stress values. Subsequentially the same multiaxial fatigue calculation is performed for both time series (predicted and simulated) of the stress tensors to evaluate the accuracy of the surrogate-based fatigue calculation. Comparable to the first step this metrics can again be used for the examination of the different damage criteria and the number of critical plane discretization.
The whole workflow of the evaluation is illustrated in figure 5, to show the stepwise calculation and validation process of the model.

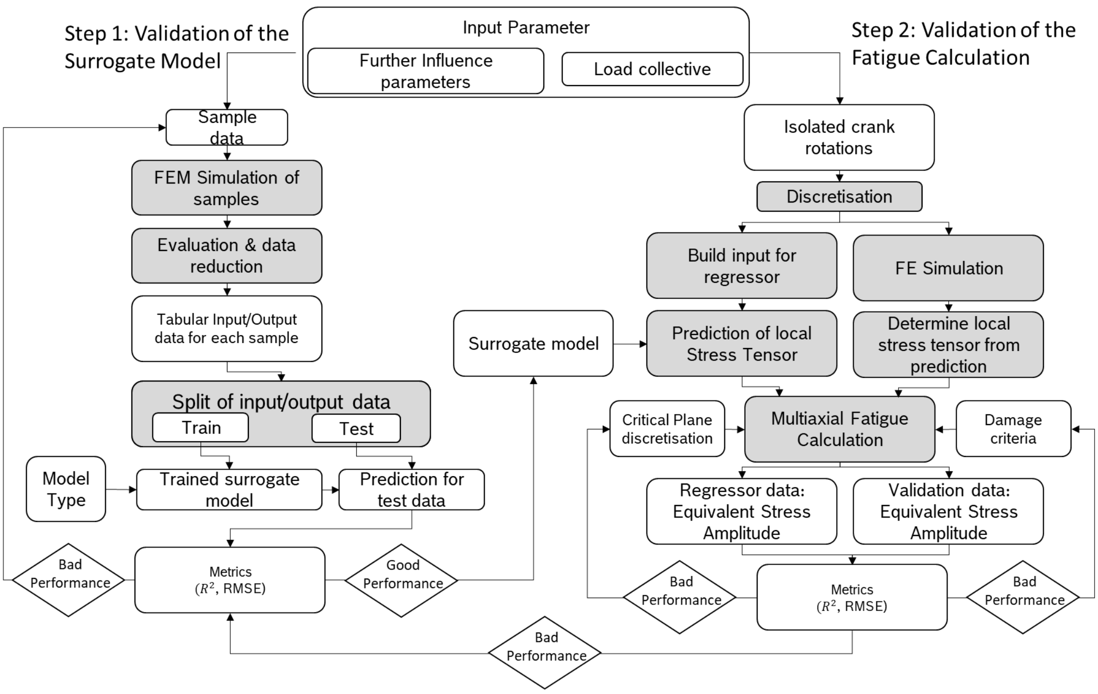


*Figure 5: Validation process of the surrogate model and the usage for the fatigue calculation*

### *3.2.1 Validation of the surrogate model*

For the validation of the surrogate model prediction a set of 400 samples was determined for a parameter space consisting of the complete measured load collectives, the crank geometry parameters and one fixed bike frame stiffness. After building the sample data set by a space filling sampling method based on the Latin Hypercube principle an automated calculation of

those FE models was performed. According to the data reduction method described in [39] a specific discretization of the output data was built to obtain a reduced but still representative mesh of the DU geometry. Subsequentially all these filtered and clustered points were transferred to suitable tabular input output data and split in typical train, test data sets for the training and the validation of the model.
For the formation of the regressor model common ML- model types like XGBoost or Random Forest Regressors were chosen. In addition, a Neural Network was trained to also represent DL- methods. The validation was performed by the determination of the RMSE and $R^2$ metrics. To avoid a selection bias and to recognize overfitting of the regressor model, those metrics were determined for a 10-fold cross validation algorithm that shuffles and reassembles the test and train data ten times to achieve robust validation metrics.
For all splits, a comparable and good $R^2$ and RMSE value could be determined, which indicates a robust regressor model. The results of this validation for one of these splits and the XG-Boost regressor can be observed in figure 6. This shows the comparison between the simulated and the predicted values for the individual stress tensors at all nodes of the reduced structural discretization.
To indicate the potential cumulative error of the errors in each tensor component, the metrics were also calculated for the Mises equivalent stress obtained for the simulated and predicted values. Altogether it can be concluded, that the regressor model is capable of interpolating the stress components at the local positions. Even though the errors increase with respect to the mises equivalent stress, a good performance can be achieved. It could be determined that the selected sample set is sufficient to represent the defined parameter space, so no adjustment of the models or the input-output pairings determined by the sample data is made.

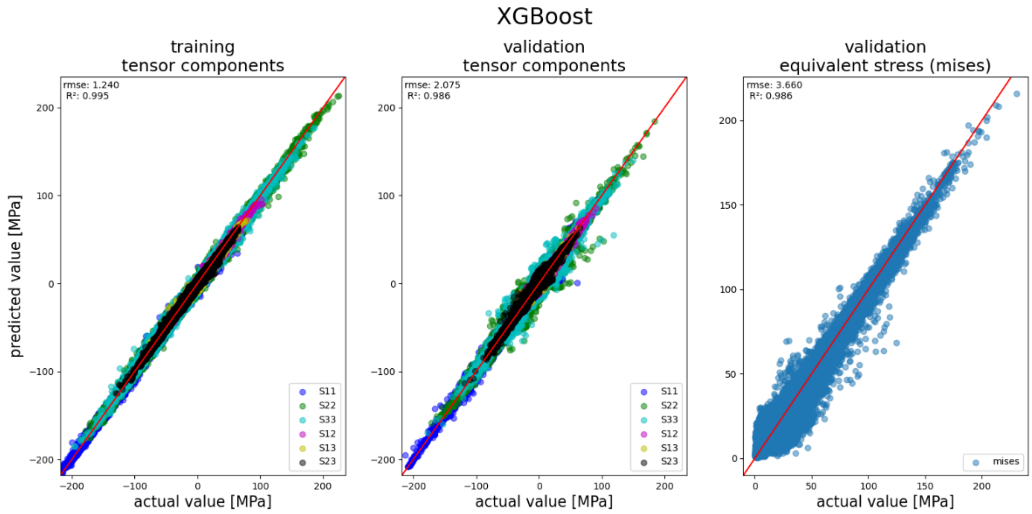


*Figure 6: Results and metrics for all tensor components and the Mises equivalent stress of the XG Boost Regressor.*

Despite their significantly longer computation time, the application of Neural Networks did not show a significant improvement compared to the tree-based algorithms XGBoost and Random Forrest, with both of them showing similar results. For this reason, the XGBoost regressor will also be used for the validation of the surrogate model-based fatigue calculation process.

### *3.2.2 Validation of the surrogate model-based fatigue calculation process*

Since the focus in the second step of the validation is on the actual fatigue values of specific load sequences, measured load time sequences have to be simulated and evaluated as a validation data set.

For this purpose, two complete crank rotations and their DU and pedal forces were recorded in the measurement runs, discretized into five-degree increments of the crank angle and fed into the FEM calculation. This discretization was performed to keep the calculation effort low. Nevertheless, 142 quasi-static calculations were required, demonstrating the enormous effort for the fatigue analysis in the time domain, even for two short load sequences.
The measured crank revolutions are assumed to be one coherent cycle and thus their complete load-time series is used for the determination of the normal and shear stress values. For this reason, the cycle counting methods mentioned in chapter 2.2 are not applied.
As shown in figure 6, this was followed by the formation of the input data for the regressor and the evaluation of its prediction to carry out the damage calculation for the FE simulated and predicted values based on the critical planes. For the validation, only the relevant input variables of the fatigue strength criteria, the normal and shear stress amplitudes in the respective critical plane are considered. These are calculated by the principle of the smallest circumscribed circle for each one of the discretized crank rotations. The results of some examples of the shear stress projection and their MCC analysis can be seen in figure 7.

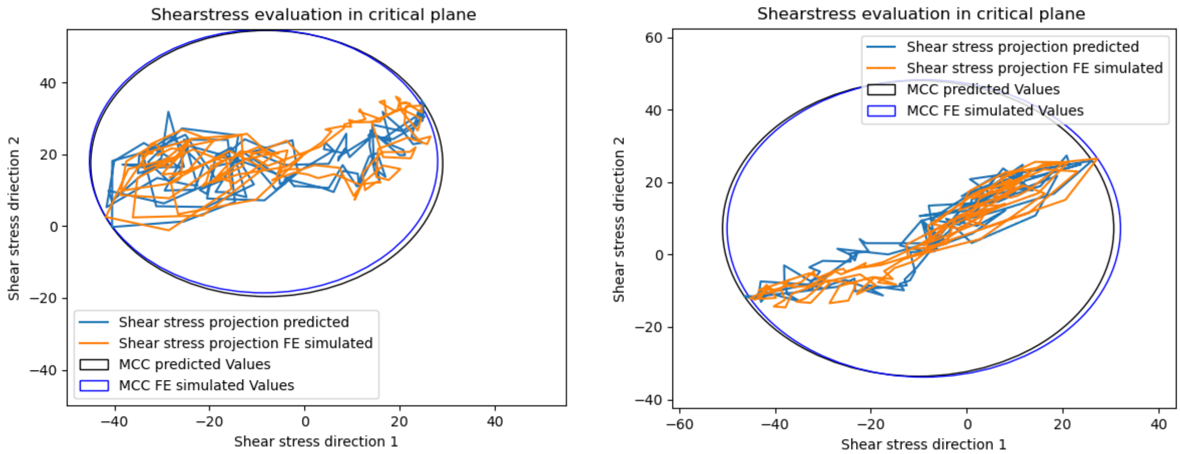


*Figure 7: Comparison of the predicted and simulated shear stress projection*

Despite clear deviations of the projected trajectory, a good agreement can be recognized for the decisive values of the circle diameter and its center for the simulated and predicted values. This behavior could be observed in general and independent of the selected crank revolution, local position of the component or the choice of plane orientation. Thus, the reasonable conclusion can be drawn that the prediction errors are decreased by reducing the analysis to the envelope of the path curve.

Since the evaluation of the shear stress projection is only a part of the overall fatigue calculation, the deviation of the equivalent amplitude calculated by a multiaxial damage criterion is still unknown. Therefore, for all local locations of the DU housing geometry that were used in the formation of the surrogate model, the equivalent stress amplitude was calculated according to the SIH (Formula 3). To validate the method, this calculation was performed for both the FE-simulated stresses and the local stresses predicted by the surrogate model.
A global perspective on these results is given in the histogram of figures 8 and 9, which shows the relative prediction error and the RMSE for the equivalent stress amplitude at each local node that was trained for the surrogate model. These results were obtained for the evaluation of one of the two simulated crank revolutions. But they both showed similar

 

metrics for the Validation. For a better understanding of these voxel pictures figure 10 also shows a normal picture of the calculated DU.

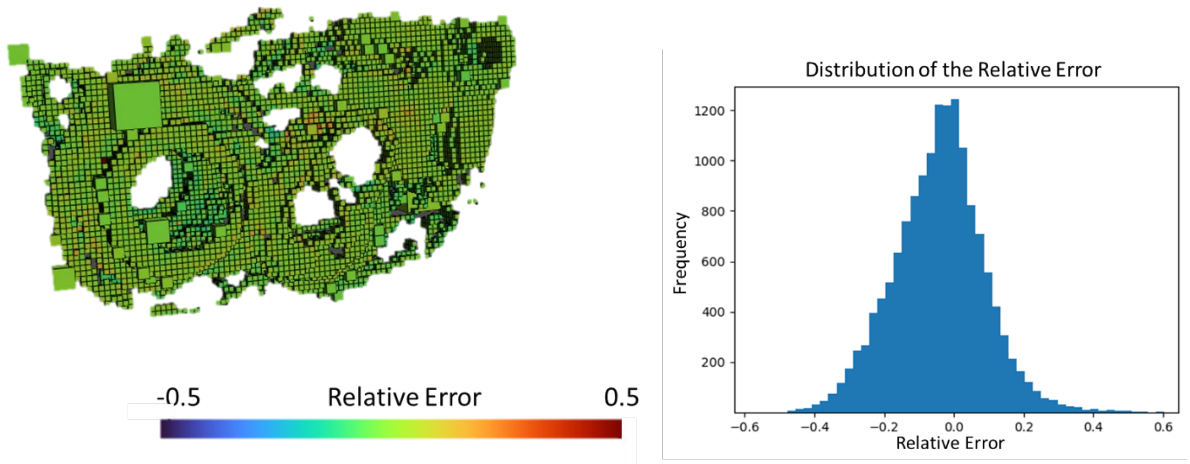


*Figure 8: Distribution of the relative error of the prediction for the equivalent stress amplitude calculated by the SIH*

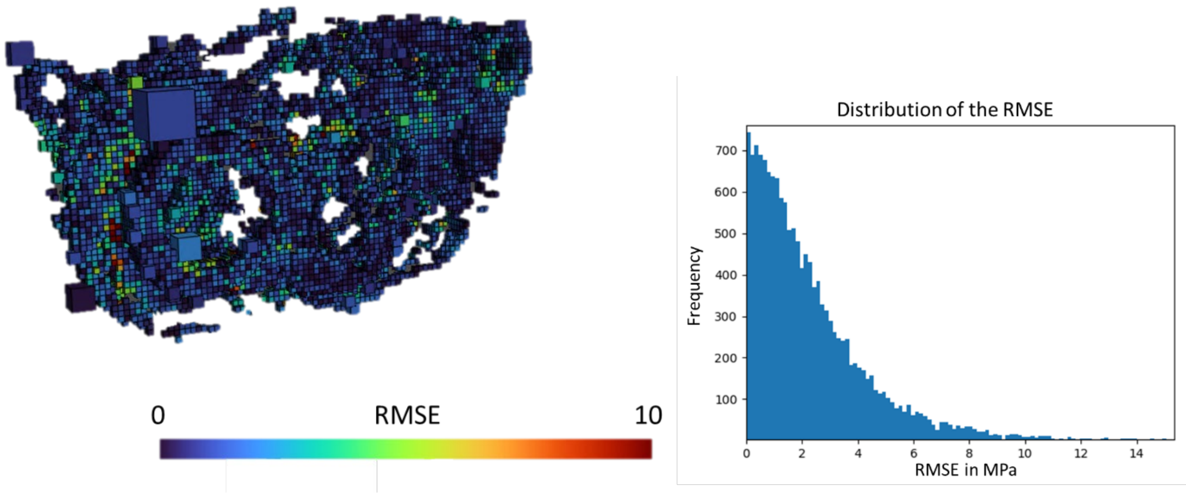


*Figure 9: Distribution of the RMSE for the equivalent stress amplitude calculated by the SIH*

*Figure 10: Illustration of the real DU component [39]*

Obviously, deviations between the simulation- and prediction-based fatigue calculation can be seen across the housing geometry. These errors could be decreased by extended sampling numbers and training data or a more intensive training of the model by e.g. a hyperparameter optimization. In other words, further improvements can be achieved by a higher computational effort. It should also be noted that the quality of the FE model regarding the numerical instabilities, which have a negative impact on the accuracy of the regression model, are nearly as crucial as the number of sample data. In general, the data-based model can only be as good as the underlying FE calculation.

Overall, these results show that the calculation of the fatigue strength can be performed with a relatively good accuracy based on the surrogate model. This enables an early consideration of real operating loads for the design process. The calculation performed on individual local points enables a global comparison of results for different loads and influencing parameters and thus a robust optimization and the formation of a Pareto Front. It is evident, however, that a direct simulation of the real operating loads in combination with additional parameter changes is not appropriate due to the enormous computational effort. The same applies for real component tests. Thus, the rather small errors of this method are practically without any alternative, which makes this method well applicable for the design process and the exploration of the design space for products with a highly variable load collective.

Due to the space-filling sampling algorithm and the good performance of the surrogate model in the cross validation, it can also be assumed that this accuracy can be transferred to other load cases and especially to the variation of the model parameters. Therefore, this methodological approach provides the basis for a robust optimization of the eBike DU or generally all products, which must be designed for different non-proportional and multiaxial load collectives. Once the surrogate model is built by an automated process, the entire geometry can be investigated for different load cases and load spectra and the effects of different design and system parameters.

Another important point that can be noted from these results is that the loads are clearly non-proportional at many locations, justifying the extensive calculation for the analysis of the component design. This can be seen by the complexity of the shear stress curve projection on a plane, that would be a straight line for proportional loading.

## 4. CONCLUSION AND OUTLOOK

After an introduction of the requirements for the calculation and robust optimization of eBike DUs, a brief insight into the state of the art of in fatigue analysis and the use of surrogate models, a suitable solution method for the calculation of the component was derived.

The method presented in this paper can solve common problems concerning computation time and effort for variable or random multiaxial loaded products like the eBike DU. In addition, a procedure for the validation of the surrogate model-based core approach of this method was also developed and applied. The validation shows that it is possible to use the surrogate model for the calculation of the component with only very small errors compared to the real FEM calculation. As a result, the approach allows an efficient exploration of the potential load and parameter combinations.

However, it must be emphasized that this method is only as good as the FEM model and the choice and number of sample data for the formation of the data-based surrogate model.

By performing the two-step validation process that is presented in this study, it is possible to ensure whether the database is sufficient for modeling. Since this process can be evaluated by clear metrics, the extension to an automatic learning process of the surrogate model is also

plausible. As the database can be parameterized and calculated in FE simulation, it can be used in an automatic process to generate more samples step by step until a certain convergence criterion or a certain value of the metrics is reached. Therefore, this automation would be a suitable extension where some manual operations can be avoided.
For a reliable prediction of fatigue strength, in addition to model accuracy, it is equally important to provide proper material data and to choose a appropriate strength hypothesis. Therefore, a comparison with real component tests and the parameter adjustment of the damage models and material data must be carried out in a third validation loop.

The possibility to test either measured or synthetic load collectives directly for different system and design parameters provides the developer with an efficient design tool. This enables a requirements-oriented product design with less uncertainties right from the start.
With regard to the eBike application, the fatigue calculation of the DU can also provide information about the fit of other eBike components such as the frame and its stiffness which can be assessed based on the multiaxial load collective.  Based on the results, the frame design can be assigned tolerances to ensure the safety of the DU and the overall system. Similarly, mounting positions and crank setups can be validated for their use in combination with the Drive Unit. Production and assembly steps and their tolerance-dependent static loads can also be incorporated and adjusted at an early stage of design. Overall, this method enables effective computations for all eBike DU related domains.
The decisive advantage here, besides the reduced computation time, is the reference to the discretization of the entire component geometry and the associated possibility to obtain results linked to a certain location of the geometry. This is a key factor for the evaluation and optimization of the geometry and to enable comparisons between designs with different geometry.
The ability to consider a variety of real load cases and their combination in a computationally efficient way can be seen as a great advantage for the robust design of structural components.

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