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MULTIVARIABLE STABILITY ANALYSIS OF POSITION-CONTROLLED PAYLOADS TO SUPPORT SHRINK IN SEMICONDUCTOR MANUFACTURING

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ABSTRACT

This paper addresses the challenge of ever-smaller structures in the semiconductor industry and the resulting requirements for high-performance mechatronic systems, especially wafer scanners, in lithography processes in the context of mechatronic system design and analysis. As a result, the development of sophisticated methods for modeling, control, and analysis of control systems has become necessary. To meet this need, advanced analysis methods of multivariable control systems are investigated, in particular the combination of classical stability analysis methods like the Nyquist criteria and use of Individual Channel Analysis and Design (ICAD) methods.

For this purpose, the requirements for the analysis of multivariable control systems are summarized and put in the context of classical methods of system analysis, for example, the use of Nyquist methods to evaluate the stability of the control loop. Subsequently, the paper provides a rationale for why the use of Single-Input Single-Output (SISO) methods to assess stability and robustness is not sufficient and how these can be extended to Multiple-Input Multiple Output (MIMO) methods to meet the requirements. A set of tailored Nyquist-like MIMO analysis methods are theoretically derived, including the ICAD method and classical Nyquist stability analysis and its use in the analysis of multivariable control system is explained. A coupling ratio parameter, quantifying the coupling of multivariable systems, is derived from the extended ICAD method. The iterative design process is explained, which allows conclusions to be drawn about individual system parameters and how to optimize them to achieve high performance. To compare the methods, a model of a mechanical payload with variable eigenfrequencies is derived. Subsequently, the suitability of the respective method for multivariable stability analysis is tested in different system configurations.

In conclusion, this paper provides insight into the analysis of stability and robustness of multivariable control systems and presents the challenges and opportunities of using these advanced methods to design high-performance mechatronics in the context of increasing requirements due to the shrink in semiconductor manufacturing. This provides a valuable contribution to the design of high-performance mechatronic systems.

Index Terms – Stability analysis, Multivariable control, Multivariable analysis, Multivariable systems

1. INTRODUCTION

Moore's Law is an empirical observation in semiconductor technology that states the number of transistors on a chip tends to double approximately every two years. It has driven the exponential growth of computing power and miniaturization since it was initially stated in 1965 by Gordon Moore [1]. This trend arises from advancements in lithography and semiconductor manufacturing techniques. As transistor dimensions approach atomic scales, maintaining the pace of Moore's Law becomes increasingly challenging, necessitating innovation in materials, device architectures, and fabrication methods to continue pushing the limits of semiconductor technology. Traditional practices in the development and design of semiconductor manufacturing technology must be augmented with advanced modeling, simulation, and optimization techniques. An integral part of that is the model-based analysis and design of high-performance mechatronics. In the following, a method in assessment of system stability will be presented.

In the field of control system design and analysis, stability is a crucial system property that must be ensured before evaluating other properties such as performance or robustness. Control engineers have access to various methods to analyze the stability of single-input single-output (SISO) systems, as documented in e.g. [2].

A widely used approach to achieve high-performance position control for mechanical and optical payloads with up to six degrees of freedom (DoFs) is static decoupling through scheduling matrices [3]. This method allows the decoupling of the system behavior into six independent SISO degrees of freedom, ensuring that each input only affects one output. In general, the dynamic properties of the mechanical and optical payloads are determined by the occurrence of flexible eigenmodes above a certain frequency range.

To improve the performance of the position control, the following strategies can be used: increasing the bandwidth to extend the frequency range in which the control loop is effective and reducing the mass of the mechanical and optical payloads to achieve higher accelerations with the same actuator forces. However, lightweight designs usually result in lower stiffness, which leads to the occurrence of resonances and flexible dynamics at low frequencies. As a result, static decoupling is less effective over a wider range as the flexible eigenmodes shift and the bandwidths are higher, making decoupling non-trivial.

For these reasons, it is assumed that static decoupling alone cannot meet the ever-increasing demands for high-performance position control. Consequently, the consideration of multiple-input multiple-output (MIMO) modelling, control and analysis techniques is imperative to ensure a more accurate and advanced design of mechatronic systems.

In the model-based development of mechatronic systems, the selection of an appropriate analysis method is crucial for mechatronic systems engineering. Therefore, this paper focuses on the derivation and comparative evaluation of different Nyquist-based methods for MIMO stability analysis. To compare the methods, a multi-mass oscillator is modeled, which allows the representation and investigation of different system behaviors. Therefore, this paper extends and adapts the performance analysis discussed in [4]. For a more comprehensive understanding of MIMO system analysis, see e.g. [5].

2. MULTIVARIABLE STABILITY ANALYSIS

In the analysis of stability for Multiple-Input Multiple-Output (MIMO) systems, control engineers have access to a variety of methods, similar to those available for Single-Input Single-Output (SISO) systems [6]. While many of these methods provide a binary assessment of stability (i.e. stable or unstable), obtaining more detailed information about the structural stability of the system comparable to the Nyquist criterion requires the use of advanced methods. For instance, it is often challenging to determine which individual Transfer Functions (TFs) contribute to instability or have the least robust performance. This paper presents classical and well-known methods for MIMO stability analysis and extends them to provide techniques for acquiring structural system stability information.

2.1 Requirements for stability analysis of multivariable systems

A variety of methods exist for investigating the stability of multivariable control systems, see e.g. [2] and [6]. For the iterative design process in the development of high-performance mechatronic systems, it is important that the methods can provide as much information as possible about the system behavior. The following requirements are in general placed on the methods for the system analysis:

1. Applicability to multivariable control systems
2. Correct assessment of multivariable system stability and robustness
3. Detailed structural information about system behavior, e.g. in which transfer path and frequency an instability or poor robustness is present
4. Numerical feasibility and effort (no focus on in this paper)

2.2 Definition of stability for feedback control systems

In this section, the definition of stability for feedback control systems is established and the basis for subsequent stability analysis is created. Throughout this paper, scalar quantities are denoted by thin formula symbols, vector quantities by vector arrows, and matrices by capital bold formula symbols.

Consider the standard feedback control loop illustrated in Figure 1. The open-loop Transfer Function Matrix (TFM) of the $m \times m$ MIMO system is defined as $\boldsymbol{L} = \boldsymbol{GC}$, where $\boldsymbol{C}$ represents the input controller and $\boldsymbol{G}$ denotes the plant. Using this definition, the sensitivity matrix $\boldsymbol{S} = (\boldsymbol{I} + \boldsymbol{L})^{-1}$, the complementary sensitivity matrix $\boldsymbol{T} = (\boldsymbol{I} + \boldsymbol{L})^{-1}\boldsymbol{L}$ and $\boldsymbol{I}$ as the identity matrix of appropriate dimension are introduced. To define stability, the general valid MIMO definitions are applied to the SISO case ($\boldsymbol{S} \rightarrow S$ and $\boldsymbol{T} \rightarrow T$). Thereby it is important to notice, that the poles of S and the poles of T in $\mathbb{C} \cup \{\infty\}$ are exactly given by the zeros of $1 + L$ in $\mathbb{C} \cup \{\infty\}$, assuming no unstable pole-zero cancellation occurs in L [7]. With this, $(1 + L)^{-1}$ is stable if

- it is proper and
- has only poles in the open left-half plane (real part < 0).

Instead of analyzing the poles of $(1 + L)^{-1}$, the zeros of $1 + L$ can be evaluated since they correspond to each other. Therefore, $(1 + L)^{-1}$ is stable if no zeros of $1 + L$ lie in the closed Right Half Plane (RHP) (i.e. real part ≥ 0).

This leads to the conclusion that in order to analyze the stability of the feedback control system, it is necessary to find the zeros of the TF and evaluate whether one of them is in the closed RHP.

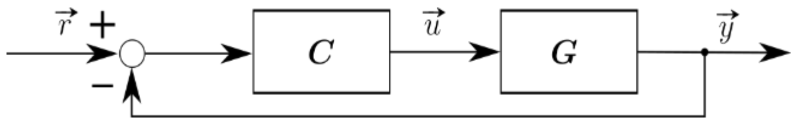

Figure 1. Standard feedback control loop

2.3 The Nyquist Criterion and extension to MIMO systems

In this section, the Nyquist criterion as a classical graphical test for checking the stability of the closed-loop system using the open-loop TF is derived [8]. This method has the advantage that robustness information can be obtained additionally to the pure stability information.

As described in Section 2.1, the stability is evaluated by checking whether any zeros of $1 + L$ lie in the closed RHP. To evaluate all unstable poles of $(1 + L)^{-1}$, a contour that encircles all of them is needed. This contour is called Nyquist Contour $\mathcal{N}$, shown in Figure 2. It passes along the $j\omega$ axis from $-j\infty$ to $j\infty$ and closes by a significantly large semicircle $R \rightarrow \infty$ to encircle all poles of L. To enclose poles on the $j\omega$ axis, small semicircles r are inserted in the Left Half Plane (LHP) [7]. The plot defined as $s \in \mathbb{C}$ travesers $\mathcal{N}$ is called Nyquist plot $\Gamma_{\mathcal{N}}$.

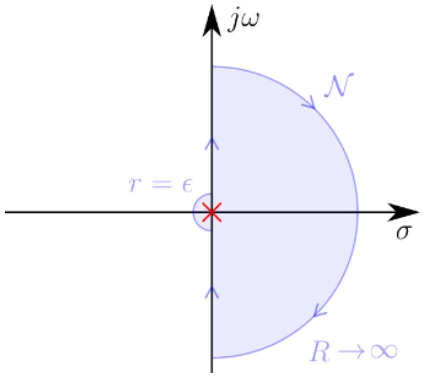

Figure 2. Definition of the Nyquist Contour $\mathcal{N}$

The mathematical background for the following theorems is given by the *Argument Principle* combined with *Cauchy's Theorem* and can be read in [9]. Here use the definition of the Nyquist criterion and the Nyquist Contour $\mathcal{N}$ from [7] is used.

Theorem 1 (Generalized Nyquist Criterion) *Let P denote the number of unstable poles in L. The closed-loop system with open-loop TF L and negative feedback is stable if and only if the Nyquist plot $\Gamma_{\mathcal{N}}$ of $L(s)$ does not pass through the critical point (-1,0) $\in \mathbb{C}$ and makes P counter-clockwise encirclements of (-1,0) $\in \mathbb{C}$ as s traverses $\mathcal{N}$ in clockwise direction, assuming no unstable zero pole cancellation takes place.*

The proof follows from the derivation of the Nyquist Criterion and can be looked up in [7]. Theorem 1 is extended to MIMO-systems ($1 + L \ \rightarrow \ \boldsymbol{I} + \boldsymbol{L}$). Therefore, the closed-loop poles are now solutions to the Equation

$$\det(\boldsymbol{I} + \boldsymbol{L}(s)) = 0 \ \ \forall s \in \mathbb{C}. \tag{1}$$

Note that in the SISO case, $\det(\boldsymbol{I} + \boldsymbol{L}(s))$ equals $1 + L(s)$. As a consequence of (1) the critical point shifts to (0,0). This leads to the Generalized MIMO Nyquist Criterion according to [6].

Theorem 2 (Generalized Nyquist Criterion for MIMO systems) *Let P denote the number of unstable poles in* $\boldsymbol{L}$. *The closed-loop MIMO system with open-loop TFM* $\boldsymbol{L}$ *and negative feedback is stable if and only if the Nyquist plot $\Gamma_{\mathcal{N}}$ of* $\det(\boldsymbol{I} + \boldsymbol{L}(s))$ *does not pass through the critical point (0,0) and makes P counter-clockwise encirclements of (0,0) as s traverses $\mathcal{N}$ in clockwise direction, assuming no unstable zero pole cancellation takes place.*

Since in this case only a single function $\det(\boldsymbol{I} + \boldsymbol{L}(s))$ is evaluated and analyzed in the Nyquist plot, only a binary statement about the stability of the MIMO System can be made. However, this limited approach does not provide any additional information about the concrete locations of instabilities within the system. It therefore does not provide a suitable method to comprehensively analyze or specifically optimize a high-performance mechatronic system.
For this reason, the following methods are presented below, which aim to decompose the determinant into individual products. This allows the stability assessment to be maintained while providing additional structural information.

2.4 Characteristic Loci

The application of Characteristic Loci for stability analysis is an extension of Theorem 2, providing an if-and-only-if statement regarding the stability of the $m \times m$ MIMO system using the eigenvalues of the open-loop TFM [10]. The main idea behind this technique is, that the determinant of a matrix is equal to the product of all its eigenvalues: $\det(\boldsymbol{I} + \boldsymbol{L}(s)) =$

$\prod_{i=1}^{m}(1+\lambda_{L_i}(s))$. Therefore, rather than checking $\det(\boldsymbol{I}+\boldsymbol{L}(s))$ with the MIMO Nyquist Criterion, it is equally valid to check each of the eigenvalues $\lambda_{L_i}(s)$ with the SISO Nyquist Criterion as defined in Theorem 1. This leads to the following theorem.

Theorem 3 (Nyquist Criterion with Characteristic Loci) *Let P denote the number of unstable poles in* $\boldsymbol{L}$*. The closed-loop MIMO system with open-loop TFM* $\boldsymbol{L}$ *and negative feedback is stable if and only if the Nyquist plots* $\Gamma_{\mathcal{N}_i}$ *of the Characteristic Loci* λ_{L_i} *do not pass through the critical point (-1,0) and drawn together make P counter-clockwise encirclements of (-1,0) as s traverses* $\mathcal{N}$ *clockwise, assuming no unstable zero pole cancellation takes place.*

2.5 Concept of equivalent plants

The Individual Channel Analysis and Design (ICAD) is an approach to handle multivariable control problems with SISO techniques by forming new equivalent SISO channels under consideration of the multivariable system behavior, the so called equivalent plants (EPs). O'Reilly and Leithead presented the ICAD as a general analysis and design framework for multivariable control problems and especially considered 2 x 2 systems in [11]. In this paper, the concept of equivalent plants and ICAD is linked to the classical Nyquist stability analysis and thus further Nyquist-like stability theorems are defined.

Consider an open-loop TFM $\boldsymbol{L}$ for a 2×2 MIMO system with a diagonal control scheme. The closed control loop with negative feedback is represented in terms of a block diagram in Fig. 3.

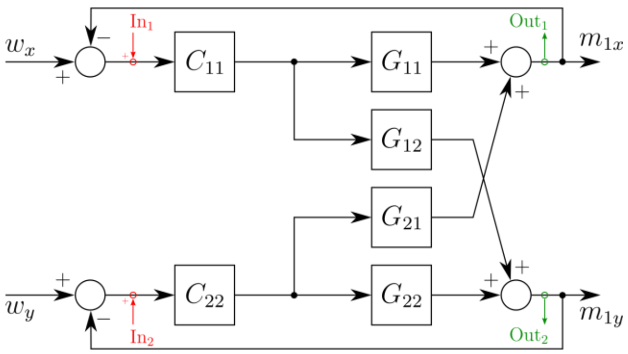

Figure 3. 2x2 feedback control structure

With the ICAD method, SISO TFs are derived for the two defined inputs In_1, In_2 and the two defined outputs Out_1, Out_2:

$$L_{11}^{EP}=\frac{\mathrm{Out}_1}{\mathrm{In}_1}=C_{11}\cdot{}_2\mathrm{EP}_{11}=C_{11}\cdot G_{11}(1-\xi\cdot h_2) \tag{2}$$

$$L_{22}^{EP}=\frac{\mathrm{Out}_2}{\mathrm{In}_2}=C_{22}\cdot{}_2\mathrm{EP}_{22}=C_{22}\cdot G_{22}(1-\xi\cdot h_1) \tag{3}$$

In (2, 3), the EPs for the 2×2 system are obtained as

$${}_2EP_{11}=G_{11}-G_{12}\cdot\frac{C_{22}}{1+G_{22}C_{22}}\cdot G_{21} \tag{4}$$

$${}_2EP_{22}=G_{22}-G_{21}\cdot\frac{C_{11}}{1+G_{11}C_{11}}\cdot G_{12} \tag{5}$$

and by rearranging (2, 3), the right-hand side is defined

$$h_k=\frac{C_{kk}\cdot G_{kk}}{1+C_{kk}\cdot G_{kk}},\qquad \xi=\frac{G_{12}\cdot G_{21}}{G_{11}\cdot G_{22}} \tag{6a, 6b}$$

with $k=\{1,2\}$. The TFs h_1 and h_2 form the closed-loop TF of channel 1 and 2 respectively. ξ is called the Multivariable Structure Function (MSF) and describes the multivariable nature of

L_{11}^{EP} and L_{22}^{EP} and of the underlying system [11]. The product of ξ and h_k is a weighted product with diagonal and off-diagonal elements and therefore describes a measure of cross-coupling. If $\xi(s)$ is small in magnitude, loop signal interaction is low and the two loops act almost independently for that $s \in \mathbb{C}$. Vice versa, loop interaction is high if $\xi(s)$ is large in magnitude. Consequently, ξ can be used to define a frequency dependent measure of the coupling ratio of MIMO systems. Furthermore, $\max_{\omega \in [0,\infty]} |\xi(j\omega)|$ can be evaluated to assess the maximum cross-coupling and the underlying frequency.

2.6 Stability analysis based on individual channel analysis and determinant decomposition

The SISO TFs defined in (2, 3) can be used to evaluate the stability of the closed-loop system with open-loop TFM. For this investigation it is important to notice, that $\det(\boldsymbol{I}+\boldsymbol{L})$ can be rearranged in such a way, that the TFs h_1 and h_2 and the MSF ξ occur:

$$\det(\boldsymbol{I}+\boldsymbol{L}) = (1+G_{11}C_{11})(1+G_{22}C_{22})(1-\xi h_1 h_2) \tag{7}$$

Therefore, stability of the multivariable system depends on the stability of each SISO loop $(1+C_{11}G_{11})$ and $(1+C_{22}G_{22})$ and of the multivariable coupling described by $(1-\xi h_1 h_2)$. This can be rewritten to a generally valid case requiring

$$\det(\boldsymbol{I}+\boldsymbol{L}) = \prod_{i=1}^{m}(1+l_i)\cdot\left(1-\xi\prod_{i=1}^{m}h_i\right) \tag{8}$$

not having zeros in the RHP for asymptotic stability of a $m \times m$ feedback control system, assuming no unstable pole zero cancellation takes place. In Eq. (8), $l_i = C_{ii}G_{ii}$ are the diagonal elements of $\boldsymbol{L}$**.** Consequently, an alternative version of Theorem 2 with the advantage that instabilities can be attributed to one of the diagonal or off-diagonal elements can be derived.

Theorem 4 (Nyquist Criterion of determinant decomposition)

Let P denote the number of unstable poles in $\boldsymbol{L}$*. The closed-loop* $m \times m$ *MIMO system with loop TFM* $\boldsymbol{L}$ *and negative feedback is stable if and only if the* m *Nyquist plots* $\Gamma_{\mathcal{N}}(l_i(s)), i = \{1,\dots,m\}$ *and the Nyquist plot of* $-\xi(s)\cdot\prod_{i=1}^{m}h_i(s)$ *do not pass through the critical point (-1,0) and the net sum of counter-clockwise encirclements of (-1,0) equals P as s traverses* $\mathcal{N}$ *in clockwise direction, assuming no unstable zero pole cancellation takes place.*

The proof is straightforward and can be done by calculating the determinant of a $m \times m$ matrix and applying the Nyquist criterion on each of the factors [5].

Even though similar functions occur in Eq. (8), no clear connection between the ICAD functions (2, 3) can be determined as the individual channels L_{ii}^{EP} are not used. It can be shown, that $\det(\boldsymbol{I}+\boldsymbol{L}) \neq \prod_{i=1}^{m}(1+L_{ii}^{EP})$ and therefore no if and only if statement on stability is possible by considering all L_{ii}^{EP}. However, a sufficient condition for stability of a multivariable system is that each of the functions L_{ii}^{EP} are stable. Nevertheless, there is a distinct connection between the individual channels and the determinant, which is derived in the following and extends the concept of EPs in [11] to $m \times m$ systems.

Let $\boldsymbol{L}$ be the open-loop TFM of a $m \times m$ MIMO system with a diagonal controller $\boldsymbol{C} := \mathrm{diag}(C_{11},\dots,C_{mm})$ and a fully populated plant matrix $\boldsymbol{G} := \left(G_{ij}\right)_{ij}$.

Consider the square submatrix $\boldsymbol{G}_{(1:n;1:n)}$ defined by deleting each row and column of $\boldsymbol{G}$ greater than n with $n \in \{1,2,\dots,m\}$. Let ${}_nEP_{nn}$ be the EP TF of the submatrix $\boldsymbol{G}_{(1:n;1:n)}$ defined in a similar way as in (4) and (5) and suppose that no pole zero cancellations take place within the multiplications. With that, the equivalent open-loop TFs L_n are defined for each EP of the submatrices with the corresponding controller according to [12]:

$$\det(\boldsymbol{I}+\boldsymbol{L}) = \prod_{i=1}^{m}(1+l_i)\cdot\left(1-\xi\prod_{i=1}^{m}h_i\right) \tag{8}$$

$$L_n = C_{nn}\cdot {}_nEP_{nn} \quad \forall\, n\in\{1,2,\dots,m\}. \tag{9}$$

Theorem 5 (Stability of MIMO Systems using ICAD)
With the definition of L_n in Equation (12), the following holds:

$$\det(I+L) = \prod_{n=1}^{m}(1+L_n). \tag{10}$$

Thus, the stability of the MIMO system can be assessed by considering the open-loop TFs L_n using SISO Nyquist criterion. An equivalent statement about the number of unstable poles can be made as in Theorem 2 by counting the net sum of encirclements of the critical point (-1,0).

Since the proof of Theorem 5 is not as simple as the previous ones, it is presented below.
Proof: A lower-upper decomposition $\boldsymbol{\mathcal{LU}}$ of the matrix $\boldsymbol{A} \coloneqq \boldsymbol{I}+\boldsymbol{GC}$ is done [13]. The result of the $\boldsymbol{\mathcal{LU}}$ decomposition are two matrices, $\boldsymbol{\mathcal{L}}$ as a lower triangular matrix with only ones on the diagonal elements and $\boldsymbol{\mathcal{U}}$ as an upper triangular matrix. Because $\det(\boldsymbol{\mathcal{L}})=1$ and $\boldsymbol{\mathcal{U}}$ is an upper triangular matrix, $\det(\boldsymbol{A})$ is equal to the product of all diagonal elements of $\boldsymbol{\mathcal{U}}$. The diagonal elements of $\boldsymbol{\mathcal{U}}$ are given by u_{ii} and therefore

$$\det(\boldsymbol{A}) = \det(\boldsymbol{\mathcal{LU}}) = \det(\boldsymbol{\mathcal{U}}) = \prod_{i=1}^{m} u_{ii}\,. \tag{11}$$

By applying the $\boldsymbol{\mathcal{LU}}$ decomposition on $\boldsymbol{A}$, $\boldsymbol{\mathcal{U}}$ is given by

$$\boldsymbol{\mathcal{U}} = \begin{bmatrix} 1+G_{11}C_{11} & {}_1G_{12}C_{22} & \cdots & {}_1G_{1m}C_{mm} \\ 0 & 1+{}_2G_{22}C_{22} & \cdots & {}_2G_{2m}C_{mm} \\ \vdots\ \vdots & & \ddots & \vdots \\ 0\quad 0 & & \cdots & 1+{}_mG_{mm}C_{mm} \end{bmatrix} \tag{12}$$

$${}_nG_{nj} = {}_1G_{nj} - \sum_{i=1}^{n-1}\frac{{}_iG_{ij}\ {}_iG_{ni}C_{ii}}{1+{}_iG_{ii}C_{ii}} \ \forall n,j\in\{1,\dots,m\} \tag{13}$$

Interestingly, for $j=n$ Equation (13) equals the EP of the submatrix $\boldsymbol{G}_{(1:n;1:n)}$ and therefore

$$L_n = C_{nn}\left({}_1G_{nn} - \sum_{i=1}^{n-1}\frac{{}_iG_{in}\ {}_iG_{ni}C_{ii}}{1+{}_iG_{ii}C_{ii}}\right) \tag{14}$$

As the $\boldsymbol{\mathcal{LU}}$ decomposition is unique, Theorem 5 is proven for the general $m\times m$ case since

$$\det(\boldsymbol{I}+\boldsymbol{L}) = \prod_{i=1}^{m} u_{ii} = \prod_{n=1}^{m}(1+L_n). \tag{15} \blacksquare$$

Note that with Equation (13), a closed expression for the EPs of a $m\times m$ ICAD system can be defined.

3. MODELLING OF THE CONTROLLED MECHANICAL SYSTEM

Following the discussion of the different methods of multivariable stability analysis, a comparison of these methods will now be presented. To facilitate the comparative analysis, a model of a dynamically coupled Multiple-Input Multiple-Output (MIMO) system is required. The dynamic behavior of actuated and controlled payloads, incorporating flexible eigenmodes, is examined using existing simulations and literature. Based on this information, the dynamic behavior is replicated using mechanical elements characterized by concentrated parameters, such as masses, dampers, and springs. As this is a very advantageous way of generating a model for the comparative analysis of stability methods, the example system is described in more detail below.
The example system, as shown in Figure 2, comprises two fundamental components:

1. The payload mechanical element, consisting of four masses ($M_1 - M_4$). Mass M_1 is controlled in two DoFs, namely x and y. To emulate the eigendynamics and flexible eigenmodes of the payload, a three-mass oscillator with masses $M_2 - M_4$ is positioned on mass M_1. By adjusting the parameters of these masses, interconnecting springs, and dampers, the dynamic behavior of a payload with flexible eigenmodes can be replicated in two DoFs. The cross coupling between x and y can be modified depending on the angles ψ and φ.
2. The masses M_{A1} and M_{A2} represent actuators to which actuation forces f_{A1} and f_{A2} are applied. These actuators are connected to mass M_1 via a spring under angles α and β to actuate mass M_1 in x and y direction. To mitigate resulting resonances, Tuned Mass Dampers (TMDs) are attached to each actuator masses. If resulting resonances are desired for the analysis of the stability method, the TMDs can also be omitted. Similar to the payload mechanical system, the cross coupling of x and y depend on α and β and can therefore be adjusted.

Utilizing this modeling framework, the system is described by a second-order Ordinary Differential Equation (ODE) system

$$\boldsymbol{M} \cdot \ddot{\vec{q_i}}(t) + \boldsymbol{C} \cdot \dot{\vec{q_i}}(t) + \boldsymbol{K} \cdot \vec{q_i}(t) = \vec{f_i}(t) \tag{16}$$

with the mass matrix $\boldsymbol{M}$, the damping matrix $\boldsymbol{C}$, the stiffness matrix $\boldsymbol{K}$, the actuation force vector $\vec{f_i}(t)$ as input vector and the displacement vector $\vec{q_i}(t)$ as output vector.

By applying the Laplace transform to (16) and tuning a variant of a classical PID controller with a diagonal control scheme as reported in [3], the open loop TFM of the example system is defined to

$$\boldsymbol{L} = \boldsymbol{GC} = \begin{bmatrix} G_{11}C_{11} & G_{12}C_{22} \\ G_{21}C_{11} & G_{22}C_{22} \end{bmatrix}. \tag{17}$$

As stated earlier, the cross-coupling in the system is determined by the angles $\alpha, \beta, \psi, \varphi$. Consequently, the (MSF) ξ, introduced in Section 2.4, of the example system becomes a function of these angles $\xi = \xi(\alpha, \beta, \psi, \varphi)$. By adjusting these angles, the multivariable behavior and cross coupling of the system can be influenced and adapted. This flexibility allows for the creation of various system configurations by modifying α, β, ψ and φ. As a result, the methods presented in Chapter 2 can be explored and comparatively evaluated with different system characteristics, such as strongly or weakly coupled systems.

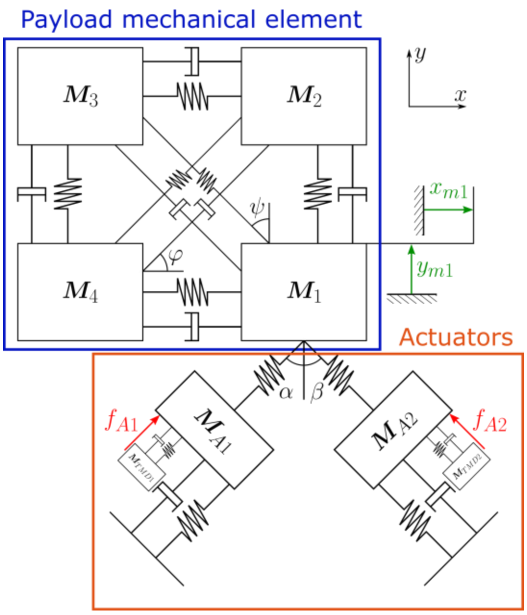

Figure 2. An actuated payload with flexible eigenmodes

4. COMPARISON OF THE METHODS AND RECOMMENDATIONS

In this chapter, the comparative analysis results of the methods examined in Chapter 2 using the example system introduced in Chapter 3 is presented. To assess the stability methods, various system configurations with distinct behaviors are modeled. By deliberately inducing instabilities in specific individual Transfer Functions (TFs) of the system's Transfer Function Matrix (TFM), the strengths and weaknesses of the methods become evident, enabling a comprehensive evaluation.

The MIMO Nyquist as introduced in Theorem 2 provides a necessary and sufficient condition for stability through evaluating $\det(\boldsymbol{I}+\boldsymbol{L})$ with the advantage that only one Nyquist plot needs to be evaluated to assess stability. However, this method cannot be used to draw conclusions about the individual physical axes and does therefore not deliver structural stability system information. Consequently, the use of this method is recommended, if only pure stability information of the system needs to be evaluated.

The m Characteristic Loci as described in Theorem 3 deliver valid stability information in all cases as the eigenvalues of $\boldsymbol{I}+\boldsymbol{L}$ are directly connected to the determinant of this matrix. Similar to the MIMO Nyquist, the Characteristic Loci do not provide information about the individual physical axes and no structural system information is obtained. The robustness margins of the Characteristic Locis correspond to SISO margins for a simultaneously change in all channels and do therefore deliver additional robustness information [9].

The Nyquist of determinant decomposition as introduced in Theorem 4 provides valid stability information in all cases as seen in Equation (9). The determinant of $\boldsymbol{I}+\boldsymbol{L}$ is split into m SISO loops $1+l_i$ and one additional loop $1-\xi(s)\cdot\prod_{i=1}^{m} h_i$ considering the cross coupling of the system with the MSF ξ. This enables to find out whether there are instabilities or critical points on one of the SISO loops or in the coupling of these loops. Therefore, the use of this method is recommended if stability and structural stability information needs to be obtained.

Similarly, the SISO Nyquist using ICAD functions as presented in Theorem 5 is an individual interpretation of the MIMO Nyquist and exactly matches the determinant of the matrix as shown in the proof of Theorem 5. Consequently, the stability information obtained with this method is valid in all cases. An advantage of this method is that information about the individual physical axes is obtained, e.g. occurrence of instabilities. To gain full information about the individual axes, each permutation of the individual channel combination can be evaluated. The use of this method is recommended, if not only pure stability information needs to be obtained, but also structural information about the system behavior is of interest.

Additionally, with Equation (13) a closed expression for the EP as introduced in the ICAD is defined. The following table summarizes the comparison results for the individual stability analysis methods.

Method	Stability statement	Structural information	Number of functions for $\boldsymbol{m}$ x $\boldsymbol{m}$ system
MIMO Nyquist.	yes	no	1
Characteristic Loci	yes	no	m
Determinant Decomposition	yes	yes	$m+1$
ICAD Nyquist	yes	yes	m

5. CONCLUSIONS

In this paper Nyquist based methods of multivariable stability analysis are evaluated comparatively. For this purpose, stability is defined and the different methods are theoretically derived. By combining the concept of EPs and ICAD, individual interpretations of the Nyquist

Criterion are derived and stated. Subsequently, an example system is modelled that represents an actuated payload system and enables a comparative analysis.
Afterwards the methods are applied to this system and the results of each method are compared. It is observed that all investigated methods enable valid stability assessments. This is also proven theoretically. However, they differ in terms of the information they provide. While the MIMO Nyquist method provides a pure statement of stability, the characteristic loci method offers additional robustness information. To obtain structural stability information for MIMO systems, the concept of EPs from [11] is extended and integrated with the Nyquist criterion, resulting in two alternative Nyquist-like stability theorems. These theorems provide valuable structural stability information for MIMO systems and thus allow a statement to be made about which individual transmission paths of the MIMO system are subject to instability.
With these alternative interpretations of the Nyquist criterion, multivariable stability analysis can be significantly improved, and additional structural system information can be obtained. For future work, the robustness analysis of multivariable systems can be examined, for which a variety of sophisticated methods is available, see e.g. [14].

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