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Corrigendum: Correlations of indistinguishable particles in non-Hermitian lattices

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An inadvertent indexing error has occurred in the derivation of the quantum Langevin equation presented in appendix A of our paper (2013 *New J. Phys.* **15** 033008). In the previous version, the summation over the system modes $\hat{a}_q^\dagger$ and the reservoir modes $\hat{b}_r^\dagger$ was executed with independent indices. However, as each system mode is coupled to a different set of reservoir modes, the summation of the reservoir modes should rather depend on the system mode. Hence, we introduce a set of reservoir summation indices $\{r_q\}_{q=1,\dots,M}$, which are assigned to the individual system modes $\hat{a}_q^\dagger$.

While this modification slightly alters equations (A.1)–(A.5) and (B.4)–(B.6), it has no implications to the results of the derivations, nor does it affect any result or conclusion in the main part of the paper.

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Appendix A. Quantum Langevin equation for coupled systems

In the following, we deduce the quantum Langevin equation for our systems from a general approach by principally following similar considerations as in [1]. Regard the total Hamiltonian of the overall configuration under the assumption of weak coupling between reservoir and loss-free system with strengths g_{m,n_m},

$$\begin{aligned}\hat{H}_{\text{total}} &= \hat{H}_0 + \hat{H}_{\text{reservoir}} + \hat{H}_{\text{interaction}} \\ &= \hbar \sum_{m=1}^{M} \left\{ \beta_m \hat{a}_m^\dagger \hat{a}_m + \sum_{j=1}^{M} C_{j,m} \hat{a}_j^\dagger \hat{a}_m \right\} + \hbar \sum_{m,n_m} \bar{\beta}_{n_m} \hat{b}_{n_m}^\dagger \hat{b}_{n_m} \\ &+ \hbar \sum_{m,n_m} \left\{ g_{m,n_m} \hat{a}_m^\dagger \hat{b}_{n_m} + g_{m,n_m}^* \hat{a}_m \hat{b}_{n_m}^\dagger \right\}, \end{aligned} \tag{A.1}$$

where $\bar{\beta}_{n_m}$ denotes the propagation constants of the reservoir modes and β_m and $C_{j,m}$ are defined in the main text. Thus, we find the Heisenberg equations for the system field operators $\hat{a}_q^\dagger$ and the reservoir field operators $\hat{b}_{r_q}^\dagger$ as coupled differential equations:

$$\frac{\mathrm{d}\hat{a}_q^\dagger(z)}{\mathrm{d}z} = -\frac{\mathrm{i}}{\hbar}[\hat{a}_q^\dagger(z),\ \hat{H}_{\text{total}}] = -\frac{\mathrm{i}}{\hbar}[\hat{a}_q^\dagger(z),\ \hat{H}_0] + \mathrm{i}\sum_{r_q} g_{q,r_q}^* \hat{b}_{r_q}^\dagger, \tag{A.2}$$

$$\frac{\mathrm{d}\hat{b}_{r_q}^\dagger(z)}{\mathrm{d}z} = -\frac{\mathrm{i}}{\hbar}[\hat{b}_{r_q}^\dagger(z),\ \hat{H}_{\text{total}}] = \mathrm{i}\bar{\beta}_{r_q} \hat{b}_{r_q}^\dagger(z) + \mathrm{i} g_{q,r_q} \hat{a}_q^\dagger(z). \tag{A.3}$$

Integrating equation (A.3) and plugging into equation (A.2) yields

$$\frac{\mathrm{d}\hat{a}_q^\dagger(z)}{\mathrm{d}z} = -\frac{\mathrm{i}}{\hbar}[\hat{a}_q^\dagger(z),\ \hat{H}_0] + \mathrm{i}\sum_{r_q} g_{q,r_q}^* \hat{b}_{r_q}^\dagger(0)\, \mathrm{e}^{\mathrm{i}\bar{\beta}_{r_q} z} - \int_0^z \mathrm{d}\zeta \sum_{r_q} |g_{q,r_q}|^2 \mathrm{e}^{\mathrm{i}\bar{\beta}_{r_q}(z-\zeta)} \hat{a}_q^\dagger(\zeta), \tag{A.4}$$

where the second term on the rhs acts as a noise operator $\hat{f}_q^\dagger = \mathrm{i}\sum_{r_q} g_{q,r_q}^* \hat{b}_{r_q}^\dagger(0)\mathrm{e}^{\mathrm{i}\bar{\beta}_{r_q} z}$. For the last term on the rhs we replace the summation by an integral $\sum_{r_q} \to \frac{V}{(2\pi)^3}\int \mathrm{d}^3\bar{k}_{r_q} \to \int \mathrm{d}\bar{\beta}_{r_q} D(\bar{\beta}_{r_q})$ with the spectral density of states $D(\beta)$:

$$\begin{aligned}\frac{\mathrm{d}\hat{a}_q^\dagger(z)}{\mathrm{d}z} &= -\frac{\mathrm{i}}{\hbar}[\hat{a}_q^\dagger(z),\ \hat{H}_0] - \int_0^z \mathrm{d}\zeta \int \mathrm{d}\bar{\beta}_{r_q} D(\bar{\beta}_{r_q}) |g_q(\bar{\beta}_{r_q})|^2 \mathrm{e}^{\mathrm{i}(\bar{\beta}_{r_q}-\beta_q)(z-\zeta)} \hat{a}_q^\dagger(\zeta) \mathrm{e}^{\mathrm{i}\beta_q(z-\zeta)} + \hat{f}_q^\dagger(z) \\ &= -\frac{\mathrm{i}}{\hbar}[\hat{a}_q^\dagger(z),\ \hat{H}_0] - \int_0^z \mathrm{d}\zeta\, D(\beta_q)|g_q(\beta_q)|^2 \pi\delta(z-\zeta)\, \hat{a}_q^\dagger(\zeta)\mathrm{e}^{\mathrm{i}\beta_q(z-\zeta)} + \hat{f}_q^\dagger(z) \\ &= \frac{\mathrm{i}}{\hbar}[\hat{a}_q^\dagger(z),\ \hat{H}_0] - \pi D(\beta_q)|g_q(\beta_q)|^2 \hat{a}_q^\dagger(z) + \hat{f}_q^\dagger(z) \\ &= -\frac{\mathrm{i}}{\hbar}[\hat{a}_q^\dagger(z),\ \hat{H}_0] - \frac{\gamma_q}{2}\hat{a}_q^\dagger(z) + \hat{f}_q^\dagger(z). \end{aligned} \tag{A.5}$$

With Fermi's golden rule decay rates $\gamma_q = 2\pi D(\beta_q)|g_q(\beta_q)|^2$ and the introduction of $\hat{H}_{\text{system}} = \hat{H}_0 + \mathrm{i}\hat{A}_{\text{loss}} = \hat{H}_0 + \mathrm{i}\hbar\sum_{m=1}^{M}\frac{\gamma_m}{2}\,\hat{a}_m^\dagger\hat{a}_m$ one obtains the quantum Langevin equation:

$$\frac{\mathrm{d}\,\hat{a}_q^\dagger(z)}{\mathrm{d}z} = -\frac{\mathrm{i}}{\hbar}[\hat{a}_q^\dagger(z),\ \hat{H}_{\text{system}}] + \hat{f}_q^\dagger(z) = \mathrm{i}\sum_{j=1}^{M}\tilde{C}_{q,j}\,\hat{a}_j^\dagger(z) + \hat{f}_q^\dagger(z), \tag{A.6}$$

where $\tilde{C}_{q,j} = (\beta_j + \mathrm{i}\frac{\gamma_j}{2})\delta_{j,q} + C_{q,j}$. With the linear propagation operator $U_{q,j}(z) = (\mathrm{e}^{\mathrm{i}\hat{\tilde{C}}z})_{q,j}$ the formal solution of (A.6) is

$$\hat{a}_q^\dagger(z) = \sum_{j=1}^{M} U_{q,j}(z)\,\hat{a}_j^\dagger(0) + \int_0^z \mathrm{d}\zeta \sum_{j=1}^{M} U_{q,j}(z-\zeta)\,\hat{f}_q^\dagger(\zeta). \tag{A.7}$$

Appendix B. Calculation of the particle number correlation

Here we present an extensive derivation of the particle number correlation and prove that this quantity does not depend on the noise operators. Calculating the particle number correlations by using (A.7)

$$\Gamma_{q,r}(z) = \langle \hat{a}_q^\dagger(z)\,\hat{a}_r^\dagger(z)\hat{a}_r(z)\hat{a}_q(z)\rangle = \sum_{\alpha=1}^{16}\Gamma_{q,r}^{(\alpha)}(z) \tag{B.1}$$

yields 16 terms $\Gamma_{q,r}^{(\alpha)}$. They can be categorized in how often a noise operator appears. One finds contributions without noise operator ($\Gamma_{q,r}^{(1)}$), with one ($\Gamma_{q,r}^{(2\ldots5)}$), two ($\Gamma_{q,r}^{(6\ldots11)}$), three ($\Gamma_{q,r}^{(12\ldots15)}$) and four ($\Gamma_{q,r}^{(16)}$) noise operators. In the following we will show exemplarily the contribution of each group by an extended calculation for one member. For the sake of clarity we slip the z-argument of the propagation operator elements $U_{j,k}$. Any other dependence will be written explicitly. Taking into account that the noise operators are Markovian and the average value of the noise equals zero as well as the zero temperature approximation (ZTA) for the reservoir, we get:

$\alpha = 1$:

$$\Gamma_{q,r}^{(1)} = \sum_{j,k,l,p=1}^{M} U_{q,j}U_{r,k}U_{r,l}^{*}U_{q,p}^{*}\langle\hat{a}_j^\dagger(0)\,\hat{a}_k^\dagger(0)\hat{a}_l(0)\hat{a}_p(0)\rangle. \tag{B.2}$$

$\alpha = 2, \ldots, 5$:

$$\begin{aligned}\Gamma_{q,r}^{(2)} &= \sum_{k,l,p=1}^{M} U_{r,k}U_{r,l}^{*}U_{q,p}^{*}\int_0^z \mathrm{d}\zeta \sum_{j=1}^{M} U_{q,j}(z-\zeta)\langle\hat{f}_q^\dagger(\zeta)\,\hat{a}_k^\dagger(0)\hat{a}_l(0)\hat{a}_p(0)\rangle \\ &= \sum_{k,l,p=1}^{M} U_{r,k}U_{r,l}^{*}U_{q,p}^{*}\int_0^z \mathrm{d}\zeta \sum_{j=1}^{M} U_{q,j}(z-\zeta)\,\underbrace{\langle\hat{f}_q^\dagger(\zeta)\rangle}_{=0}\langle\hat{a}_k^\dagger(0)\hat{a}_l(0)\hat{a}_p(0)\rangle \\ &= 0.\end{aligned} \tag{B.3}$$

$\alpha = 6, \ldots, 11$:

$$\begin{aligned}
\Gamma^{(6)}_{q,r} &= \sum_{l,p=1}^{M} U^*_{r,l} U^*_{q,p} \int_0^z \mathrm{d}\zeta \int_0^z \mathrm{d}\zeta' \sum_{j,k=1}^{M} U_{q,j}(z-\zeta) U_{r,k}(z-\zeta') \langle \hat{f}^\dagger_q(\zeta)\, \hat{f}^\dagger_r(\zeta') \hat{a}_l(0) \hat{a}_p(0)\rangle \\
&= \sum_{l,p=1}^{M} U^*_{r,l} U^*_{q,p} \int_0^z \mathrm{d}\zeta \int_0^z \zeta\, \mathrm{d}\zeta' \sum_{j,k=1}^{M} U_{q,j}(z-\zeta) U_{r,k}(z-\zeta') \langle \hat{f}^\dagger_q(\zeta)\, \hat{f}^\dagger_r(\zeta')\rangle \langle \hat{a}_l(0) \hat{a}_p(0)\rangle \\
&= \sum_{l,p=1}^{M} U^*_{r,l} U^*_{q,p} \int_0^z \mathrm{d}\zeta \int_0^z \mathrm{d}\zeta' \sum_{j,k=1}^{M} U_{q,j}(z-\zeta) U_{r,k}(z-\zeta') \\
&\quad \times \sum_{\iota_q,\kappa_r} g^*_{q,\iota_q} g^*_{r,\kappa_r} \underbrace{\langle \hat{b}^\dagger_{\iota_q}(0)\, \hat{b}^\dagger_{\kappa_r}(0) \langle}_{=0} \mathrm{e}^{\mathrm{i}\bar{\beta}_{\iota_q}\zeta} \mathrm{e}^{\mathrm{i}\bar{\beta}_{\kappa_r}\zeta'} \langle \hat{a}_l(0) \hat{a}_p(0)\rangle \\
&= 0.
\end{aligned} \tag{B.4}$$

$\alpha = 12, \ldots, 15$:

$$\begin{aligned}
\Gamma^{(12)}_{q,r} &= \sum_{p=1}^{M} U^*_{q,p} \int_0^z \mathrm{d}\zeta \int_0^z \mathrm{d}\zeta' \int_0^z \mathrm{d}\zeta'' \sum_{j,k,l=1}^{M} U_{q,j}(z-\zeta) U_{r,k}(z-\zeta') U^*_{r,l}(z-\zeta'') \\
&\quad \times \langle \hat{f}^\dagger_q(\zeta)\, \hat{f}^\dagger_r(\zeta')\, \hat{f}_r(\zeta'') \hat{a}_p(0)\rangle \\
&= \sum_{p=1}^{M} U^*_{q,p} \int_0^z \mathrm{d}\zeta \int_0^z \mathrm{d}\zeta' \int_0^z \mathrm{d}\zeta'' \sum_{j,k,l=1}^{M} U_{q,j}(z-\zeta) U_{r,k}(z-\zeta') U^*_{r,l}(z-\zeta'') \\
&\quad \times \langle \hat{f}^\dagger_q(\zeta)\, \hat{f}^\dagger_r(\zeta')\, \hat{f}_r(\zeta'')\rangle \langle \hat{a}_p(0)\rangle \\
&= \sum_{p=1}^{M} U^*_{q,p} \int_0^z \mathrm{d}\zeta \int_0^z \mathrm{d}\zeta' \int_0^z \mathrm{d}\zeta'' \sum_{j,k,l=1}^{M} U_{q,j}(z-\zeta) U_{r,k}(z-\zeta') U^*_{r,l}(z-\zeta'') \\
&\quad \times \sum_{\iota_q,\kappa_r,\lambda_r} g^*_{q,\iota_q} g^*_{r,\kappa_r} g_{r,\lambda_r} \underbrace{\langle \hat{b}^\dagger_{\iota_q}(0)\, \hat{b}^\dagger_{\kappa_r}(0) \hat{b}_{\lambda_r}(0)\rangle}_{=0} \mathrm{e}^{\mathrm{i}\bar{\beta}_{\iota_q}\zeta} \mathrm{e}^{\mathrm{i}\bar{\beta}_{\kappa_r}\zeta'} \mathrm{e}^{-\mathrm{i}\bar{\beta}_{\lambda_r}\zeta''} \langle \hat{a}_p(0)\rangle \\
&= 0.
\end{aligned} \tag{B.5}$$

$\alpha = 16$:

$$\begin{aligned}
\Gamma^{(16)}_{q,r} &= \int_0^z \mathrm{d}\zeta \int_0^z \mathrm{d}\zeta' \int_0^z \mathrm{d}\zeta'' \int_0^z \mathrm{d}\zeta''' \sum_{j,k,l,p=1}^{M} U_{q,j}(z-\zeta) U_{r,k}(z-\zeta') U^*_{r,l}(z-\zeta'') U^*_{q,p}(z-\zeta''') \\
&\quad \times \langle \hat{f}^\dagger_q(\zeta)\, \hat{f}^\dagger_r(\zeta')\, \hat{f}_r(\zeta'')\, \hat{f}_q(\zeta''')\rangle \\
&= \int_0^z \mathrm{d}\zeta \int_0^z \mathrm{d}\zeta' \int_0^z \mathrm{d}\zeta'' \int_0^z \mathrm{d}\zeta''' \sum_{j,k,l,p=1}^{M} U_{q,j}(z-\zeta) U_{r,k}(z-\zeta') U^*_{r,l}(z-\zeta'') U^*_{q,p}(z-\zeta''') \\
&\quad \times \sum_{\iota_q,\kappa_r,\lambda_r,\pi_q} g^*_{q,\iota_q} g^*_{r,\kappa_r} g_{r,\lambda_r} g_{q,\pi_q} \underbrace{\langle \hat{b}^\dagger_{\iota_q}(0)\, \hat{b}^\dagger_{\kappa_r}(0) \hat{b}_{\lambda_r}(0) \hat{b}_{\pi_q}(0)\rangle}_{=0\ (\mathrm{ZTA})} \mathrm{e}^{\mathrm{i}\bar{\beta}_{\iota_q}\zeta} \mathrm{e}^{\mathrm{i}\bar{\beta}_{\kappa_r}\zeta'} \mathrm{e}^{-\mathrm{i}\bar{\beta}_{\lambda_r}\zeta''} \mathrm{e}^{-\mathrm{i}\bar{\beta}_{\pi_q}\zeta'''} \\
&= 0.
\end{aligned} \tag{B.6}$$

Note, $\langle \hat{b}^{\dagger}_{\iota_q}(0)\,\hat{b}^{\dagger}_{\kappa_r}(0)\hat{b}_{\lambda_r}(0)\hat{b}_{\pi_q}(0)\rangle = 0$ because in the ZTA the average particle number for each reservoir mode vanishes.

As all terms with $\alpha > 1$ do not contribute it holds for the particle number correlation:

$$\Gamma_{q,r}(z) = \sum_{j,k,l,p=1}^{M} U_{q,j}(z)U_{r,k}(z)U^{*}_{r,l}(z)U^{*}_{q,p}(z)\langle \hat{a}^{\dagger}_{j}(0)\,\hat{a}^{\dagger}_{k}(0)\hat{a}_{l}(0)\hat{a}_{p}(0)\rangle \qquad \text{(B.7)}$$

which is independent of any noise operators.

Reference

[1] Yamamoto Y and İmamoğlu A 1999 *Mesoscopic Quantum Optics* (New York: Wiley)