

# Diffractive Optics for Gravitational Wave Detectors

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**Abstract.** All-reflective interferometry based on nano-structured diffraction gratings offers new possibilities for gravitational wave detection. We investigate an all-reflective Fabry-Perot interferometer concept in 2nd order Littrow mount. The input-output relations for such a resonator are derived treating the grating coupler by means of a scattering matrix formalism. A low loss dielectric reflection grating has been designed and manufactured to test the properties of such a grating cavity.

## 1. Introduction

Laser interferometric gravitational wave detectors employ partly transmissive mirrors as 50/50 beam splitters and couplers to cavities. To avoid thermal effects associated with laser power absorption in transmitted mirror substrates all-reflective interferometer topologies can be used [1]. All-reflective interferometers have the additional advantage that opaque materials with potentially superior mechanical properties, e.g. silicon [2], can be used as mirror substrates.

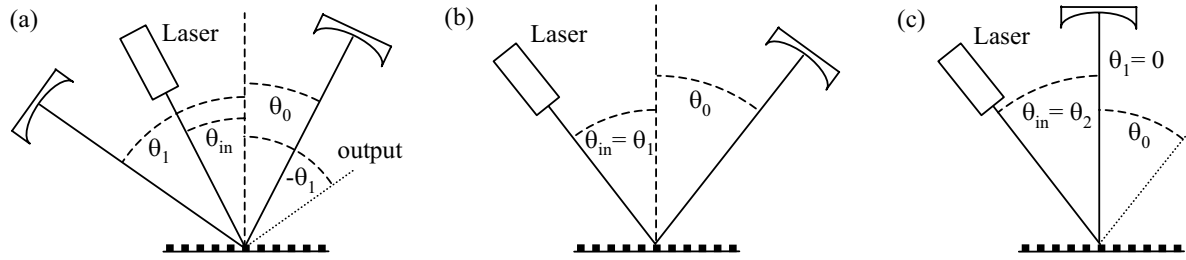
Previously realized all-reflective cavity concepts require high 1st order diffraction efficiency for high finesse cavities. Here we report on the investigation of an all-reflective cavity concept based on low 1st order diffraction efficiency gratings that was successfully used to construct a high finesse cavity.

The paper is organized as follows: after a short summary of the basic principles of grating beam splitters and all-reflective interferometer concepts a theoretical description of a cavity concept in 2nd order Littrow mount is given. The design and fabrication of the grating is explained briefly followed by a comparison of the experimental cavity properties with theoretical results.

## 2. Basic all-reflective interferometer concepts

Transmissive beam splitters are traditionally used to split and recombine optical beams in interferometers. If transmission through optical substrates is unfavorable, reflection gratings can serve as beam splitters. For a laser beam of wavelength  $\lambda$  incident onto a grating, the output angle of the  $m$ th diffracted order is given by the grating equation

$$d(\sin \theta_m + \sin \theta_{\text{in}}) = m\lambda, \quad (1)$$



**Figure 1.** (a) grating in non Littrow configuration with two existing orders can be used as a beam splitter for a Michelson interferometer; (b) linear grating Fabry-Perot interferometer in 1st order Littrow mount; (c) 2nd order Littrow mount.

where  $d$  is the grating period and  $\theta_{\text{in}}$  is the angle of incidence. The number of existing diffraction orders depends on the choice of  $d$ ,  $\lambda$  and  $\theta_{\text{in}}$ .

To obtain an analog to a transmissive beam splitter the parameters are chosen such that only one additional diffraction order to the  $m = 0$  order is present (see Fig. 1(a)). Michelson and Sagnac interferometers can be formed when the grating is used in a non Littrow mount and the power of an incoming beam is split equally into the two orders.

A linear Fabry-Perot interferometer with a coupler in 1st order Littrow mount ( $\theta_{\text{in}} = \theta_1$ ) is formed when a mirror is placed to retro-reflect the 0th order to the grating (see Fig. 1(b)). The finesse of such a cavity is limited by the 1st order diffraction efficiency of the grating. If a grating is used in 2nd order Littrow mount and a mirror is used to retro-reflect the 1st order, the finesse of the resulting linear Fabry-Perot cavity (see Fig. 1(c)) is only limited by the reflectance of the grating for normal incidence. Since high reflectance values are unequally easier to achieve than high diffraction efficiency values, 2nd order Littrow mounting is likely to be the more appropriate concept for all-reflective coupling to high-finesse Fabry-Perot interferometers.

### 3. All-reflective Fabry-Perot cavity in 2nd order Littrow mount

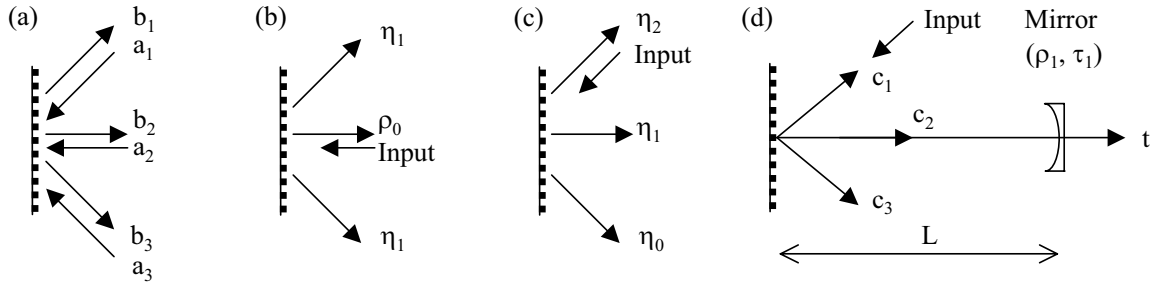
Conventional transmitting beam splitters always couple one input beam to two output beams. The input-output relations of a conventional two mirror Fabry-Perot interferometer follow directly from the phase relation of the reflected and transmitted beams. If the length  $L$  of the cavity is expressed as a tuning  $\phi = \omega L/c$ , where  $\omega$  is the angular frequency of the light and  $c$  is the speed of light, the amplitude reflectance  $r_{\text{FP}}$  and transmittance  $t_{\text{FP}}$  of a cavity can be written as

$$r_{\text{FP}} = [\rho_0 - \rho_1 \exp(2i\phi)]d, \quad (2)$$

$$t_{\text{FP}} = -\tau_0\tau_1 \exp(-i\phi)d, \quad (3)$$

where  $\rho_{0,1}$  and  $\tau_{0,1}$  denote the reflectance and transmittance of the two cavity mirrors respectively, and we have introduced the resonance factor  $d = [1 - \rho_0\rho_1 \exp(2i\phi)]^{-1}$ .

In contrast to a transmissive beam splitter the 2nd order Littrow grating beam splitter couples one input always to three outputs. For normal incidence it couples to the orders -1, 0, +1 and for 2nd order Littrow incidence  $\theta_{\text{in}} = \arcsin(\lambda/d)$  it couples to the orders 0, 1, 2. The corresponding amplitude diffraction efficiencies are termed  $\eta_1, \rho_0, \eta_1$  and  $\eta_0, \eta_1, \eta_2$ , respectively, as depicted in Fig. 2. The -1st and 1st order for normal incidence have the same diffraction coefficient for a symmetric grating structure. For loss-less gratings  $\rho_0^2 + 2\eta_1^2 = 1$  and  $\eta_0^2 + \eta_1^2 + \eta_2^2 = 1$  hold. Coupling to three instead of two output ports results in more complex phase relations which lead to different cavity properties compared to a conventional cavity. The phase relations of the three ports are described by means of a scattering matrix [3] formalism in which a complex



**Figure 2.** 3-port reflection grating: (a) labelling of the input and output ports; (b) amplitudes of reflection coefficients for normal incidence; (c) for 2nd order Littrow incidence; (d) grating cavity in 2nd order Littrow mount and the amplitudes of back reflected light  $c_1$ , intra-cavity field  $c_2$ , forward reflected light  $c_3$  and the transmitted light  $t$ .

valued  $3 \times 3$  scattering matrix  $\mathbf{S}$  represents the 3 port beam splitter. The 3 input ports are represented by a vector  $\mathbf{a}$  with components  $a_i$  that are the complex amplitudes of the incoming waves at the  $i$ th port. The outgoing amplitudes  $b_i$  are represented by vector  $\mathbf{b}$ . The input and output ports are coupled via  $\mathbf{b} = \mathbf{S} \times \mathbf{a}$ . The grating matrix can be written as [4]

$$S_{3p} = \begin{pmatrix} \eta_2 \exp(i\phi_2) & \eta_1 \exp(i\phi_1) & \eta_0 \exp(i\phi_0) \\ \eta_1 \exp(i\phi_1) & \rho_0 \exp(i\phi_0) & \eta_1 \exp(i\phi_1) \\ \eta_0 \exp(i\phi_0) & \eta_1 \exp(i\phi_1) & \eta_2 \exp(i\phi_2) \end{pmatrix}, \quad (4)$$

where  $\phi_0, \phi_1, \phi_2$  is the phase shift for 0th, 1st, 2nd order diffraction respectively. For a loss less grating  $\mathbf{S}$  must be unitary and  $|S_{ij}| = |S_{ji}|$  holds for the matrix elements due to reciprocity of the device. There is no unique solution for the phases  $\phi_i$  since one can choose different reference planes for the various input and output ports. If without loss of generality we assume that specular reflection is associated with no phase change one gets

$$\phi_0 = 0, \quad (5)$$

$$\phi_1 = -(1/2) \arccos[(\eta_1^2 - 2\eta_0^2)/(2\rho_0\eta_0)], \quad (6)$$

$$\phi_2 = \arccos[-\eta_1^2/(2\eta_2\eta_0)]. \quad (7)$$

For a given normal incidence reflectivity  $\rho_0$  there are limits for  $\eta_0$  and  $\eta_2$ , namely

$$\eta_{0,\min}^{\max} = \eta_{2,\min}^{\max} = (1 \pm \rho_0)/2. \quad (8)$$

It should be noted that these limits are fundamental in the sense that a reflection grating can only be designed and manufactured having diffraction efficiencies within these boundaries.

A cavity in 2nd order Littrow mount with an end mirror reflectivity  $\rho_1$ , transmittance  $\tau_1$  and unity input in port one, as depicted in Fig. 2 is described by

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = S_{3p} \times \begin{pmatrix} 1 \\ \rho_1 c_2 \exp(2i\phi) \\ 0 \end{pmatrix}, \quad (9)$$

where  $c_1$  is the amplitude of the field reflected back to the laser,  $c_2$  the intra cavity field and  $c_3$  the field of the forward reflected port. Solving for the amplitudes yields

$$c_1 = \eta_2 \exp(i\phi_2) + \eta_1^2 \exp[2i(\phi_1 + \phi)]d, \quad (10)$$

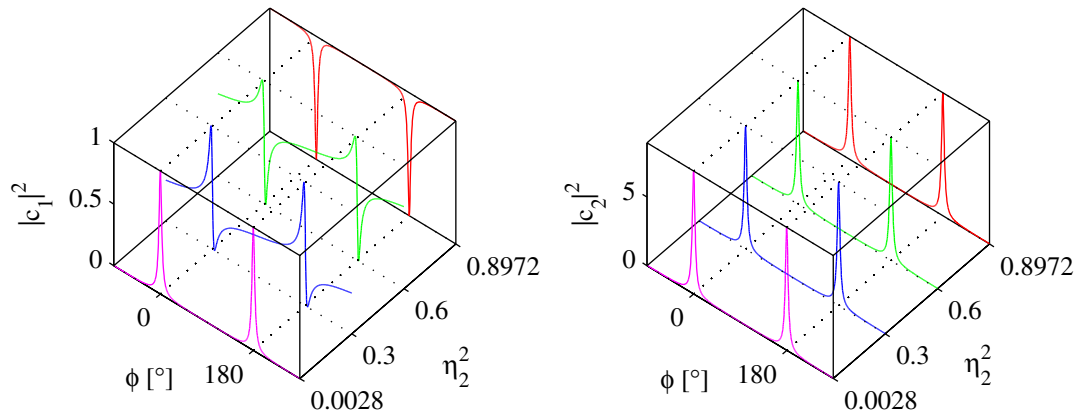
$$c_2 = \eta_1 \exp(i\phi_1)d, \quad (11)$$

$$c_3 = \eta_0 + \eta_1^2 \exp[2i(\phi_1 + \phi)]d, \quad (12)$$

$$t = i\tau_1 c_2 \exp(i\phi) \quad (13)$$

and  $t$  is the amplitude of the light transmitted through the cavity.

The only grating parameter that determines the finesse of such a grating cavity is  $\rho_0$ . For given values of  $\rho_0$  and  $\eta_1$  the intra cavity power  $|c_2|^2$  and the transmitted power  $|t|^2$  do not depend on  $\eta_0$  and  $\eta_2$ . The power of the two reflection ports  $|c_1|^2$  and  $|c_3|^2$  however strongly depend on the values of  $\eta_0$  and  $\eta_2$ . It is therefore possible to tune the cavity properties of the two reflecting ports by means of controlling the 0th and 2nd order diffraction efficiency in the grating production process. Fig. 3 illustrates how the power reflectance  $|c_1|^2$  out of the back

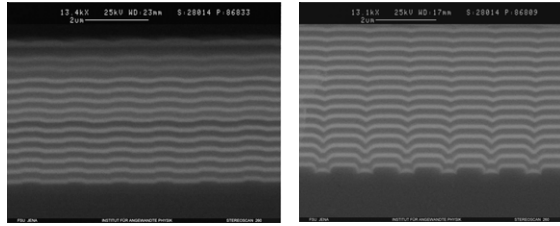


**Figure 3.** left: Power reflectance  $|c_1|^2$  of cavity back reflecting port for a grating cavity with end mirror reflectivity  $\rho_1 = 1$  and cavity coupling  $\eta_1^2 = 0.1$  for selected values of  $\eta_2$ ; right: power inside the cavity  $|c_2|^2$ .

reflecting port varies as a function of  $\eta_2$  and the tuning  $\phi$  of the cavity. For simplicity a cavity with a perfect end mirror  $\rho_1 = 1$  is assumed. The coupling to the cavity is  $\eta_1^2 = 0.1$ . For a coupler with  $\eta_2^2 = \eta_{2,\max}^2 \approx 0.8972$ , the cavity does not reflect any light back to the laser for a tuning of  $\phi = 0$ . This corresponds to an impedance matched cavity that transmits all the light on resonance. For a coupler with  $\eta_2^2 = \eta_{2,\min}^2 \approx 0.0028$ , the situation is reversed and all the light is reflected back to the laser. For all other values of  $\eta_2$  the back-reflected power has intermediate values and as a significant difference to conventional cavities: the intensity as a function of cavity-tuning is no longer symmetric to the  $\phi = 0$  axis.

#### 4. Grating design and fabrication

The dielectric grating used as 3-port input coupler unite low diffraction efficiency and high reflectivity in a single component. A common approach to produce dielectric high diffraction efficiency grating is to etch a periodic structure in the top layer of a dielectric multilayer stack [5]. Here we used a different approach. We first etched the grating into a fused silica substrate and then overcoated it such that the dielectric layers effectively form a volume grating as can be seen in Fig. 4. A grating period of  $d = 1450$  nm was used corresponding to a 2nd order Littrow angle  $\theta_{\text{in}} = \lambda/d \approx 47.2^\circ$  for the Nd:YAG laser wavelength of  $\lambda = 1064$  nm used. A shallow binary structure with a depth of 40-50 nm, a ridge width of 840 nm was generated by electron beam lithography and reactive ion beam etching on top of a fused silica substrate. The applied multilayer stack was composed of 32 alternating layers of silica ( $\text{SiO}_2$ ) and tantalum pentoxide ( $\text{Ta}_2\text{O}_5$ ). We used a power meter to measure a diffraction efficiency of  $\eta_1^2 = 0.58\%$  and  $\eta_2^2 = 0.13\%$  for 1064 nm light with a polarization plane parallel to the grating grooves and perpendicular to the plane of incidence ( $s$ -polarization). The grating allowed for a construction

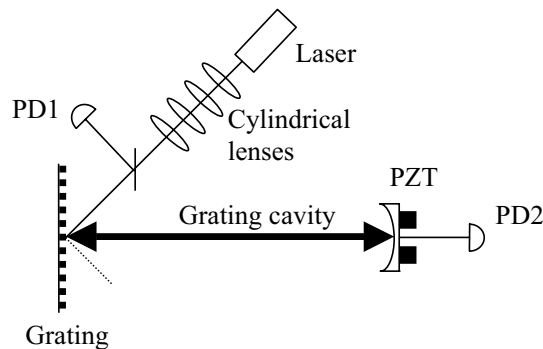


**Figure 4.** Cross sections of overcoated binary gratings (SEM-images) with  $d = 1450$  nm; left: groove depth of 40 – 50 nm; right: groove depth 150 nm. The rectangular pattern visible at the bottom is washed out towards the top of the grating.

of a cavity with a finesse of 400 from which we could deduce the normal incidence reflectivity  $\rho_0^2 = 98.5$  of the grating [6].

## 5. Experimental results

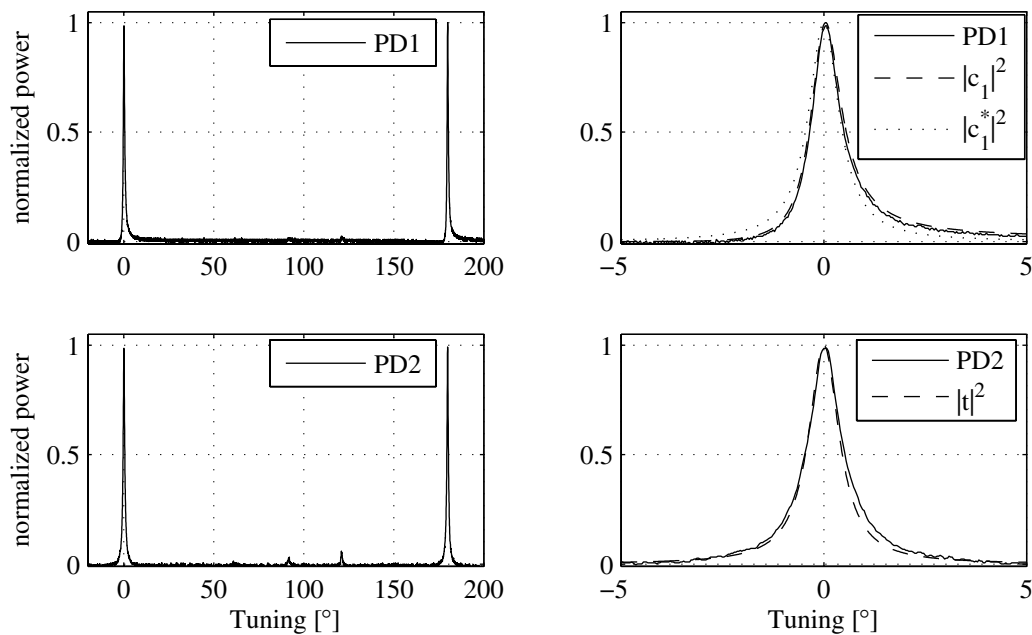
Fig. 5 shows the experimental setup for the all-reflective Fabry-Perot cavity. An end mirror with  $\rho_1^2 \approx 0.99$  and a radius of curvature of 1.5 m mounted on a piezoelectric transducer (PZT) to allow for cavity length control was placed parallel to the grating surface at a distance of 43 cm. An  $s$ -polarized beam of 50 mW from a 1.2 W, 1064 nm diode pumped Nd:YAG laser was used.



**Figure 5.** Experimental setup for the demonstrated grating Fabry-Perot cavity: PZT, piezoelectric transducer; PD, photo diode.

Photo detector PD1 is used to monitor the back reflected light from the cavity and PD2 is used to monitor the light transmitted through the cavity. To match the eigenmode of the grating cavity the incoming beam must have an elliptical beam profile which is generated by two pairs of cylindrical lenses.

Fig. 6 shows measured PD signals normalized to unity as the cavity length is scanned. On the left hand side a scan over one free spectral range of the cavity is shown. The right hand side is a zoom around one resonance peak. For comparison the normalized theoretical curves for  $|c_1|^2$  and  $|t|^2$  are shown. The lower curve is the well known transmission peak of a Fabry Perot cavity symmetric to the  $\phi = 0$  axis. The upper curve however is not symmetric to zero tuning. To emphasize the asymmetry we have also plotted  $|c_1^*|^2 \equiv |c_1(\phi, \eta_{2,\min})|^2$  which is the reflected power if  $\eta_2$  had its minimal allowed value of  $\eta_{2,\min}$ . Note that the observed asymmetry is not as pronounced as for the exemplary curves in Fig. 3 since  $\eta_2$  is relatively close to  $\eta_{2,\min}$ . The theoretical curves agree well with the experimental data which confirms the input-output relations of the grating cavity in 2nd order Littrow mount.



**Figure 6.** Normalized Measured power at PD1 and PD2 and corresponding theoretical curves.

## 6. Conclusion

The input-output relations of an all-reflective cavity concept that relies on low diffraction efficiency gratings only have been derived. A first test using a shallow dielectric grating with a groove depth of 40-50 nm experimentally confirms these relations. For a complete test we will design and manufacture several gratings with constant  $\eta_1$  but different values for  $\eta_2$  and  $\eta_0$  thereby tuning the properties of the two reflected ports.

Additionally we will dedicate more research to the design and manufacturing of all-reflective cavity couplers for 1st order Littrow mount and 50/50 beam splitters. The reduction of the overall optical loss of the gratings due to transmission and scattering [7] and a precise control of the diffraction efficiency of the various diffraction orders are our main goals since they are two requirements for using diffractive optics in future generations of gravitational wave detectors. Moreover we will investigate new grating interferometer topologies as well as practical implementation issues.

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