

# High mechanical Q-factor measurements on silicon bulk samples

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**Abstract.** Future gravitational wave detectors will be limited by different kinds of noise. Thermal noise from the coatings and the substrate material will be a serious noise contribution within the detection band of these detectors. Cooling and the use of a high mechanical Q-factor material as a substrate material will reduce the thermal noise contribution from the substrates. Silicon is one of the most interesting materials for a third generation cryogenic detector. Due to the fact that the coefficient of thermal expansion vanishes at 18 and 125 K the thermoelastic contribution to the thermal noise will disappear. We present a systematic analysis of the mechanical Q-factor at low temperatures between 5 and 300 K on bulk silicon (100) samples which are boron doped. The thickness of the cylindrical samples is varied between 6, 12, 24, and 75 mm with a constant diameter of 3 inches. For the 75 mm substrate a comparison between the (100) and the (111) orientation is presented. In order to obtain the mechanical Q-factor a ring-down measurement is performed. Thus, the substrate is excited to resonant vibrations by means of an electrostatic driving plate and the subsequent ring-down is recorded using a Michelson-like interferometer. The substrate itself is suspended as a pendulum by means of a tungsten wire loop. All measurements are carried out in a special cryostat which provides a temperature stability of better than 0.1 K between 5 and 300 K during the experiment. The influence of the suspension on the measurements is experimentally investigated and discussed. At 5.8 K a highest Q-factor of  $4.5 \times 10^8$  was achieved for the 14.9 kHz mode of a silicon (100) substrate with a diameter of 3 inches and a thickness of 12 mm.

## 1. Introduction

Current interferometric gravitational wave detectors like LIGO [1], VIRGO [2], GEO600 [3], or TAMA [4] are among the most sensitive instruments for detecting length changes. Their design resolution for displacement measurements is in the order of  $\Delta L/L \approx 10^{-23} \dots 10^{-21}$  within the detection band between about 100 and 1000 Hz. Different kinds of noise are limiting this resolution. In the low frequency band the seismic noise is the dominating noise contribution. In the high frequency range the photon shot noise limits the sensitivity. The detectors have

reached their design noise limit in the mid-frequency range. In this region the thermal noise of the optical components plays a dominating role.

Cooling of the crucial parts of the interferometers can provide a reduced thermal noise level. In addition, the use of high-Q materials offers the opportunity to further reduce thermal noise within the detection band. The Q-factor is a measure for the internal losses in a system. The higher the Q-factor the lower the internal dissipation. The Q-factor of a system at a given frequency  $\omega$  is related to the losses  $\phi$  by [5]:

$$Q(\omega) = \frac{1}{\phi(\omega)}. \quad (1)$$

The fluctuation-dissipation theorem [6] connects the thermal noise with the mechanical Q-factor. The spectral power density of a one-dimensional harmonic oscillator far away from its resonances is given by [7]:

$$\langle x^2 \rangle \propto \frac{T}{Q}. \quad (2)$$

That means that the reduction of the thermal noise within the detection band requires low temperatures and high-Q materials. Unfortunately, the mechanical Q-factor is a strongly temperature dependent value [8]. Fused silica – as the material currently used as the optical substrate material of the detectors – can not be used at cryogenic temperatures. Its Q-factor decreases by about 3 orders of magnitude by cooling from room temperature down to about 10 K. This can be understood by its amorphous state [9]. Thus, the thermal noise within the detection band of the gravitational wave detector would be increased although the temperature is reduced by a factor of 30.

Therefore, other materials need to be investigated concerning the temperature dependence of their Q-factor. Promising candidates as substitutes for the current materials would be crystalline materials [10]. One of the most interesting materials is silicon. Silicon provides a high mechanical Q-factor [11], excellent thermal properties and is available in sizes needed for future gravitational wave detectors.

We present systematic cryogenic measurements on cylindrical samples with a diameter of 3 inches and a varying thickness of 6, 12, 24 and 75 mm. The crystal orientation was (100) for the investigation of the different thicknesses. For the 75 mm substrates a comparison between the (100) and the (111) orientation is presented.

## 2. Experimental setup and measuring technique

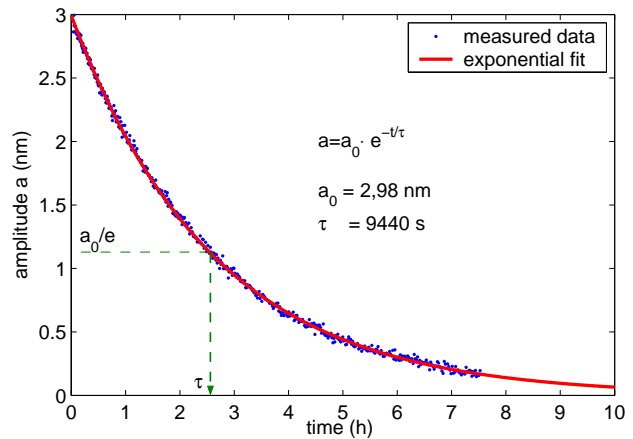
The experiments were carried out in a special built cryostat providing a temperature range between 5 and 300 K with a temperature stability of better than 0.1 K. The experimental set-up is placed in a special probe-chamber which can be evacuated to a residual pressure of  $10^{-3}$  Pa. The substrate is suspended as a pendulum by means of polished tungsten wires with a diameter between 25 and  $75 \mu\text{m}$  depending on the mass of the substrate. To reduce the influence of the suspension the thinnest possible wire was used. A second substrate is used for thermometry (see fig. 1). More details of the cryostat and the set-up can be found elsewhere [8].

In order to determine the mechanical Q-factor at the desired temperature a ring-down technique is used. The substrate is excited to resonant vibrations by means of an inhomogeneous sinusoidal electrical field oscillating with the resonant frequency of the substrate. After reaching a sufficiently high amplitude  $a_0$  (typically some 10 nm) the exciter is switched off and the subsequent ring-down is recorded. The substrate vibrations are sensed by a Michelson-like interferometer. The free-decaying amplitude  $a(t)$  follows an exponential law (see fig. 2):

$$a(t) = a_0 \times e^{-t/\tau}. \quad (3)$$



**Figure 1.** Schematic view of the experimental setup inside the special built cryostat.



**Figure 2.** Exponential ring-down of the fundamental drum mode at 14885 Hz of a Si(100) substrate ( $\varnothing 76.2 \text{ mm} \times 12 \text{ mm}$ ) at 5.6 K. The measured values result in a Q-factor of  $4.5 \times 10^8$ .

The characteristic ring-down time  $\tau$  is used to determine the Q-factor:

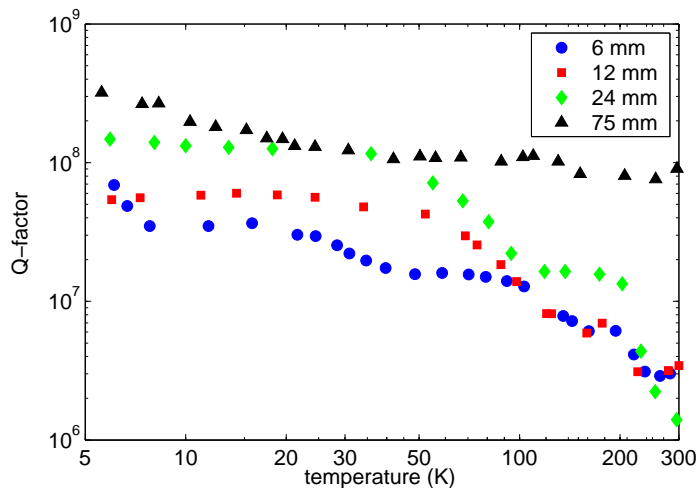
$$Q = \pi f_0 \tau, \quad (4)$$

with the temperature dependent value  $f_0$  as the resonant frequency. The Q-factor is determined at each temperature step which is repeated at least three times.

### 3. Results

#### 3.1. Comparison of the Q-factor of substrates with different thicknesses

Fig. 3 represents the measured cryogenic Q-factors for different substrates with a thickness of 6, 12, 24 and 75 mm and a constant diameter of 3 inches.

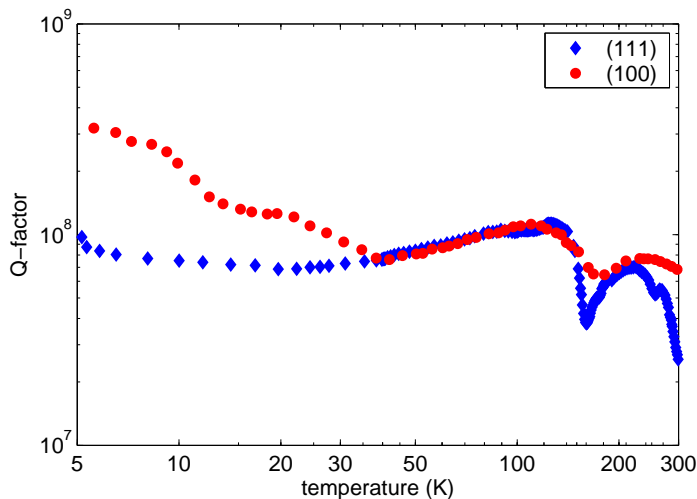


**Figure 3.** Dependence of the mechanical Q-factor on the temperature for different substrate thicknesses. The fundamental drum mode was measured for each substrate.

Within a temperature range from 300 K down to about 100 K the 6 to 24 mm substrates show a strong increase of the Q-factor. Below 70 K the increase is smaller reaching nearly a plateau below 50 K. The 75 mm substrate shows a different behavior. No strong increase is visible. The Q-factor starts at  $9 \times 10^7$  at room temperature and reaches  $3.2 \times 10^8$  at 5.6 K. Up to now the different behavior is not fully understood although the crystals consist of the same material from the same ingot (boron doped with  $4 \times 10^{14} \text{ cm}^{-3}$ , (100) orientation). One possible mechanism would be an interaction with the suspension. For the lightweight substrates this influence is much stronger than for the heavy ones. On the other hand other parameters like the surface to volume ratio are changed in addition. It is known [12] that the surface to volume ratio plays a crucial role for the damping mechanisms in substrate materials.

### 3.2. Comparison of the Q-factor of substrates with different orientations

The dependence of the mechanical Q-factor on temperature for two different crystal orientations is shown in fig. 4. The doping properties are the same for both crystals (Czochralski method,  $4 \times 10^{14} \text{ cm}^{-3}$  boron doping).



**Figure 4.** Dependence of the mechanical Q-factor on temperature for different substrate orientations. The fundamental drum mode was measured for each substrate.

Between room temperature and 100 K both orientations show a characteristic loss peak. This peak is narrower and deeper for the (111) orientation. From 100 K down to 40 K both orientations have the same Q-factor. Below 40 K – which is the most important temperature region for future detectors – the (100) orientation shows a strongly increasing Q-factor whereas the Q-factor of the (111) orientation remains nearly constant. Only at the lowest temperatures reached in the experiment (below 7 K) the (111) orientation shows a small increase of the mechanical Q-factor. For both substrates different lengths of the suspension were used to investigate the loss peak around 150 K. By coincidence it might happen that the resonant frequency of the substrate reaches a harmonic of the suspension violin mode (see e.g. [13]). Thus, an extraction of energy would occur resulting in a similar loss peak as shown in fig. 4. By using different suspension lengths the frequencies of the violin modes are changed and this kind of resonance effect can be excluded. The loss peak has its origin in an internal mechanism.

### 3.3. Analysis of Q-measurements

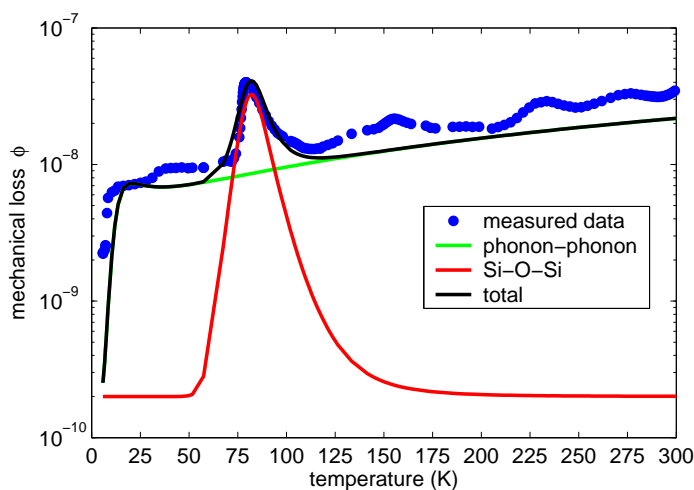
It is necessary to investigate the internal loss mechanisms of the bulk materials more in detail to understand their temperature dependence. A single relaxation process causes mechanical losses in form of a Debye peak:

$$\phi(\omega) = \Delta \frac{\omega^2 \tau_R^2}{1 + \omega^2 \tau_R^2}, \quad (5)$$

where  $\Delta$  is the relaxation strength,  $\omega/2\pi$  the frequency, and  $\tau_R$  the relaxation time. Most relaxation mechanisms can be described using a double-well potential [14]. Two nearly identical states (e.g. different positions of impurity atoms) are separated by an activation energy  $E_a$ . The relaxation time follows an Arrhenius-like law:

$$\tau_R = \tau_{R,0} e^{\frac{E_a}{k_B T}}. \quad (6)$$

Here,  $\tau_{R,0}$  is the relaxation constant and  $E_a$  the activation energy. Both parameters are characteristic for the microscopical process and can be used for its identification.



**Figure 5.** Data analysis of the measured mechanical losses  $\phi$  of a silicon (100) substrate ( $\varnothing 76.2 \text{ mm} \times 12 \text{ mm}$ ,  $\omega/2\pi = 14 \text{ kHz}$ ).

Fig. 5 represents a data analysis of a silicon test sample. The mechanical loss  $\phi$  was obtained using eq. 1. The measured data was exemplarily decomposed into single relaxation processes by means of eq. 5 and eq. 6. The phonon-phonon-interaction [15, 16] dominates over the full temperature range. It is described by a temperature dependent relaxation strength  $\Delta$ . Around 80 K an additional strong loss peak is visible with the characteristic parameters  $E_a = 140 \text{ meV}$ ,  $\tau_{R,0} = 2 \times 10^{-12} \text{ s}$ . These parameters correspond to a vibration of a Si-O-Si complex [17]. This result is plausible because the silicon was grown using the Czochralski method. These crystals are known to have a high concentration of oxygen ( $7 \times 10^{17} \text{ cm}^{-3}$ ).

#### 4. Summary

The mechanical Q-factors of several silicon samples were measured at different temperatures between 5 and 300 K. The measurements confirm that silicon is a high-Q material well suited as an optical material for future gravitational wave detectors. Q-factors up to  $4.5 \times 10^8$  at 5.6 K were reached. Furthermore, the measurements revealed that below 40 K Si(100) seems to have a higher mechanical Q-factor as Si(111). Detailed measurements of the mechanical Q-factors at low temperatures yield an insight to elementary processes which cause mechanical loss. With a better understanding of these processes it is possible to reduce the thermal noise of the crucial parts of the detectors by at least one order of magnitude following eq. 2.

#### Acknowledgments

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