The origin of multibranch IV-characteristics of shunted Josephson junction arrays

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Abstract. To model the IV-characteristics of shunted intrinsic Josephson junctions we calculated phase dynamics of a chain of two identical junctions covered by a resistive shunt and with a resistive crosspiece between the shunt and the junctions. The coherent solution of dynamic equations for this system is unstable at some bias currents above critical currents of junctions. We have shown that this instability leads to the multibranch structure of IV-curves in the hysteretic region. The multibranch structure appears due to the random switching of junctions to the superconducting state above their critical currents. We found that the resistance of the interface between a covering shunt and the junctions can regulate the multibranch behaviour.

1. Introduction

Recently, intensive investigations are made to study electrical properties of intrinsic Josephson junctions in high temperature superconductors (HTSC) [1, 2, 3]. It was found that the IV-characteristic of a chain of Josephson junctions loaded by a resistive shunt can show the multibranch behaviour in some regions of bias currents *above* the critical currents of junctions [4]. Such a behaviour of IV-characteristics show that there are instabilities at some bias currents above critical currents. This behaviour is not investigated until the present time. The origin of unstable dynamic states in the resistively shunted Josephson junctions is as follows. When the chain of junctions is loaded by a resistive shunt, the ac current $\tilde{i}$ produced by the voltages over junctions is flowing in the circuit consisted of all the chain and the load. To fulfill the current conservation conditions, this current must be subtracted from the bias current i [5], and the total current through the chain $i_{tot} = i - \tilde{i}$ decreases. In some regions of bias currents the values of i_{tot} become lower than the critical currents of junctions, so some of junctions can switch to the superconducting state. However, in the superconducting state these junctions do not produce their ac current, so the total current increases again. Junctions then switch back to the voltage state etc. This unstable dynamical state of the whole system is the subject of our present investigation. The analyses of stability by means of Floquet exponents [6] allows to reveal unstable regions of the IV-characteristics but it does not give the information about the shape of IV-characteristics. The aim of the present work is to investigate the shape of IV-characteristics within unstable regions. For this we applied the same approach as for the stability analyses, i.e. we added random perturbations to solutions of dynamic equations and then solved the dynamic equations without any additional random current terms and found IV-characteristics. Because the equations of phase dynamics are differential equations of the second order, we can introduce perturbations in two ways. One of them is the addition of random perturbations of phase differences over junctions and another is the addition of random voltage drops over junctions. We added random perturbations to voltages over junctions because these additions have the straight physical meaning of noise produced by random electromagnetic fields. To refine the effects connected with the instability, we considered only two shunted *identical* junctions and decreased voltage perturbations to the lowest magnitude of 10^{-15} in units of voltages normalized to the critical voltage of the junction.

2. The Model

The high-frequency scheme of the system is shown in Fig. 1 a. For the shunted chain of Josephson junctions investigated in the experiment [4], the resistance R_1 represents the gold shunting cover and R_2 is resistance of the interfaces between the shunt and the chain of junctions. We suppose all critical currents of junctions are equal to each other and all the resistances of junctions are the same. The dynamic equations within a capacitively extended model of a resistively shunted junction [5] read

$$\beta_C \ddot{\varphi}_k + \dot{\varphi}_k + \sin\varphi_k + \widetilde{i_k}(\tau) = i,$$
$$\widetilde{i_k}(\tau) = \frac{1}{r_1(1+2\gamma)}\left[\dot{\varphi}_k + \gamma\left(\dot{\varphi}_1 + \dot{\varphi}_2\right)\right], \tag{1}$$

where $k = 1, 2$, φ_k, $\dot{\varphi}_k$ and $\ddot{\varphi}_k$ are the phase difference across the k-th junction and its first and second derivatives used with respect to dimensionless time $\tau = \frac{2\pi RI_c t}{\Phi_0}$ with Φ_0 is the quantum of magnetic flux, I_c is a critical current, R is the resistance of the junction, $\beta_C = \frac{2\pi I_c C R^2}{\Phi_0}$ is the McCumber parameter of the junction with C as the capacitance of the junction, i and $\widetilde{i_k}$ are the bias current and the current flowing through the k-th junction normalized with respect to I_c, r_1 and r_2 are resistances of the load normalized with respect to R and $\gamma = \frac{r_2}{r_1}$. Voltages across the k-th junction measured in units of RI_c are $\nu_k = \dot{\varphi}_k$. Equations (1) were solved by the standard method of Runge-Kutta. Initial conditions for each bias current were values of variables obtained during the last time step at the previous bias current (conditions with the memory). For the investigation of IV-characteristics of junctions in the perturbed state, we perturbed *only initial conditions* of voltages, i.e. we added random values of the order of magnitude 10^{-15} to ac voltages $\dot{\varphi}_k$ of junctions. We applied also initial conditions without the memory (i. e. voltages were randomly perturbed around zero values at every new bias current).

3. Results and Discussion

Considering currents in the shunt for the system of equivalent junctions without voltage perturbations, we conclude that in the coherent state the IV-characteristics of junctions should be the same for all possible values of r_2 because the equal currents flow through this resistance in opposite directions. IV-characteristics of systems with arbitrary values of r_2 should coincide with each other and IV-characteristics of both junctions should be equal to each other.

However, calculations with randomly perturbed initial conditions for voltages show quite different results. To make the particularities of the curves clear we plot curves for the increase of the bias current only.

IV-characteristics of both junctions are the same for $r_2 = 0.1$ (Fig. 1 b) and coincide with those for the coherent solution (the value $\overline{\nu}$ in the plot is the averaged over time voltage across the junction). The increase of the resistance r_2 to 1 and further to 10 leads to the appearance of branches of the voltage state (the active state) and the zero voltage (passive) state (Figs. 1 c, d) above the critical currents of the junctions. If we apply the initial conditions without the memory, the switching of junctions between active and passive states becomes randomly (Fig. 1 e). The

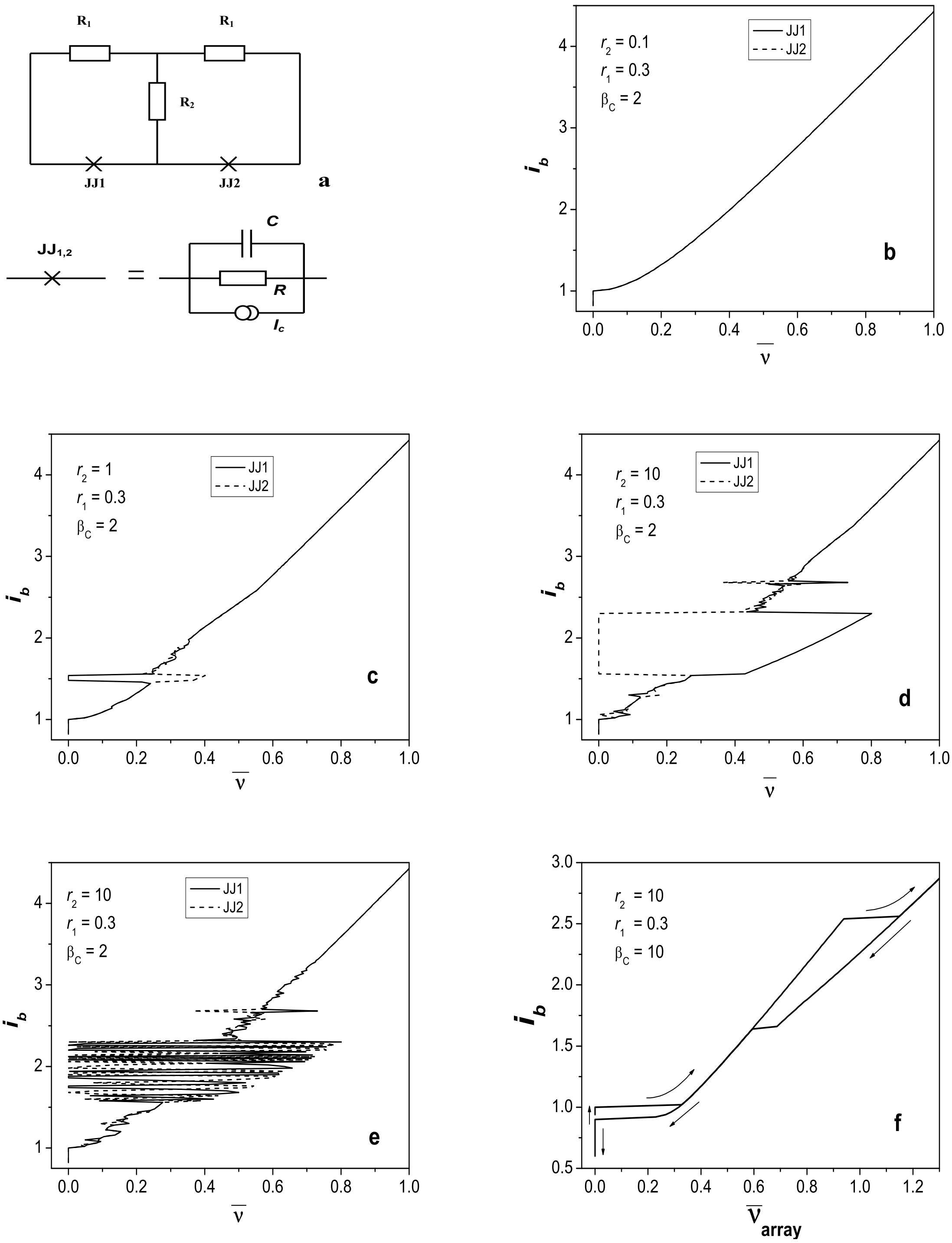

Figure 1. The high-frequency scheme of the system (a), individual IV-characteristics of two junctions with equal critical currents and $\beta_C = 2$ loaded by the shunt with $r_1 = 0.3$ and $r_2 = 0.1$ (b), $r_2 = 1$ (c), $r_2 = 10$ (d), $r_2 = 10$ and with initial conditions without memory (e); the IV-characteristic of the whole system for $\beta_C = 10$, $r_2 = 10$ and with initial conditions with memory (f). Arrows in (f) show the change of the movement of the bias current.

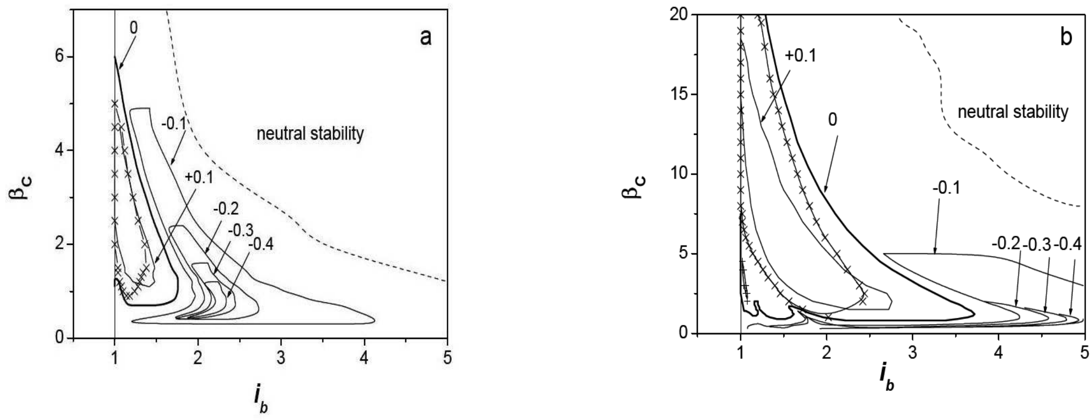

Figure 2. Contours of the largest real part of the Floquet exponents λ on planes $i_b - \beta_C$ for the array of two junctions loaded by a matched resistor (a) and the shunt shown in Fig. 1 a with $r_1 = 0.3$ (b). Contours of regions of multibranch IV-characteristics are marked by the line with crosses. Numbers on the plots are values of λ.

regions of the random switching are placed inside the regions of the unstable coherent solutions (see Fig. 2, where we marked the areas of random switching by crosses). This proves that the origin of the random switching is the instability of the coherent solutions. The ambiguity of switching leads to the appearance of the additional branch on the IV-characteristic of the whole array (Fig. 1 f, initial conditions with the memory).

The resistance r_2 plays the role of the interaction coefficient. Due to the resistance r_2 junctions exchange perturbations with each other. If $r_2 < r_1$, the perturbations are small. The strongest interaction between junctions provides the shunt with $r_2 >> r_1$.

4. Conclusions

We showed that in the resistively shunted array of junctions random perturbations of voltages across junctions lead to the switching of their IV-characteristics from the voltage state to the superconducting state above critical currents that can result in the multibranch structure of IV-characteristics of the whole system. Thus, the multibranch structure appears due to instability of the coherent solution of dynamic equations. The resistance r_2 between the shunt and junctions is the parameter which regulates the splitting of the IV-curves. If $r_2 >> r_1$ the splitting is largest and if $r_2 = 0$ there is no splitting. To avoid the multibranch and switching behaviour the resistance r_2 has to be as small as possible.

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