Nonlinear vibration energy generators with mechanical stops for low-frequency broadband applications

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Abstract. A variety of energy-autonomous sensor systems are excited by a spectrum of excitation frequencies. Especially in the low-frequency range, an optimal system design is important because the available energy for conversion is relatively low. Because the maximum deflection for multimodal excitations is dependent on the stochastic phase, mechanical stops must be implemented to prevent an overloading of the springs. Due to the nonlinear effect of the mechanical stops, only an expected value of the mean output power can be specified. The standard deviation increases with the occurrence of contacts between the seismic mass and the stops. This paper presents a comparison between a linear approach and nonlinear simulations.

1. Introduction

With the increase of widespread decentralised sensors in industrial processes and in daily life, benefits in the fields of monitoring, ease of use, security and conservation of energy and resources are expected. Especially at hard-to-access places alternative power supplies are required because of the finite life time of batteries. A viable option is to use vibration energy generators which convert kinetic energy available in the vicinity of the sensor system into electrical energy. Depending on the magnitude of the required output power, the mechanical-electrical conversion is carried out by micro-mechanical or mesoscopic systems.

Most of the vibration energy generators are considered as resonant systems for sinusoidal excitation with fixed excitation frequency [1, 2]. However, in practical applications this assumption is not true. In so-called broadband or multimodal excitations, different properties can be distinguished. Based on the characterisation of various application scenarios, a linear approach and a nonlinear calculation method for energy generators with mechanical stops are presented and compared. They allow the calculation of system parameters depending on the required mean output power, the excitation spectrum and the permitted deflection.

2. Characterisation of excitation spectrums

In order to characterize specific applications, 3D acceleration measurements were made. The first application scenario is the realisation of energy-independent sensors close to the wheel of a car. The excitations were measured on different vehicles, test routes and speeds. The vertical acceleration shows a continuous power spectral density (PSD) in low-frequency range up to 30 Hz excited by road unevenness. This result agrees very well with the specified frequency range between zero and about 25 Hz given in [3]. Figure 1 shows the sensor on the wheel suspension and the vertical acceleration spectrum measured in a test vehicle driving at 120 km/h on a highway.

A second application scenario is a sensor for sportspersons. The study of the vertical acceleration of human ankles while jogging on a tartan track exhibits a discrete spectrum. The maximum acceleration values can reach multiple of g (gravity of earth). Human activities are also an example for low-frequency excitations. Figure 2 shows the measurement setup and found PSD distribution.

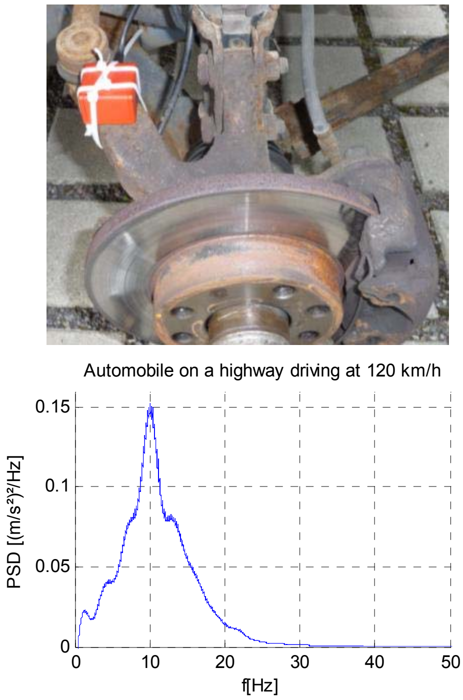

Figure 1. Accelerometer on a wheel suspension and vertical excitation spectrum while driving at 120 km/h on a highway

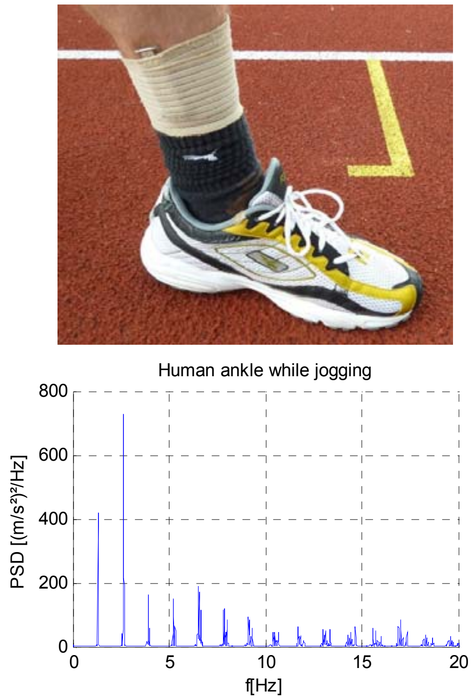

Figure 2. Accelerometer on human ankle and vertical excitation spectrum while jogging on a tartan track

Both applications show frequency spectrums with different characteristics. Broadband excitations in our study are vibrations, which results from a superposition of different frequency components. It does not refer to sinusoidal excitations with variable frequency.

A differentiation between continuous and discrete spectrums is difficult based on the time signals. The description of excitations with the help of power spectral density allows an easy comparison between stochastic signals. The PSD of acceleration (PSD_A) can be calculated as

$$PSD_A(f_i) = \frac{a_i^2}{2\Delta f} \tag{1}$$

where a_i denotes amplitude at the frequency f_i and discretisation Δf. The principal advantage of using PSD is that the power content of a bandwidth can be directly calculated by integrating the PSD over the frequency range. In contrast to the time behaviour

$$a(t) = \sum_{i=1}^{n} \left(a_i \sin(2\pi f_i t + \varphi_i)\right), \tag{2}$$

the PSD does not deliver characterisation of the maximum acceleration because of the absence of the information about the phase of the different frequency components φ_i. Therefore, the results of calculations based on PSD do not fully describe the behaviour in the time domain.

3. Design strategy for linear systems

System parameters for energy harvesters with multimodal excitation can easy calculated, when a linear spring with a constant stiffness is assumed. In an inertial mass-spring-damping system, the seismic mass m is connected with the housing by a spring with stiffness k and the mechanical (dm) and electrical (de) damping (shown in Figure 3).

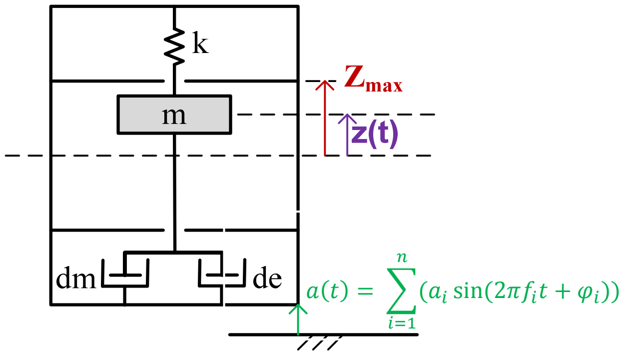

Figure 3. Mass-spring-damping system with a multimodal acceleration $a(t)$

System parameters are dimensioned using the limitation of a root mean square (RMS) of the deflection, expressed by the integral of the power spectral density of the deflection (PSD_Z):

$$RMS_Z = \sqrt{\frac{1}{T}\int_{to}^{t0+T} z^2(t)\mathrm{d}t} = \sqrt{\int_f PSD_Z\,\mathrm{d}f} \tag{3}$$

Because of the linear spring stiffness, PSD of deflection (PSD_Z) and PSD of velocity (PSD_V) can be directly expressed by transfer function and the PSD of acceleration as

$$PSD_Z = \left|\frac{m}{ms^2 + (dm + de)s + k}\right|^2 PSD_A \tag{4}$$

and

$$PSD_V = \left|\frac{ms}{ms^2 + (dm + de)s + k}\right|^2 PSD_A. \tag{6}$$

For a given excitation spectrum PSD_A, mass m and an assumed mechanical damping dm the optimal spring stiffness and the needed electrical damping by the conversion system can be calculated, so that the mean output power in steady state

$$P_e = de\sqrt{\int_f PSD_V\,\mathrm{d}f} \tag{7}$$

is maximized. With the calculated system parameters, the design parameters can be adjusted.

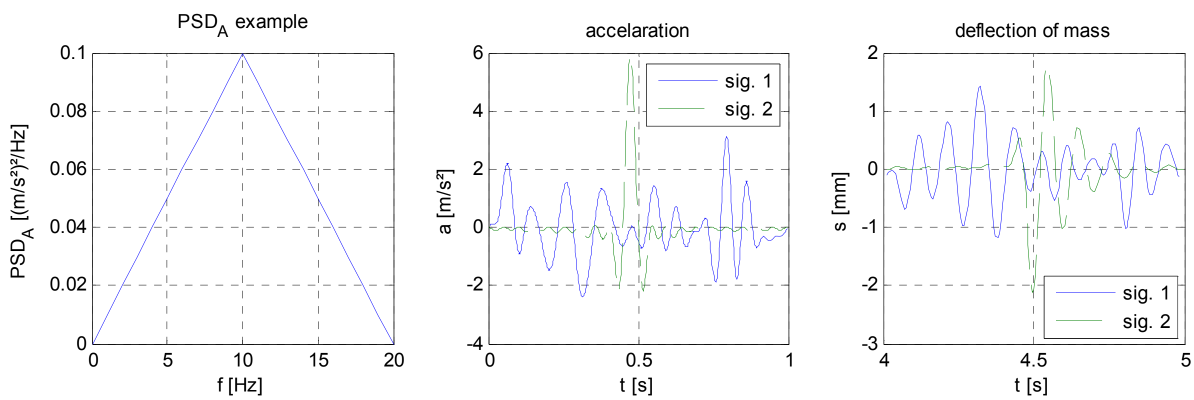

Figure 4. Demonstration example: different time signals (sig. 1 and sig. 2) for a given PSD_A. RMS_Z (= 0.5 mm) and mean output power in steady state (= 1.7 mW) of both signals are equal.

However, only the root mean square can be constrained. Figure 4 shows an example with two signals having the same PSD_A distribution. Although RMS_Z and the mean output power are equal, different maximum deflections are reached. Since the maximum deflection cannot be predicted, the springs have to be protected against the overloading by an implementation of mechanical stops.

4. Nonlinear system with mechanical stops

In [4], mechanical stoppers are examined for a realization of a wideband vibration energy generator. Besides nonlinear springs and bi-stable structures, the mechanical stoppers give also a nonlinear opportunity to increase the bandwidth of the power spectrum. Simulations of the limited model enable to find optimal system parameters. However, the mean output power cannot be calculated in the same way as in the linear case solely by the PSD characterisation of the excitation, because of the only piecewise-linear stiffness.

The excitation must be further described by a phase distribution. In our study, a uniform distribution of the excitation is assumed and stochastic excitation time signals are generated with MATLAB®. The mechanical stops are implemented as springs with a few times higher stiffness at the limit of deflectionZ_{Stop}. An increase of the mechanical damping at stop has not yet been taken into account and needs to be investigated experimentally. The mean output power in steady state for each random excitation is evaluated in the time domain by numerical simulations using Simulink®. The implemented equations can be found in Figure 5. Simulations have been started 1000 times to enable analysis of the expected value and the standard deviation of the mean output power.

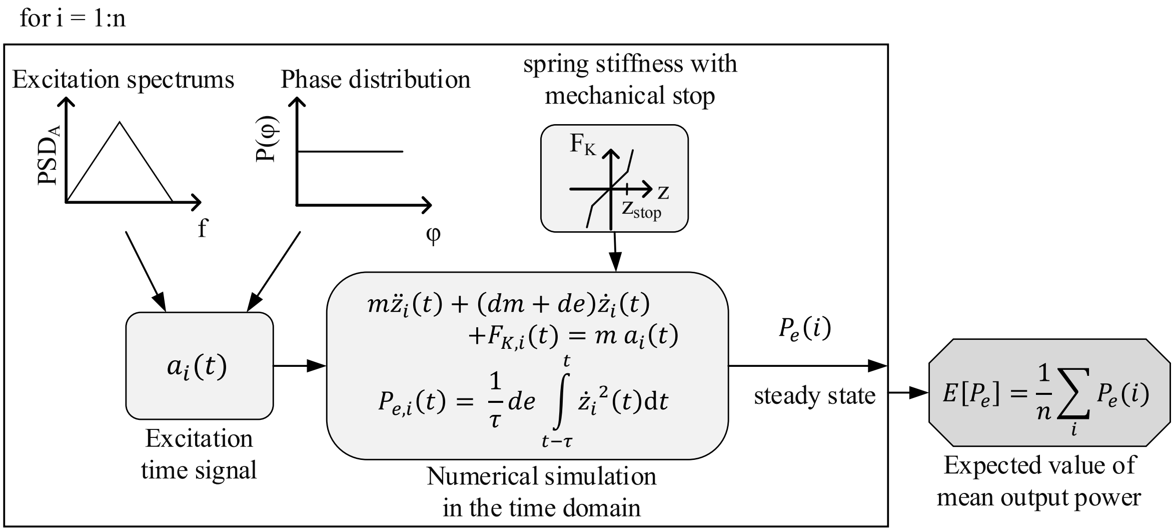

Figure 5. Principle steps of calculations of the expected value of mean output power for a nonlinear vibration energy generator with multimodal excitation and mechanical stops

5. Results for a demonstrator and comparison

For a new multimodal excited electromagnetic demonstrator with a given mass of 50 g and the limitation of deflection to 1.4 mm the optimal electric damping and spring stiffness are searched. The mechanical damping of the system is assumed to 0.1 kg/s. The excitation spectrum is approximated by a triangular shaped profile in a range of 0 to 20 Hz with a maximum value 0.12 $(m/s^2)^2/Hz$ at 10 Hz.

First, an approximate solution is determined using linear approach. In this case, a RMS value of the deflection of 0.5 mm (about one-third of the maximum deflection) is assumed. Figure 6 shows the dependence of mean output power on the spring stiffness, respectively the natural frequency. In addition, the required electrical damping values are added. For the given boundary conditions, the maximum output power is reached at the natural frequency of 10.2 Hz and the electrical damping value of 0.98 kg/s.

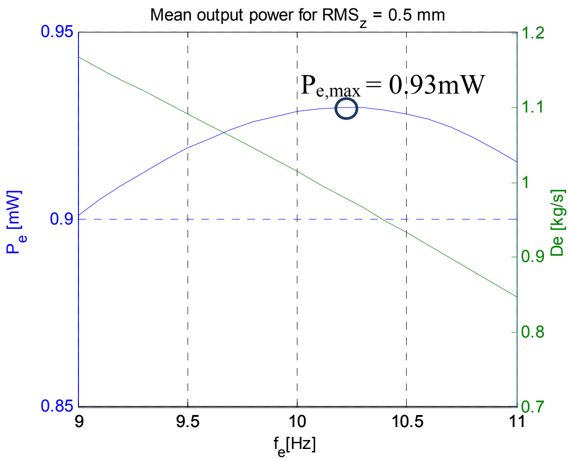

Figure 6. Maximum output power of the linear approach without mechanical stops depending on the natural frequency and the corresponding electrical damping

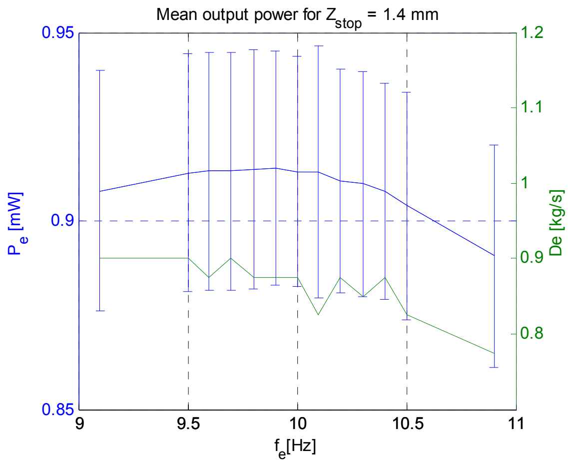

Figure 7. Expected value and standard deviation of the nonlinear calculations depending on the natural frequency and the corresponding electrical damping

Figure 7 shows results of the nonlinear study with mechanical stops at ± 1.4 mm. The maximum mean output power in steady state has a good agreement with the estimation based on the linear approach. The standard deviation is relative high compared with the differences in consequence of the variation of spring stiffness. It is a result of a frequent occurrence of contacts between the seismic mass and the stops. For the system design using linear dimensioning with RMS_Z significant less than one-third of the maximum deflection, the expected value corresponds to the linear result. However, the found power density of the system in this case is not optimal.

6. Conclusions

In the paper, the need for a proper design strategy for multimodal excited energy generators is presented. As a result of a stochastic distribution of phase shifts between the individual frequency components, only the expected value and the standard deviation can be specified for the mean output power in steady state. In our study, the system parameters could be well approximated by a linear approach using a ratio between the maximum deflection and RMS value of 2.8. However, this ratio is not fixed, but depends on the allowable deflection. In ongoing studies these effects will be studied in more details to allow an approximation of the suitable system parameters using the linear method instead of very time consuming nonlinear dimensioning.

Acknowledgement

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