Coherent emission of intrinsic Josephson junctions

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The model of synchronization of intrinsic Josephson junctions in a mesa structure of a high-temperature superconductor is developed. According to our model, the mesa structure does not consist of continuous layers but is divided into small stacks of layers. These stacks form the continuous environment which is considered as a transmission line. We showed that the conventional s-wave dynamic model for the Josephson junction and the d-wave dynamic model give nearly the same particularities in IV-characteristics. We showed also that the model describes numerically the dependence of the frequency of the coherent emission on the width of samples obtained in experiments.

1. Introduction

It was experimentally shown that systems of intrinsic Josephson junctions of high temperature superconductors radiate coherently on frequencies that coincide with frequencies of geometrical resonances of the system [1]. For the explanation of this effect the hypothesis about inhomogeneous distributions of critical currents in samples was made [1]. Structures of intrinsic Josephson junctions usually have defects like boundaries of grains and dislocations, and it is naturally to suppose that the system does not consist of continuous layers but is divided into 'grains'. At the same time, the set of grains forms a continuous environment, which has a definite refraction index for waves with the length which exceeds the size of the 'grain'. Each of the 'grains' we consider as the stack of n junctions with the linear size κ_0. Earlier we showed that our approach explains the appearance of self-induced steps in IV-characteristics of junctions, the coherent radiation which appears in vicinity of these steps and the strongly inhomogeneous distribution of the temperature along the sample [2]. In the present paper we compare calculations of self-induced steps obtained with the use of the d-wave and the s-wave dynamic model of the Josephson junction. Then we model the mesa structure with the width of 80 micrometers and the height of 1 micrometer and calculate the dependence of the frequency at which the first maximum of emitted power appears as a function of the reciprocal width of the system and compare it with the experimental data [1].

2. The model

The high-frequency scheme of the system and the meaning of indices for layers, cells and stacks in our model is shown in figure 1. Each of the stacks has the source of the direct current I_b. In the conventional s-wave resistive model of the Josephson junction [3], the junction with the indices (i, l) is

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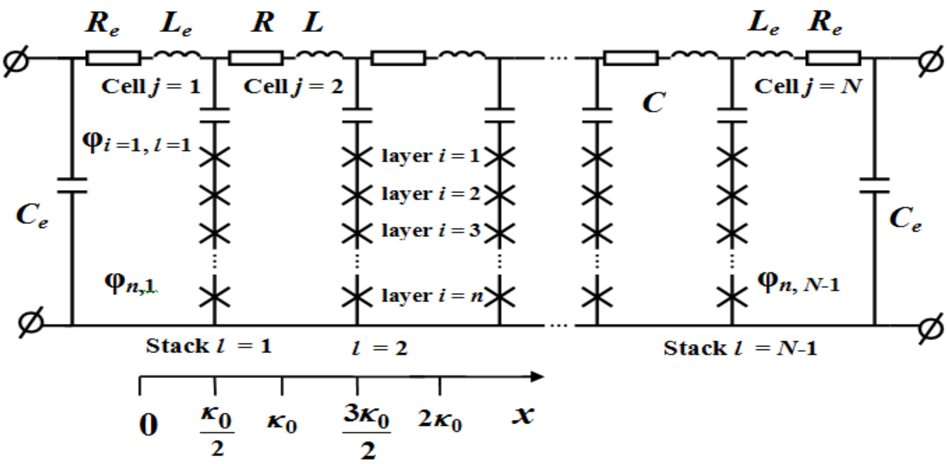

Figure 1. The high-frequency scheme of the system. Josephson junctions are marked by crosses.

represented as a parallel connection of the capacitance $C_{0i,l}$, the electric resistance $R_{0i,l}$ and the source of the Josephson current $I_{i,l} = I_{cj,l} \sin \varphi_{i,j}$, where $\varphi_{i,l}$ is the difference of phases of the order parameter of superconductor across the junction and $I_{ci,l}$ is the critical current of the junction. Dynamics of phases $\varphi_{i,l}$ of junctions in the system is described by a system of equations [2,3]:

$$\alpha C_{0i,l}\ddot{\varphi}_{i,l} + (\alpha / R_{i,l})\dot{\varphi}_{i,l} + I_{ci,l} \sin \varphi_{i,l} = I_b - \dot{q}_j + \dot{q}_{j+1}, \quad l = j = 1 \ldots N-1, \tag{1}$$

$$L\ddot{q}_j + R\dot{q}_j + (2q_j / C) = \alpha\left(\sum_{i=1}^{n} \dot{\varphi}_{i,l-1} - \sum_{i=1}^{n} \dot{\varphi}_{i,l}\right), \quad l = j = 2 \ldots N-1, \tag{2}$$

$$L_e\ddot{q}_1 + R_e\dot{q}_1 + (2q_1 / C_e) = -\alpha\sum_{i=1}^{n} \dot{\varphi}_{i,1}, \quad L_e\dot{q}_N + R_e\dot{q}_N + (2q_N / C_e) = \alpha\sum_{i=1}^{n} \dot{\varphi}_{i,N-1}, \tag{3}$$

where one or two dots over symbols mean the first or the second derivatives with respect to time, correspondingly, $\alpha = \Phi_0 / 2\pi$ with Φ_0 is the quantum of magnetic flux, q_j is the electric charge that passes through the inductance L of the j-th cell of the transmission line. It is supposed that $I_{ci,j}R_{0i,j} = V_c = const$ for all junctions. The velocity of the propagation of waves in a transmission line is equal to $c' = c_0 n'^{-1} = \kappa_0 (LC)^{-1/2}$, where c_0 is the velocity of light in vacuum, $n' \approx 4.47$ is the refraction index. Voltages across junctions are equal to $V_{i,l} = \alpha\dot{\varphi}_{i,l}$. The averaged frequency and the averaged voltage are equal to $\langle \nu \rangle = [2\pi n(N-1)]^{-1} \left\langle \sum_l \sum_i \dot{\varphi}_{i,l} \right\rangle$ and $\langle V \rangle = \Phi_0 \langle \nu \rangle$, where $\langle \ldots \rangle$ means averaging over time. The mean square of the ac voltage is $S = \left\langle \left(\sum_{l=1}^{N-1} \sum_{i=1}^{n} \left(\alpha\dot{\varphi}_{i,l} - \langle V_{i,l} \rangle \right) \right)^2 \right\rangle$. The value of S is proportional to the power of ac radiation emitted by the system. The relation of the ac power emitted in the l-th row to the averaged ac power emitted by one junction in this row is equal to $p_l = \left\langle (1/n)\left[\sum_{i=1}^{n} \left(\alpha\dot{\varphi}_{i,l} - \langle V_{i,l} \rangle \right) \right]^2 \right\rangle \left[(1/n) \left\langle \sum_{i=1}^{n} \left(\alpha\dot{\varphi}_{i,l} - \langle V_{i,l} \rangle \right)^2 \right\rangle \right]^{-1}$. If all junctions oscillate in-phase,

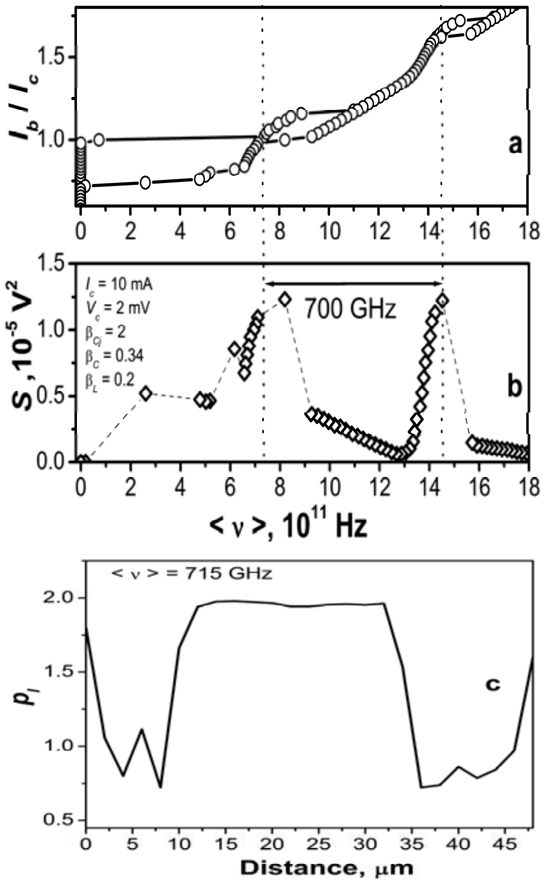

Figure 2. The s-wave model for the system of 2 layers. Dependences of $\langle \nu \rangle = f(I_b / I_c)$ (a), $S = f(\langle \nu \rangle)$ (b) and $p_l = f(x)$ (c). The width of the system is 48 μm.

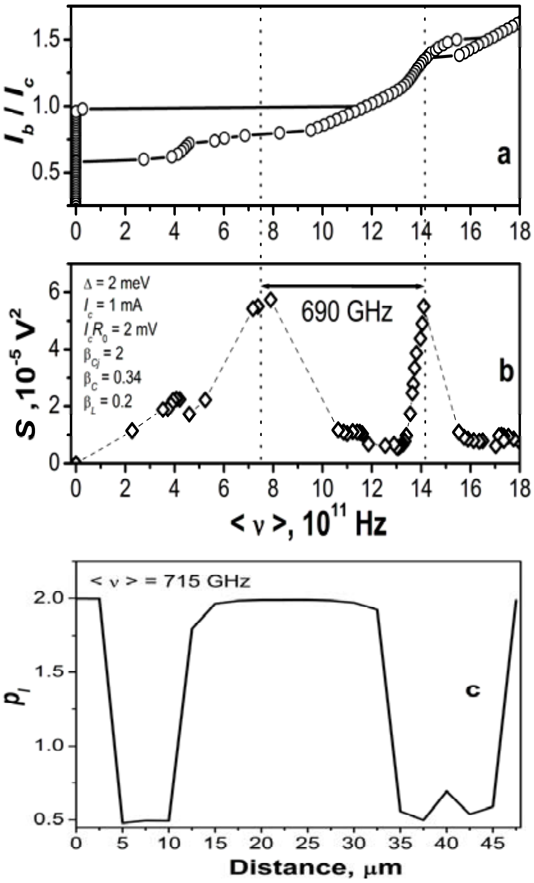

Figure 3. The d-wave model for the system of 2 layers. Dependences of $\langle \nu \rangle = f(I_b / I_c)$ (a), $S = f(\langle \nu \rangle)$ (b) and $p_l = f(x)$ (c). The width of the system is 48 μm.

then $p_l = n$. We developed also the dynamic model of $d_{x^2-y^2}$-wave symmetry of pairing [4]. In this case the second terms in equations (1) are changed to quasiparticle tunnel currents $I_q(\dot{\varphi}_{i,l})$ calculated according to standard expressions for the superconductor with $d_{x^2-y^2}$-wave symmetry of pairing [4].

3. Results of calculations

Dependences of the averaged frequency of Josephson radiation on the normalized bias current calculated with the use of the s-wave model are shown in figure 2a for the system with two layers. Valued of parameters were as follows: $I_{ci,1} = I_c(1-\delta)$, $I_{ci,2} = I_c(1+\delta)$, with I_c = 10 mA and δ = 0.02, V_c = 2 mV, C_{0il} = const = 25 pF, C = 0.28 pF, C_e = 0.22 pF , L_e = 30 pH, L = 3 fH, R = 10^{-4} Ohm, R_e =10 Ohm, κ = 2 μm. Both self-induced steps of IV-curves and maxima of S at frequencies of these steps are clearly seen in figure 2a, b. The distribution of values of p_l along the system shows that junctions are synchronized in the center of the transmission line (figure 2c). The value of the energy gap in the d-wave model was Δ = 2 meV. Values of resonant frequencies and the distribution

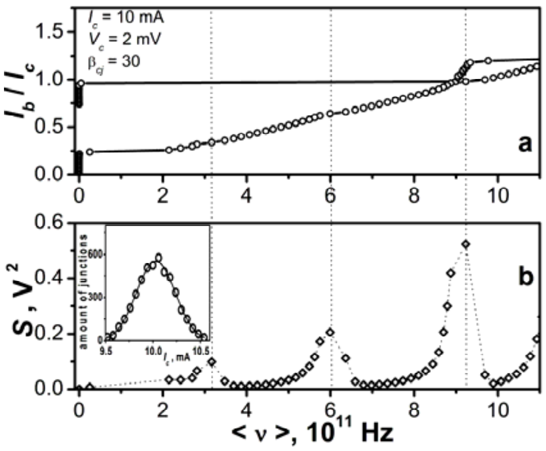

Figure 4. The s-wave model. Dependences of the averaged frequency of Josephson radiation on the normalized bias current (a) and $S = f(\langle \nu \rangle)$ (b) for the system of 80 μm width and with 667 layers. In inset the distribution of critical currents is shown.

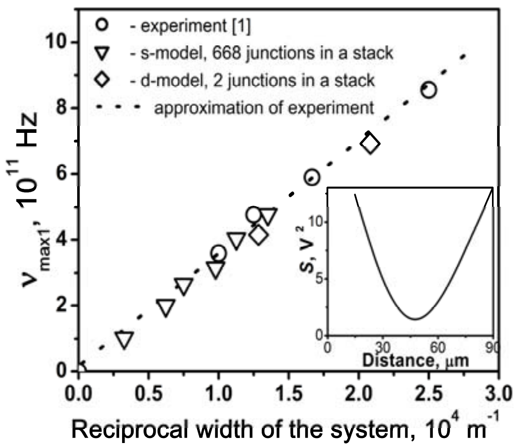

Figure 5. The dependence of $\nu_{\max 1}$ on the reciprocal width of the system: experimental data [1] (circles), the approximation of experimental data (dot line), our calculations (triangles and diamonds). In inset the distribution of S along the length for the 90 μm system and $\langle \nu \rangle$=300 GHz is shown.

of p_l (figure 3) calculated with the use of the d-wave model are nearly the same as for the s-wave model, though they obtained at different values of critical currents (see figure 2a and 3a). In figure 4 dependences of $\langle \nu \rangle = f(I_b / I_c)$ and $S = f(\langle \nu \rangle)$ are shown for 667 intrinsic junctions that corresponds to the thickness of the mesa structure of about 1 μm [1]. Parameters of calculations were as follows: $C_{0i,l}$=20 pF, L = 0.5 pF, R_e=10^4Ohm, L_e=10L, C_e=C, κ = 5 μm. We set the Gaussian random distribution of critical currents (see inset in figure 4b). We plotted frequencies of the first maximum of emitted radiation $\nu_{\max 1}$ on the reciprocal width of the system with 668 intrinsic junctions (figure 5). Parameters of calculations were the same as in figure 4. It is seen that our calculations are in good agreement with the experimental data [1]. The distribution of S along the system (see inset in figure 5) shows that in this case junctions are synchronized at ends of the system.

4. Conclusions

We developed the model of coherent emission from the mesa structure of intrinsic Josephson junctions. We showed that different models of junctions give the same particularities of IV-characteristics, i.e. that the effect does not depend on the model for the individual junction and that self-induced steps are produced under the influence of the transmission line. The calculated resonant frequencies of coherent emission are in good agreement with the experiment [1].

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