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# Measurement uncertainty consideration in the case of non-linear models for precision length measurement

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**Abstract.** The requirements on the precision lengths and precision load measurement rise due to the rapid technical development, in particular of nanotechnologies and precision optics. According to the roadmap "Dimensional metrology for micro- and nanotechnology" of the European National Metrology Institutes (iMERA)<sup>2</sup> in the next few years "traceable 2D (3D) metrology at (sub)-nm accuracy over several 100 mm range" will be required.

## 1. Introduction

Measurement uncertainties attainable with high precision instruments depend on the measuring ranges, the metrological structure, the sensors and among others on the conditions of use. So, on the one hand, low-error metrological arrangements are necessary for coordinate measurement systems with nanometer precision to achieve small uncertainties in budgeting. On the other hand, a measurement value correction is possible by detailed metrological modelling. These measurement uncertainty budgets can be created on the basis of vector models.

To achieve nanometre accuracy it is very important to use error minimum arrangements. In 3D-nano coordinate measurement machines it is necessary to fulfil the Abbe comparator principle in all three measurement axes [1, 2]. For highest performance we improve this approach by an additional extension: besides the aligned arrangement of measurement object and scale we control all angular deviations to be zero.

On the basis of this approach, we have realized a nanopositioning and nanomeasuring machine with a measuring range of 25 mm x 25 mm x 5 mm and meanwhile an improved resolution of 20 pm. To determine the measurement capability of the machine, we have analysed the measurement uncertainty of the whole machine by complex vector modelling (cf. Fig. 1) [3, 4, 5].

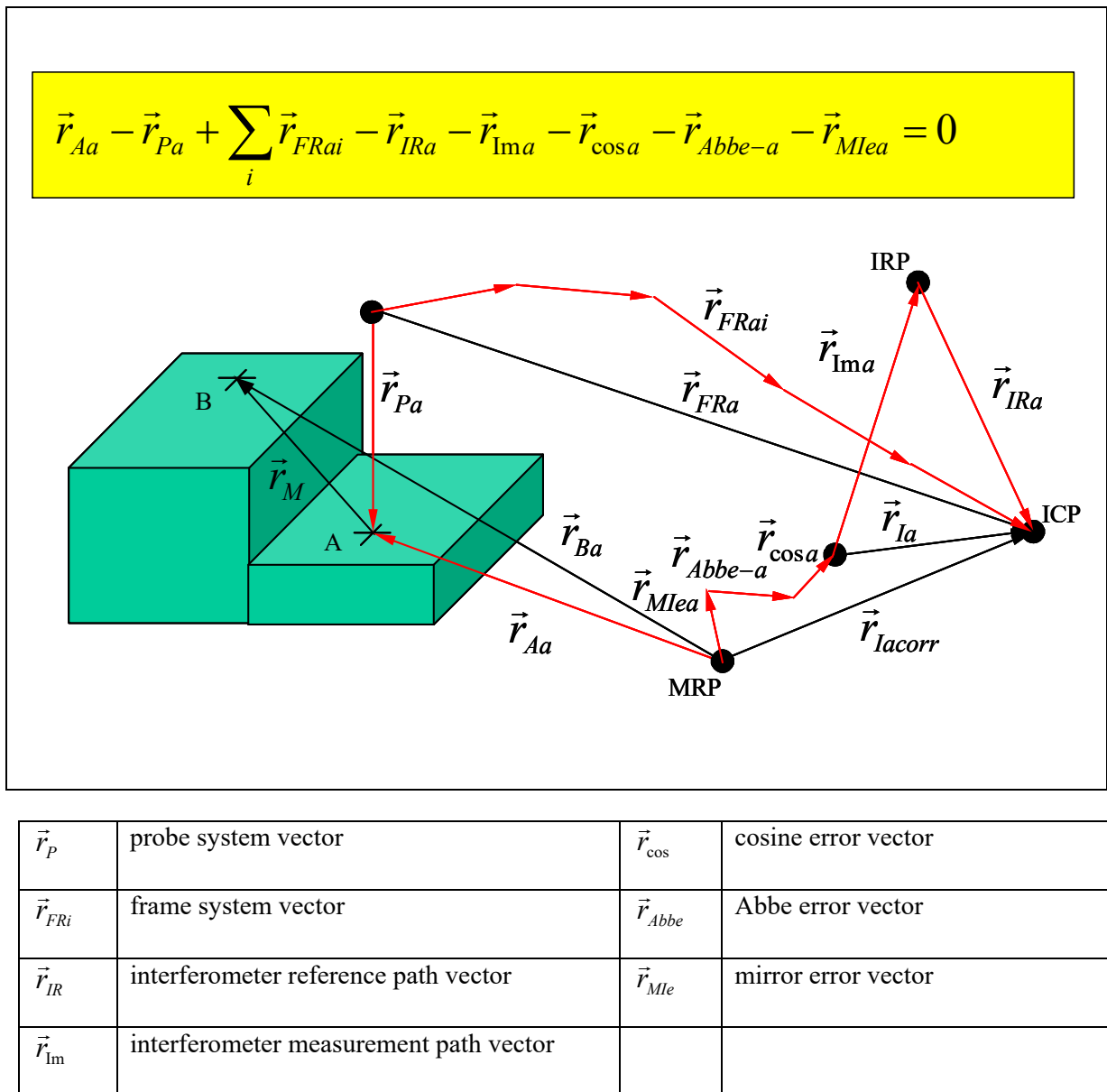
On this basis, error influences can be determined very effectively. We can derive conclusions for the design of the machine or of components as well as for operating the machine or the measuring conditions.

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<sup>2</sup> [http://www.technology-roadmaps.eu/doku.php?id=Micro\\_and\\_nano](http://www.technology-roadmaps.eu/doku.php?id=Micro_and_nano)





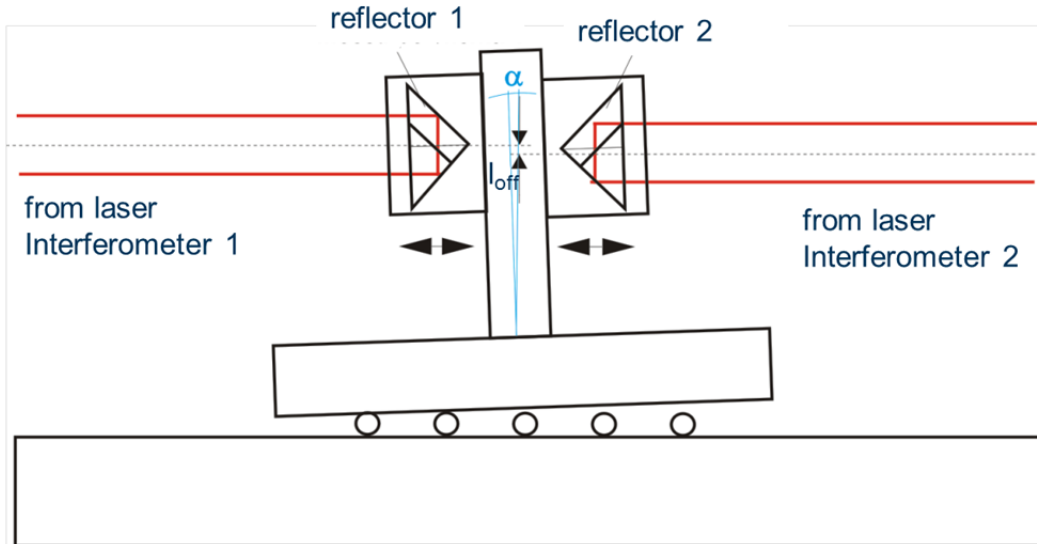
**Figure 1:** Vector chain of the measuring point A

Some partial models often lead to non-linear models, for example in the case of geometrical errors. One interesting measurement aspect arises for partial models, where the expectations of all input values are zero, which also leads to an uncertainty of zero in case of the linear GUM approach.

## 2. Abbe error

In the field of one-dimensional interferometric length measurement it is always possible to minimize the so-called Abbe-error by an aligned arrangement of object to be measured and scale. In precision measurement technology, however, it is well possible for residual sine errors to cause significant errors due to insufficient alignment. For example, if two high-precision laser interferometers are to be

compared with each other within a range of, for example, 2m, they cannot be adjusted to each other – in general – better than 500 μm (cf. Fig. 2).



**Figure 2:** Set-up for the metrological comparison of two interferometers

Thus, for determining the measurement uncertainty contribution, for this set-up the problem arising here is that in the product

$$\Delta l = l_{off} \cdot \sin \alpha$$

both mean values are zero:  $\bar{l}_{off} = \bar{\alpha} = 0$

The estimation of the measurement uncertainty according GUM leads to an incorrect solution for the uncertainty:

$$u^2(\Delta l) = \sum_{i=1}^N \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) = \bar{\alpha}^2 u^2(l_{off}) + l_{off}^2 u^2(\alpha) = 0$$

In this case, the residual Abbe error can be derived with Steiners theorem:

$$u^2(\Delta l) = \text{VAR} \{ \Delta l \} = E \{ (\Delta l - E(\Delta l))^2 \} = E \{ \Delta l^2 \} - E^2 \{ \Delta l \}$$

With  $E^2 \{ \Delta l \} = 0$  we get:

$$u^2(\Delta l) = E \{ \Delta l^2 \} = E \{ l_{off}^2 \} \cdot E \{ \hat{\alpha}^2 \}$$

and:

$$u^2(\Delta l) = (\text{VAR} \{ l_{off} \} + E^2 \{ l_{off} \}) \cdot (\text{VAR} \{ \hat{\alpha} \} + E^2 \{ \hat{\alpha} \})$$

Finally, because of  $E^2 \{ l_{off} \} = E^2 \{ \hat{\alpha} \} = 0$  we get in contrast to the GUM approach the result:

$$u^2(\Delta l) = \text{VAR} \{ l_{off} \} \cdot \text{VAR} \{ \hat{\alpha} \} = u^2(l_{off}) \cdot u^2(\hat{\alpha})$$

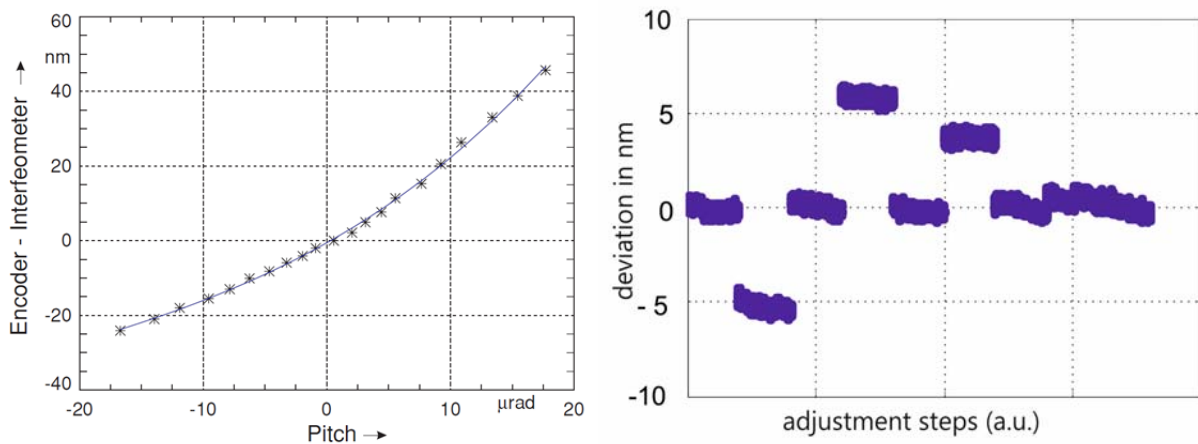
If we consider under compliance of the Abbe comparator principle realistic values of 500 μm Abbe offset and 60 arcsec angular deviation for a high precision linear stage we can calculate 100 nm as expanded combined uncertainty (at  $k = 2$ ).

For high precision measurements ( $U \leq 10\text{nm}$  at  $k = 2$ ) this leads to the following deductions:

- a) a diminution of the Abbe offset to less than 50  $\mu\text{m}$  is necessary or
- b) an additional angular measurement of the linear stage with an uncertainty of less than 6 arcsec is necessary.

In [6], a method for determining the Abbe error as well as for correcting it is described. Here, the measuring table is tilted in a specific way, and the Abbe error induced by this tilting is determined. From this, the Abbe-offset can be specified and finally minimized through iterative adjustment and measurement. However, in the verification experiments some non-linearity that should not be neglected was found. The cause of this phenomenon has not been detected so far (cf. Fig. 4, left-hand side).

The same non-linear behaviour has also been observed by the authors. In Fig. 3, right-hand side, an iteration process can be recognised in the case of which the Abbe error has been reduced through the determined tilting by a constant angle from originally 5 nm to about 1 nm.



**Figure 3:** Measurement uncertainty of two measuring systems occurring due to an Abbe error [6] (left-hand side)/ Lowering of the Abbe error through gradual reduction of Abbe offset (right-hand side)

Under certain circumstances, the iterative adjustment process can be very time-consuming. If routine comparisons of interferometers are to be made, this approach will not be suitable. Such a test procedure (Fig. 2) can be extended by an additional angular measurement made by means of an autocollimator and a piezo – angular adjustment unit so as to control in a permanent manner the angular deviations through a closed-loop control circuit.

### 3. Cosine error

In length measurement, the cosine error is regarded as error of 2nd order. However, it becomes significant for precision measurements. For a detailed analysis of this error, the measuring arrangement must be inspected very carefully. While tactile length measurements lead in general to a positive measuring uncertainty, interference-optical measurements mostly present negative uncertainties:

$$\Delta l_{\text{mech}} = l \left( \frac{1}{\cos \alpha} - 1 \right) \quad \text{and} \quad \Delta l_{\text{interf}} = l (\cos \alpha - 1)$$

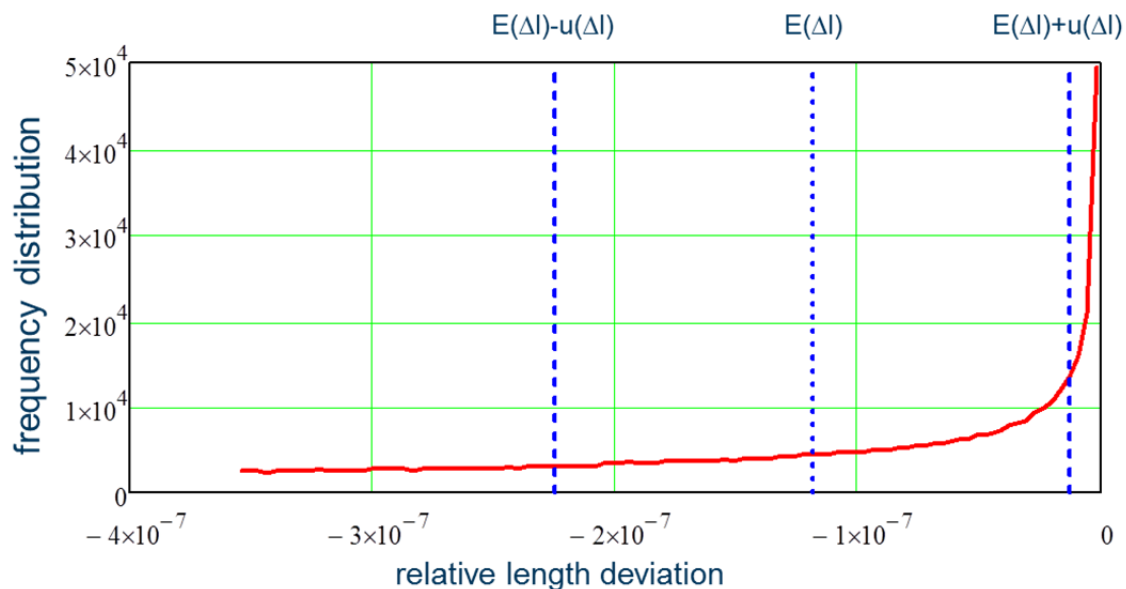
In both cases, the uncertainties represent unbalanced distributions. Figure 4 shows the frequency distribution for an interferometric relative cosine error. As for the angular deviation, it is a rectangle-distributed random error with a half-width of 200 arcsec. In this case, the expected value of the relative length measurement deviation amounts to  $-1.2 \cdot 10^{-7}$ .

The relative dispersion amounts to  $1 \cdot 10^{-7}$ , with the expected value being determined according to:

$$\Delta l_{\cos} = -1/2 \cdot u^2(\alpha)$$

and the dispersion of the length measurement deviation according to:

$$u(\Delta l) = \sqrt{\frac{4}{5}} \cdot \Delta l_{\cos}$$



**Figure 4:** Frequency distribution for an interferometric cosine error

Here, the relation of the fourth central moment to the fourth power of the standard measuring uncertainty amounts to 9/5 (cf. [7]). These theoretically determined values have been checked with view to their correctness by means of the Monte-Carlo-method.

Assuming symmetrical probability ( $p = 95\%$ ), measurement uncertainty can now be specified only by means of an asymmetrical overlapping interval (e.g. for a displacement path of 1m  $[-0.34\mu\text{m}, -0.0002\mu\text{m}]$ ).

For the usual indication of  $3\sigma$ -values, an upper limit of  $2 \cdot 10^{-7}$  results erroneously, i.e., of positive cosine errors which, however, cannot occur in practice. Furthermore, it is quite unusual for the expected value of the cosine error to depend on the value of the standard deviation of the angular deviation, which is usually estimated. If this standard deviation is wrongly estimated, not only a change of the measurement uncertainty will result, but also a change of the expected value of the path measured.

#### 4. Summary

In the present paper, some theoretical investigations on the two most important measurement uncertainty contributions of precision length measurement – the Abbe error and the cosine error – have been carried out. They provide non-linear sub-models in the measurement uncertainty analysis. Some practical conclusions concerning measurement arrangements with minimum errors have been drawn, and various proposals for minimizing the Abbe error in practice have been made. A possible method to improve the accuracy of linear length measurement is to additionally measure the pitch and yaw angles, combined with a stable control of the angular deviations in a closed-loop control circuit, or also by a computational correction of the measurement deviations conditioned by the tilt angle. The special feature of the cosine error consists in the asymmetrical distribution and in the correction of the expected value of the length measurement, with this correction being dependent on the value of the occurring angular deviation. Through the intensive work on the measuring uncertainty budget it is possible to derive interesting and important conclusions for further developments of new measurement strategies, arrangements and technology in the field of nanometrology.

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