Tidal polarizability effects in neutron star mergers

S Bernuzzi[1]**, A Nagar**[2]**, S Balmelli**[3]**, T Dietrich**[1] **and M Ujevic**[4]

[1]Theoretical Physics Institute, University of Jena, 07743 Jena, Germany
[2]Institut des Hautes Etudes Scientifiques, 91440 Bures-sur-Yvette, France
[3]Physik-Institut, Universität Zürich, 8057 Zürich, Switzerland
[4]Centro de Ciências Naturais e Humanas, Universidade Federal do ABC, 09210-170, Santo André, São Paulo, Brazil

Abstract. Using the analytical effective-one-body model and nonlinear 3+1 numerical relativity simulations, we investigate binary neutron star mergers. It is found that, for nonspinning binaries, both the mass-rescaled gravitational wave frequency at merger and the specific binding energy at merger almost uniquely depend on the tidal coupling constants κ_2^T, which are functions of the stars' Love numbers, compactnesses and mass ratio. These relations are quasiuniversal in the sense that there is an additional dependence on the spins, which is linear for realistic spins values $\chi \lesssim 0.1$. In the effective-one-body model, the quasiuniversality is a direct consequence of the conservative dynamics of tidally interacting bodies. In the context of gravitational wave astronomy, our findings may be used to constrain the neutron stars' equation of state using waveforms that accurately model the merger.

1. Introduction

Coalescing neutron stars are among the most promising sources for the currently operating, ground-based interferometric gravitational waves (GW) detectors [1]. In particular, starting from the next few years, the advanced configurations of the LIGO/Virgo network are expected to detect $\sim 0.4 - 400$ events per year [2]. The late part of the coalescing process is likely to provide, via measurement of the tidal polarizability, crucial information about the neutron stars' internal structure and equation of state (EOS) [3, 4, 5, 6].

At present time, an accurate modelling of binary neutron star (BNS) mergers is only possible with numerical relativity (NR). Many improvements have been done in recent years, which led, e.g., to phasing analysis for multiorbit simulations [7, 8, 9] or to studies of the gauge-invariant relation between energy and angular momentum [10, 11].

Nevertheless, some important aspects still need to be clarified. For instance, there is insufficient knowledge about the role of finite mass ratio and finite mass effects at merger. BNS-related studies (see e.g. [6]) typically neglect these effects, insofar as they estimate the end of the quasiadiabatic BNS inspiral and the amount of GW energy emitted during the coalescence by means of the simple Schwarzschild quantities $2M\Omega_{\mathrm{LSO}}^{\mathrm{Schw}} \approx 0.13608$ and $E_{b\,\mathrm{LSO}}^{\mathrm{Schw}} \approx -0.0572$ taken in correspondence of the Schwarzschild last stable orbit (LSO).

We use new multi-orbit NR data and the analytical effective-one-body (EOB) approach to further investigate neutron stars mergers. We find that the GW frequency and binding energy at the moment of merger are only characterized, besides of the spins, by certain dimensionless tidal coupling constants (a fact also empirically observed in [5] for the frequency). For more details, see Ref. [12].

2. Methods

2.1. LSO and merger within the EOB

The EOB approach maps the general relativistic 2-body problem into the dynamics of a particle with mass $\mu = M_A M_B / M$ moving in an effective metric generated by a central mass $M = M_A + M_B$ [13]. It consists of three building blocks (see e.g. Ref. [14]): (i) a Hamiltonian $H_{\rm EOB}$; (ii) a factorized gravitational waveform; and (iii) a radiation reaction force $\mathcal{F}_\varphi$. The EOB Hamiltonian $H_{\rm EOB} = M\sqrt{1 + 2\nu(\hat{H}_{\rm eff} - 1)}$ is constructed upon an effective Hamiltonian, that in the nonspinning case reads $\hat{H}_{\rm eff}(u, p_{r_*}, p_\varphi) = \sqrt{A(u;\nu)\left(1 + p_\varphi^2 u^2 + 2\nu(4 - 3\nu)u^2 p_{r_*}^4\right) + p_{r_*}^2}$. Here, $\nu = \mu/M$, $u \equiv 1/r$ (r is a dimensionless separation radius), while p_φ and $p_{r*} \equiv \sqrt{A/B}\, p_r$ are dimensionless (orbital and radial) momenta. PN results are included, in a resummed way, into the potentials $A(u;\nu)$ and $B(u;\nu)$. We implement the spin following Ref. [15], and taking the spin-orbit coupling at NNLO [16]. Finite size effects can be reproduced by adding to the point-mass contribution $A^0(u;\nu)$ of the radial potential a suitable tidal term $A^T(u;\nu)$. In $A^0(u;\nu)$, we exploit the full analytical information we posses so far, including terms up to 4PN [17], and use a (1,4)-Padé resummation. Tidal terms formally start at 5PN and are included up to NNLO order. They have the structure [18]

$$A^T(u) = -\sum_{\ell=2}^{4} \kappa_\ell^T u^{2\ell+2}(1 + \bar{\alpha}_1^{(\ell)} u + \bar{\alpha}_2^{(\ell)} u^2), \tag{1}$$

with only $\bar{\alpha}_{1,2}^{(2),(3)}$ known analytically [19]. The full EOS information is encoded into the tidal coupling constants ($\ell \geq 2$)

$$\kappa_\ell^T \equiv 2\left[\frac{1}{q}\left(\frac{X_A}{C_A}\right)^{2\ell+1} k_\ell^A + q\left(\frac{X_B}{C_B}\right)^{2\ell+1} k_\ell^B\right], \tag{2}$$

where $q = M_A/M_B \geq 1$, $X_A \equiv M_A/M = q/(1 + q)$, $X_B \equiv M_B/M = 1/(1 + q)$, while $k_\ell^{A,B}$ and $C_{A,B}$ are the dimensionless Love numbers and compactness of star A and B. We stress that, in this work, we use a fully analytic EOB model, that does not contain any parameter calibrated against NR.

For a fixed angular momentum p_φ, circular orbits satisfy $\partial_u H_{\rm eff}(u, p_\varphi, p_r \equiv 0) = 0$. Within the adiabatic ($\mathcal{F}_\varphi = 0$) EOB model, the LSO ($u_{\rm LSO}, p_{\varphi{\rm LSO}}$) is defined as the inflection point of the radial effective potential $\hat{H}_{\rm eff}(u)$, and thus must satisfy the additional equation $\partial_u^2 H_{\rm eff}(u, p_\varphi, p_r \equiv 0) = 0$. Below $u_{\rm LSO}^{-1}$, no stable circular orbit exists, while below $p_{\varphi{\rm LSO}}$ no circular orbit is possible at all. The gravitational wave frequency and the binding energy at the LSO are simply calculated as $M\Omega_{\rm LSO} = \mu^{-1}\partial_{p_\varphi} H_{\rm EOB}(u_{\rm LSO}, p_\varphi, p_r \equiv 0)|_{p_{\varphi{\rm LSO}}}$ and $E_{b\,{\rm LSO}} = (H_{\rm EOB}(u_{\rm LSO}, p_{\varphi{\rm LSO}}, p_r \equiv 0) - M)/\mu$.

A more complete approach is provided by the nonadiabatic EOB ($\mathcal{F}_\varphi \neq 0$). In this case the dynamics is continued after the LSO crossing, where it shows, as also NR simulations do, a characteristic peak in the GW amplitude. We define (for both EOB and NR) the peak of the $l = m = 2$ mode as the moment of merger.

2.2. NR simulations

Our NR simulations employ the BAM code and the method described in [20, 21], but with some different features: (i) we use the Z4c formulation of Einstein's equations [22]; (ii) GWs are extracted from an extended wavezone [23]. The binaries are equal-mass, irrotational configurations with different EOS. A $\Gamma = 2$ polytropic EOS model is employed to simulate different compactnesses. The evolutions cover about ten orbits up to merger. These are among
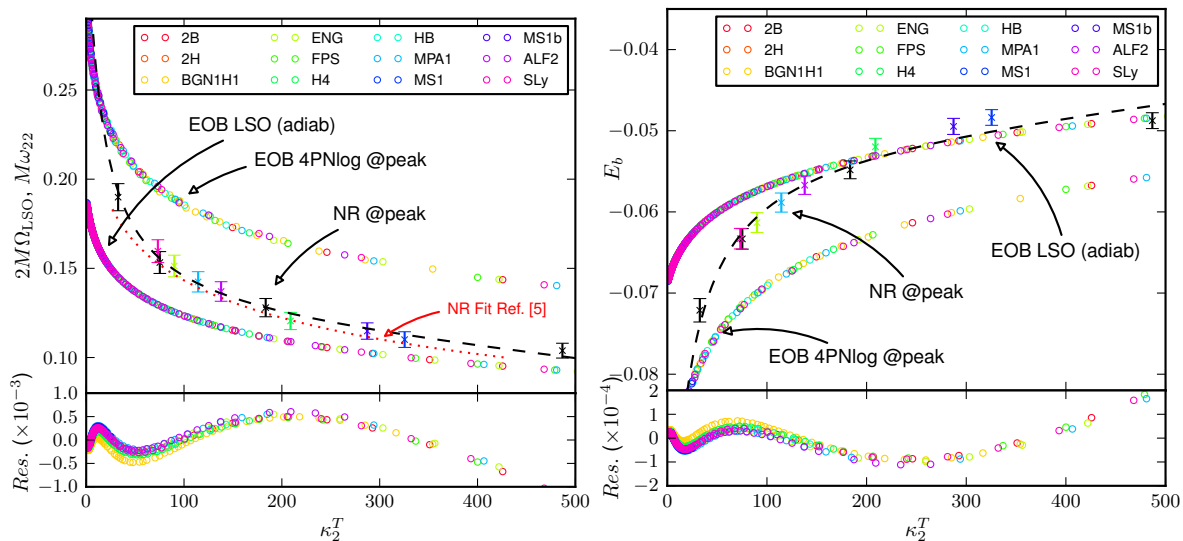
Figure 1. Top panels: GW frequency (left) and binding energy (right) at the LSO (adiabatic EOB) and at merger (nonadiabatic EOB and NR) as a function of κ_2^T for different EOS. NR data are fitted with a function of type $f(0)(1 + n_1\kappa + n_2\kappa^2)/(1 + d_1\kappa)$ (black dashed line). Bottom panels: residuals of the LSO data. The picture is taken from Ref. [12].

the longest BNS simulations ever performed, and some of the few where an error analysis is available [8, 10]. For each NR data set, we compute the binding energy per reduced mass, E_b^{NR}, subtracting the GW energy loss from the initial ADM mass, following [24, 10, 11]. Here, differently from previous works, all the multipoles are included. GW frequency and binding energy are extracted at the moment of merger. We estimate error bars due to truncation errors and waveform finite extraction uncertainties from resolution tests for fewer configurations.

3. Quasiuniversal κ_ℓ^T relations

We investigated the dependence of the GW frequency $2\Omega_{LSO}$ and of the specific binding energy $E_{b\,LSO}$ at merger on the tidal coupling constant κ_2^T for different EOS. Within the EOB approach, we considered 12 realistic EOS and varied the masses from $1.3M_\odot$ up to the maximum allowed mass. Instead, for NR simulations, we fixed the isolation mass $M = 2 \times 1.35 M_\odot$ and considered the set of compactnesses $C_A = C_B = (0.12, 0.14, 0.16, 0.18)$ for the EOS MS1, MS1b, H4, ALF2, MPA1, ENG and SLy. We found that both $2\Omega_{LSO}$ and $E_{b\,LSO}$, once parametrized by the tidal coupling constants, are essentially independent of the EOS. From Fig. 1, which shows the plots for equal mass ratio and no spin, one sees that deviations from universality are below 0.2% in the EOB-generated curves. Because of the structure given by Eq. (1), this behavior is actually not surprising. It is a consequence of the fact that, at the LSO, the term $\kappa_2^T u^6$ dominates over the higher multipoles $\kappa_\ell^T u^{2\ell+2}$, $\ell \geq 3$. As it can be clearly seen, NR results qualitatively confirm this prediction. At the quantitative level, however, there is a gap between EOB and NR that is around $20 - 30\%$ for $2\Omega_{LSO}$ and $10 - 20\%$ for $E_{b\,LSO}$. The reason for this disagreement is due to the fact that our EOB model does not include neither nonlinear tidal interactions, nor hydrodynamic effects. Despite being incomplete, the EOB is able to catch deep physical connections in such a complex process as the merger.

We then used the adiabatic EOB to further investigate the merger, adding small spins $\chi = \pm 0.1$ and considering the additional mass ratio $q = 2$. The curves turn out to be almost independent of the mass ratio (see Fig. 2), despite the fact that the interval of κ_2^T gets narrower when q deviates from 1 (which is simply due to the smaller range of possibilities of picking, in
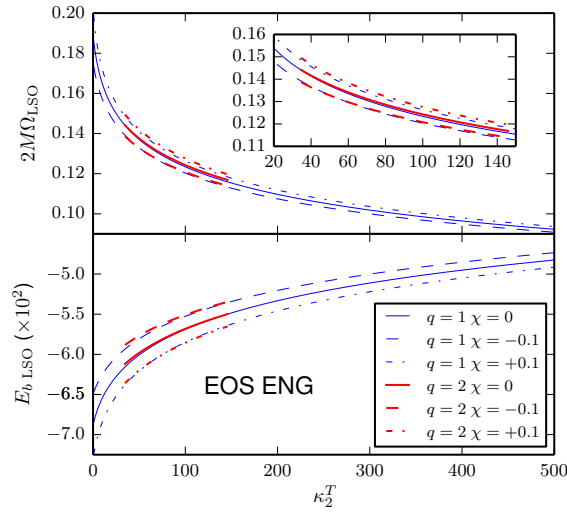
Figure 2. GW frequency (top) and binding energy (bottom) according to the adiabatic EOB model at the LSO, for different spins and mass ratios. The picture is taken from Ref. [12].
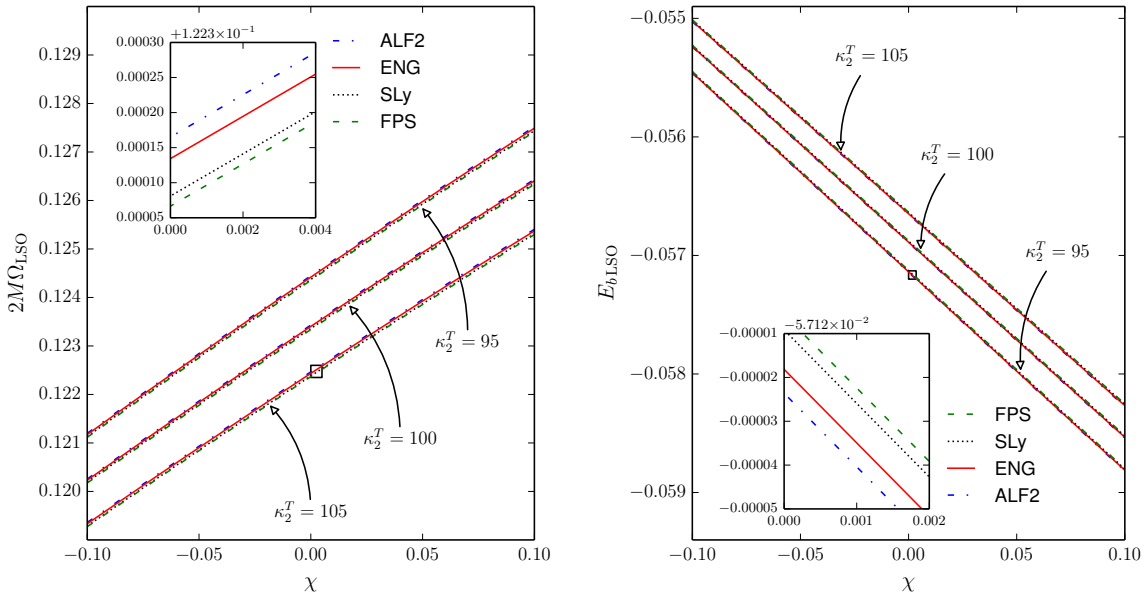

Figure 3. Linear dependence of GW frequency (left) and binding energy (right) on small spins $\chi_1 = \chi_2 \equiv \chi$ ranging from -0.1 to 0.1, for four different EOS. Both quantities are taken at the LSO of the adiabatic EOB model, with the tidal coupling contants fixed at the values $\kappa_2^T = 95, 100, 105$. The small boxes indicate the scale at which deviation from universality shows up.

the allowed interval, two masses with the wished mass ratio). The difference between $q = 1$ and $q = 2$ is $\leq 0.5\%$. By contrast, as it can be seen in Figs. 2-3, the curves are sensitive to the spin, showing an almost linear dependence. Since this implies a certain degeneracy between spin and tidal coupling constant, we denote the relations as *quasi*-universal.

4. Conclusions

We have identified the κ_ℓ^T as fundamental coupling constants of the binary tidal interactions, together with κ^T-universal relations and their physical origin. This provides, for instance, a better insight into the empirical relation found in Ref. [5].

Our findings have implications for GW astronomy. A single measurement of the GW frequency at merger allows to extract the value of κ_ℓ^T. If both masses can be determined from the earlier template, it would be consequently possible to strongly constrain the EOS. In view of this, the κ_ℓ^T relations exhibit certain analogies with the merger properties found in Ref. [25] and with the so called $I - \lambda - Q$ relations [26, 27], see also [28]. Of course, a detailed study about the possibility of constraining the EOS deserves a study on its own.

Finally, the EOB could be used to provide a simple criterion, more precise than the Schwarzschild LSO, for stopping the waveform template.

Acknowledgments

References

[1] Abadie J *et al.* (LIGO Scientific Collaboration, Virgo Collaboration) 2010 *Class.Quant.Grav.* **27** 173001
[2] Aasi J *et al.* (LIGO Scientific Collaboration, Virgo Collaboration) 2013
[3] Read J S, Markakis C, Shibata M, Uryu K, Creighton J D *et al.* 2009 *Phys.Rev.* **D79** 124033
[4] Damour T, Nagar A and Villain L 2012 *Phys.Rev.* **D85** 123007
[5] Read J S, Baiotti L, Creighton J D E, Friedman J L, Giacomazzo B *et al.* 2013 *Phys.Rev.* **D88** 044042
[6] Del Pozzo W, Li T G F, Agathos M, Broeck C V D and Vitale S 2013 *Phys. Rev. Lett.* **111** 071101
[7] Baiotti L, Damour T, Giacomazzo B, Nagar A and Rezzolla L 2010 *Phys. Rev. Lett.* **105** 261101
[8] Bernuzzi S, Thierfelder M and Brügmann B 2012 *Phys.Rev.* **D85** 104030
[9] Hotokezaka K, Kyutoku K and Shibata M 2013 *Phys.Rev.* **D87** 044001
[10] Bernuzzi S, Nagar A, Thierfelder M and Brügmann B 2012 *Phys.Rev.* **D86** 044030
[11] Bernuzzi S, Dietrich T, Tichy W and Bruegmann B 2014 *Phys.Rev.* **D89** 104021
[12] Bernuzzi S, Nagar A, Balmelli S, Dietrich T and Ujevic M 2014 *Phys.Rev.Lett.* **112** 201101
[13] Damour T and Nagar A 2009 The Effective One Body description of the Two-Body problem (*Preprint* `gr-qc/0906.1769`)
[14] Damour T, Nagar A and Bernuzzi S 2013 *Phys.Rev.* **D87** 084035
[15] Damour T 2001 *Phys. Rev.* **D64** 124013
[16] Nagar A 2011 *Phys.Rev.* **D84** 084028
[17] Bini D and Damour T 2013 *Phys.Rev.* **D87** 121501
[18] Damour T and Nagar A 2010 *Phys. Rev.* **D81** 084016
[19] Bini D, Damour T and Faye G 2012 *Phys.Rev.* **D85** 124034
[20] Brügmann B, Gonzalez J A, Hannam M, Husa S, Sperhake U *et al.* 2008 *Phys.Rev.* **D77** 024027
[21] Thierfelder M, Bernuzzi S and Brügmann B 2011 *Phys.Rev.* **D84** 044012
[22] Bernuzzi S and Hilditch D 2010 *Phys. Rev.* **D81** 084003
[23] Hilditch D, Bernuzzi S, Thierfelder M, Cao Z, Tichy W *et al.* 2013 *Phys. Rev.* **D88** 084057
[24] Damour T, Nagar A, Pollney D and Reisswig C 2012 *Phys.Rev.Lett.* **108** 131101
[25] Bauswein A and Janka H T 2012 *Phys.Rev.Lett.* **108** 011101
[26] Yagi K and Yunes N 2013 *Science* **341** 365–368
[27] Yagi K, Stein L C, Pappas G, Yunes N and Apostolatos T A 2014 *Phys. Rev. D* **90**(6) 063010
[28] Maselli A, Cardoso V, Ferrari V, Gualtieri L and Pani P 2013 *Phys. Rev. D* **88**(2) 023007