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Original published in:	Proceedings in applied mathematics and mechanics. - Weinheim [u.a.] : Wiley-VCH. - 20 (2021), 1, art. e202000083, 2 pp.
Conference:	Annual Meeting of the International Association of Applied Mathematics and Mechanics (GAMM) ; 91 (Kassel) : 2020.03.16-20
Original published:	2021-01-25
ISSN:	1617-7061
DOI:	10.1002/pamm.202000083
[*Visited:*	2022-03-25]

Received: 17 June 2020 | Accepted: 24 November 2020

DOI: 10.1002/pamm.202000083

Dynamics of a magnetic pendulum in the presence of an oscillating conducting plate

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A pendulum with an attached permanent magnet moving near a conductor is a typical experiment for the demonstration of electromagnetic braking. When the conductor itself moves, it can transfer energy to the pendulum. We study a simple but exact analytical model where the conductor is a horizontally unbounded flat plate. For this geometry, eddy currents and induced Lorentz force due to the motion of a magnetic dipole are known analytically in the quasistatic limit. A vertical oscillation of such a horizontal plate located beneath the magnet is considered. In this setup, the vertical position of the pendulum is an equilibrium point when the magnetic moment of the magnet is perpendicular to its plane of motion. Depending on the strength of the magnetic dipole moment, the frequency and amplitude of the plate as well as the distance between plate and magnet, the plate oscillation can destabilize the equilibrium. The stability limits for weak electromagnetic coupling are computed analytically using the harmonic balancing method. For stronger coupling, the stability limits are obtained numerically using Floquet analysis. Chaotic motions with finite amplitudes are also found.

1 Introduction

Our work is motivated by the analytical results obtained in Ref. [1] for the induced eddy currents and resulting Lorentz force and torque when a magnetic dipole moves slowly along or rotates slowly about an axis that is parallel to the surface of an unbounded flat plate. For slow motion, the electromagnetic induction equation (derived from Maxwell's and Ohm's laws) simplifies considerably since magnetic diffusion and stretching terms dominate. The induced currents, forces and torque are then linear functions of the magnet's translational or angular velocity. This so-called quasistatic limit applies when the magnetic Reynolds number is small [2]. The work in Ref. [1] was in part motivated by the problem of contactless inductive flow measurement in conducting liquids, e.g. molten metals. One particular method is Lorentz force velocimetry (LFV), where a magnet is placed next to the flow and the induced drag force on the magnet is measured [3]. A drag force must be present since Lenz' law states that electromagnetic induction opposes its cause, i.e. the relative motion between conducting material and the magnetic field. Since only the relative motion is important for the induction, the dipole considered in Ref. [1] can also be regarded as fixed in space while the plate is moving. It then represents a parallel conducting flow with uniform velocity. With a view to LFV, more complicated velocity distributions unfortunately do not permit analytical solutions. We have therefore decided to generalize the problem to relative motions between plate and magnet beyond those relevant for LFV, which leads naturally to the consideration of a magnetic pendulum as sketched in Fig. 1. For the additional vertical motion between plate and dipole, one can also determine the induced currents, forces and torques analytically in the quasistatic limit. However, a fixed plate only causes a damped motion of the pendulum. We therefore assume that the plate oscillates vertically and that the dipole moment is perpendicular to the plane of motion. These choices give rise to a parametric resonance behavior. The magnet is assumed to be small so that its size can be neglected. The plate surface corresponds to $z_p = l + P + Q \sin \omega t$. The nondimensional eigenfrequency and geometry parameters of the problem are

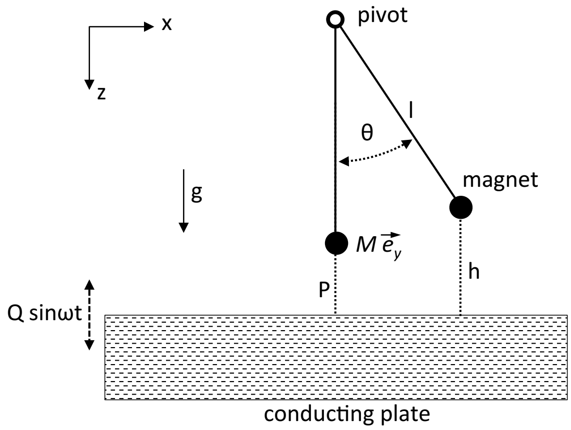

Fig. 1: Sketch of the magnetic pendulum with eigenfrequency $\omega_0 = \sqrt{|g|/l}$. The magnet with mass m_b and dipole moment M is attached to a massless inflexible rod of length l. The conducting plate with conductivity σ vibrates with angular frequency ω and amplitude Q in z-direction. The pivot point is at the origin of the coordinate system. P denotes the vertical distance between magnet and plate in equilibrium when $Q = 0$.

$$A = \frac{4\omega_0^2}{\omega^2}, \qquad B = \frac{2Q}{l}, \qquad S = \frac{2P}{l}.$$

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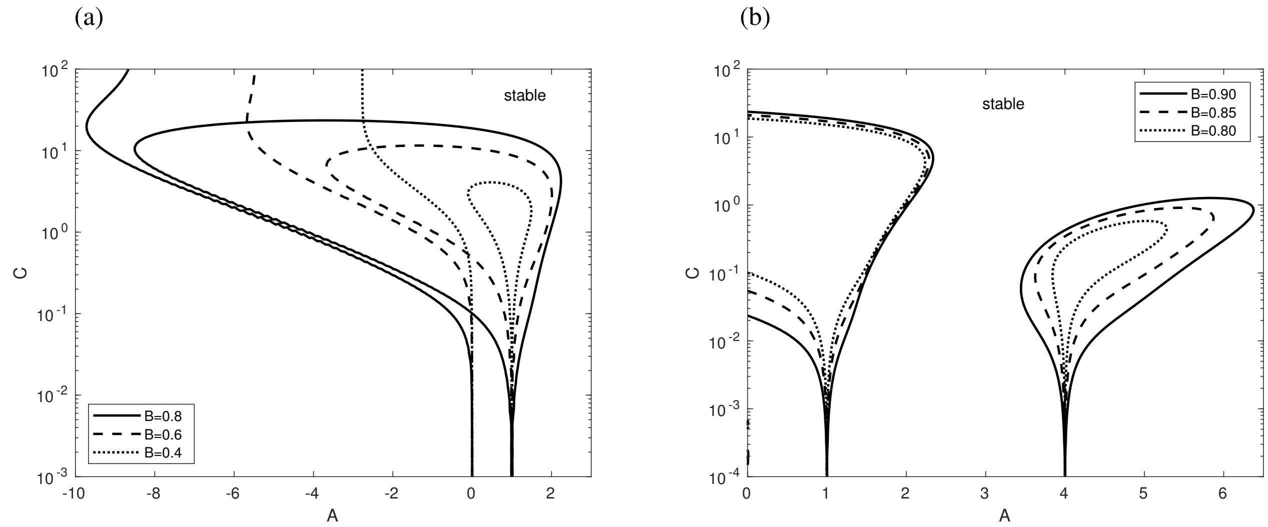

Fig. 2: Regions of instability in the (A, C)-plane for $S = 1$ from numerical Floquet analysis.

2 Equation of motion and stability results

The pendulum experiences a torque along the z-direction generated by the Lorentz force acting on the magnet. For the chosen orientation of the dipole moment there is no electromagnetic torque on the magnet itself. The nondimensional equation of motion is

$$\ddot{\theta} + \dot{\theta}\, C \frac{1 + 3\sin^2\theta}{2\left(2 - 2\cos\theta + S + B\sin 2t\right)^3} + \left(A + C\frac{2B\cos 2t}{\left(2 - 2\cos\theta + S + B\sin 2t\right)^3}\right)\sin\theta = 0, \quad C = \frac{\mu_0^2 M^2 \sigma}{16\pi m_b l^3 \omega}, \tag{1}$$

where the parameter C characterizes the ratio between electromagnetic and inertial forces. Time is nondimensional with unit $2/\omega$. To avoid contact between plate and magnet and a corresponding singularity in the coefficients, the constraint $S > B$ is imposed. To examine the stability, we linearize eq. (1) about $\theta = 0$. The linearized equation is similar to the classical Mathieu equation with linear viscous damping but both the stiffness coefficient and the damping coefficient are modulated by anharmonic functions. For small C one can use the harmonic balance method described in Ref. [4] to find conditions for instability. As for the classical damped Mathieu equation, instability occurs only near the resonance cases $A = n^2$ with integer n. In addition, analytical constraints for S and B can be derived from the Fourier expansion of the modulated stiffness and damping coefficients. These constraints agree with results obtained by a standard numerical Floquet analysis using Matlab. For $S = 1$, subharmonic instability ($A = 1$) becomes possible when $B > 0.32$ and harmonic instability ($A = 4$) when $B > 0.77$. Fig. 2 illustrates that the unstable intervals for A grow with B when C is small. There is also an upper limit for C at given B, i.e. the damping effect ultimately dominates.

The stabilization of the inverted pendulum ($\theta = \pi$) is also characterized by the linearized form of eq. (1) for negative A. In this case, the plate is located above the pivot point. The lower stability limit $A_{min} \leq 0$ in Fig. 2(a) decreases with increasing B and C except when C is very large, i.e. both effects tend to stabilize the inverted position. The behavior for finite θ requires a numerical solution of the nonlinear eq. (1). We have considered the parameters $B = 0.6$ and $S = 1$. Multiple solutions and chaotic motion occur for $0 < A \lesssim 0.3$ and $C \gtrsim 1$.

More results including a derivation of eq. (1), consideration of other orientations of the magnetic dipole moment and bifurcation diagrams of the chaotic solutions are provided in our paper [5] recently submitted for publication.

Acknowledgements We are grateful to Daniel Hernandez for verifying the analytical results for the induction problem with COMSOL. Tatiana Becker acknowledges financial support by the Deutsche Forschungsgemeinschaft (grant BE 6553/1-1).
Open access funding enabled and organized by Projekt DEAL.

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