SPARROW Applications Using Bayesian Inference Techniques

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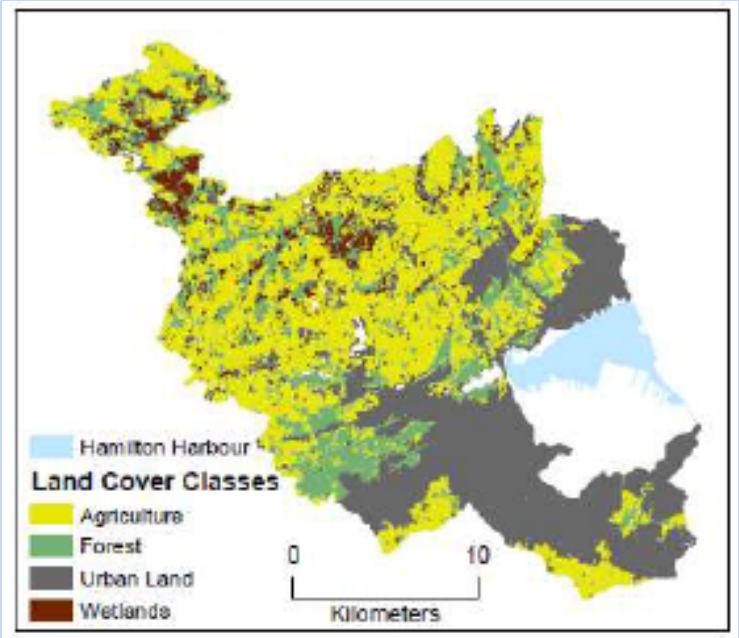
SPARROW model

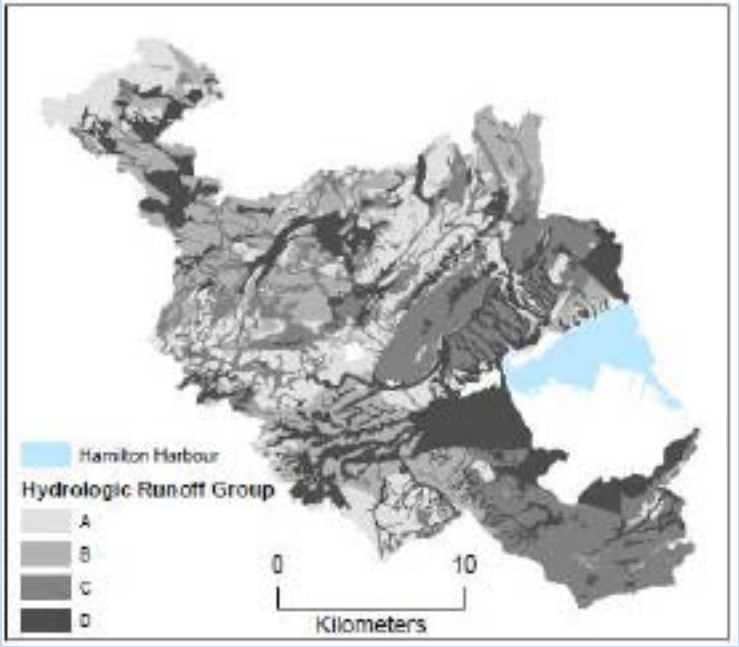
$$\overline{MAL_i} = \left(\sum_{n=1}^{N} \sum_{j=1}^{J_i} \beta_n S_{n,j} e^{(-\alpha Z_j)} H_{i,j}^{S} H_{i,j}^{R} \right)$$

Nutrient Export Coefficients

Delivery Factors

Stream and Reservoir Attenuation Terms





SPARROW application in small watersheds with limited information

- **How can we address the knowledge gaps from the studied system?**

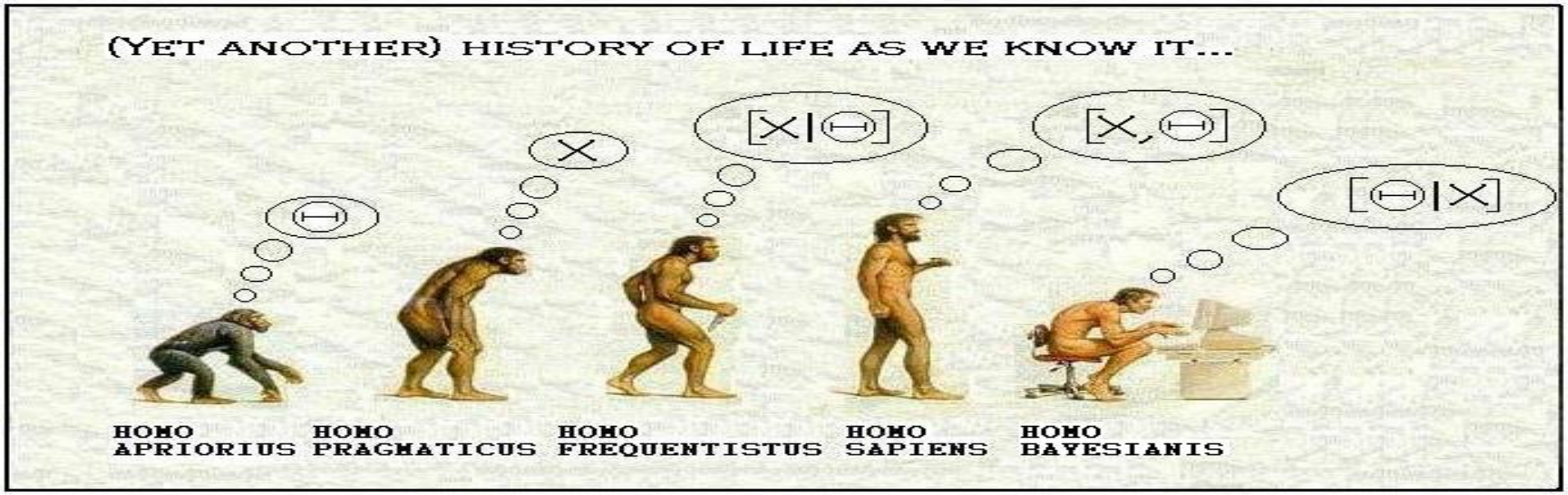
- **How can we explicitly accommodate the uncertainty pertaining to our dataset?**

Bayesian Approach

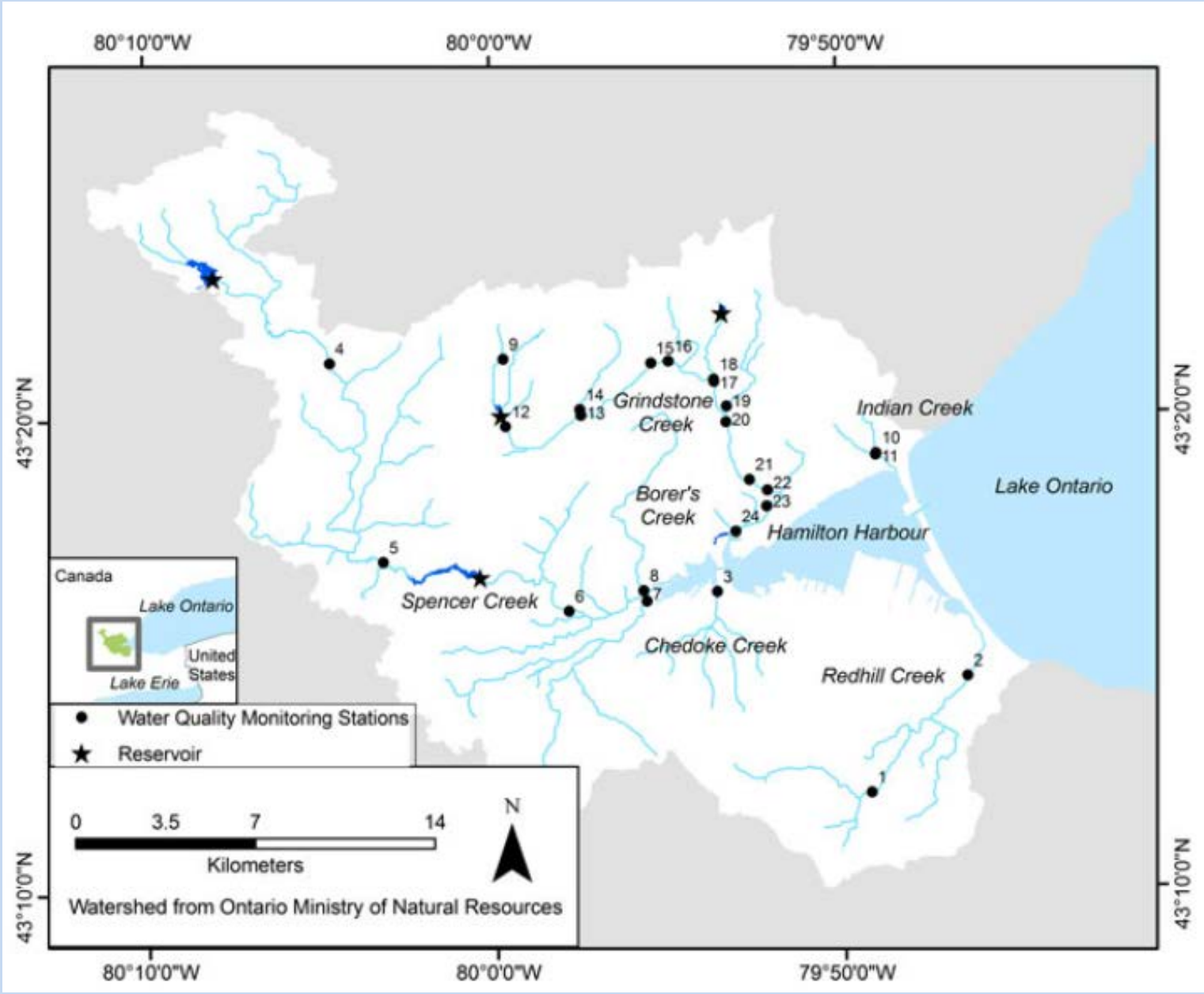
In modeling context:

$$P(\text{Future}|Data) = \frac{P(Data|Model)\,P(Model)}{P(Data)}$$

$$\propto P(\text{Present}|Model)\,P(\text{Past})$$

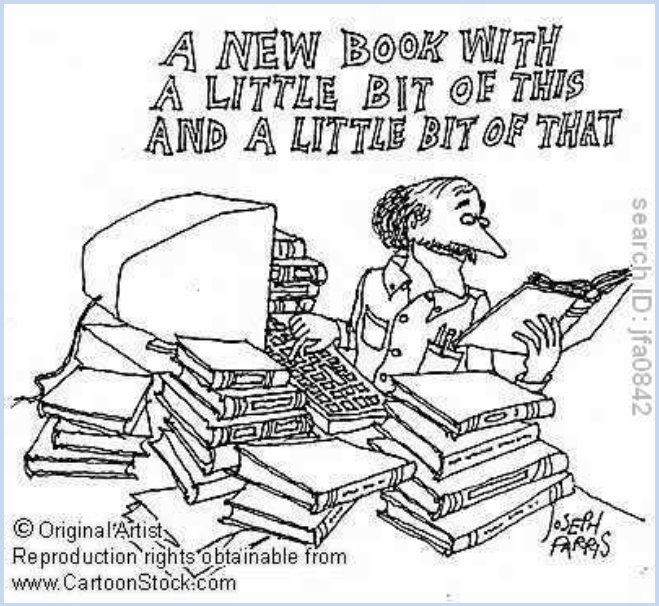


Space versus Time

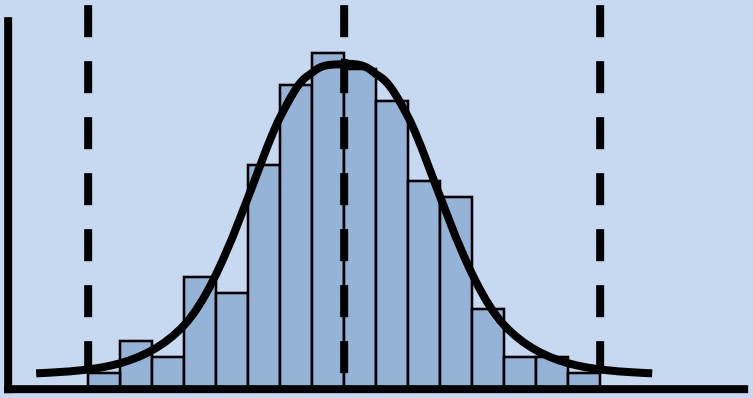


Informative priors

Prior probability



Plausible literature range of parameters (min, max)



Assignment of a probability distribution
(e.g., normal, lognormal form)

Measurement Error Model

$$Y_i \sim N(Load_i,\ \delta_i^2)$$

$$Load_i \sim N(\mu_i,\ \sigma^2)$$

$$\mu_i = \overline{MAL}_i = \ln\left(\sum_{n=1}^{N} \sum_{j=1}^{J_i} \beta_n S_{n,j} e^{(-\alpha Z_j)} H_{i,j}^{S} H_{i,j}^{R} \right)$$

Observed load ~ *N*(“Error-free” load, Measurement error)

“Error-free” load ~ *N*(Predicted load, Structural error)

Predicted load = SPARROW model

Parameter Identification

Parameters	Conventional SPARROW		Non-informative Priors		Informative Priors	
	Mean	SD	Mean	SD	Mean	SD
α	6.53	3.37	8.63	4.09	0.46	0.76
β_1	2.95	3.69	6.76	7.28	0.19	0.14
β_2	10.83	13.16	16.22	15.96	0.10	0.09
k_r	17.27	19.57	17.37	19.69	13.03	4.25
k_s	0.19	0.07	0.17	0.12	0.05	0.04
σ	0.75	0.14	0.65	0.19	0.40	0.14
Deviance	56.59	5.48	52.57	5.05	47.19	4.25
DIC	46.74		64.06		55.07	
RMSE	0.67		0.68		0.60	
WRMSE	1.03		0.83		0.51	

Observed

Predicted

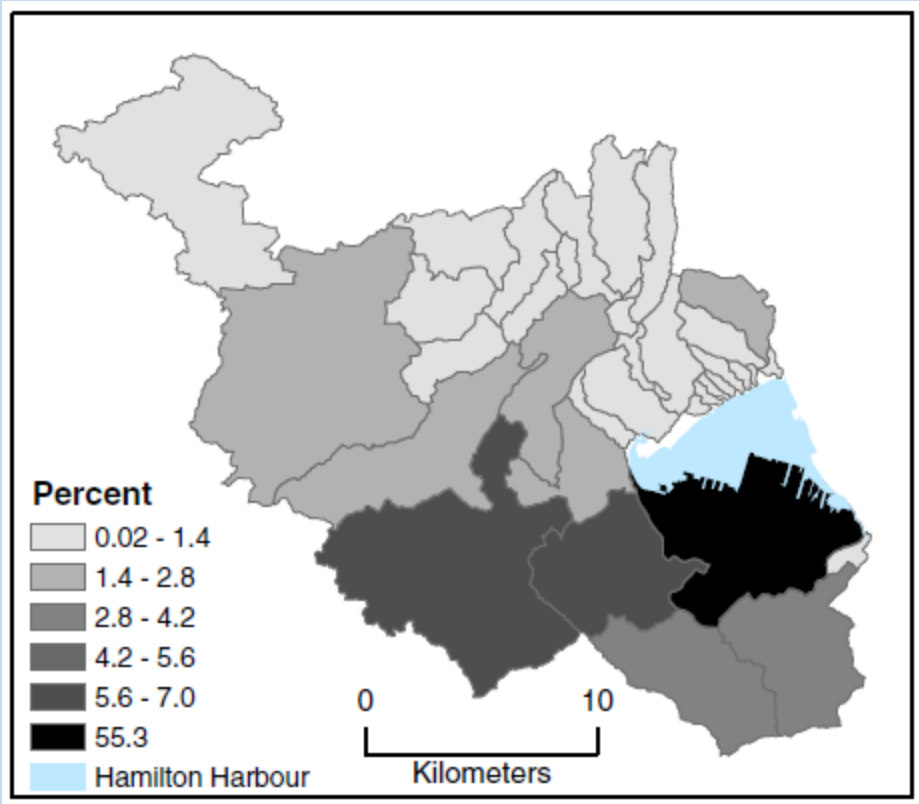
0.23 - 0.54
0.54 - 1.06
1.06 - 2.10
2.10 - 4.10
4.10 - 8.00

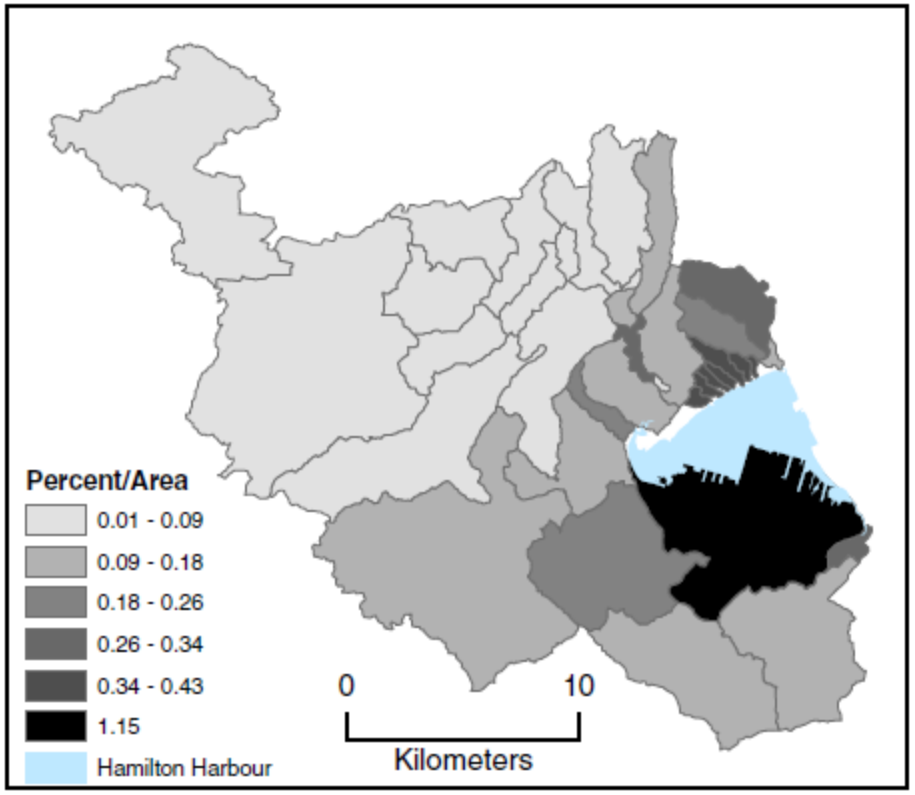
Tons year^{-1}

Annual P loading

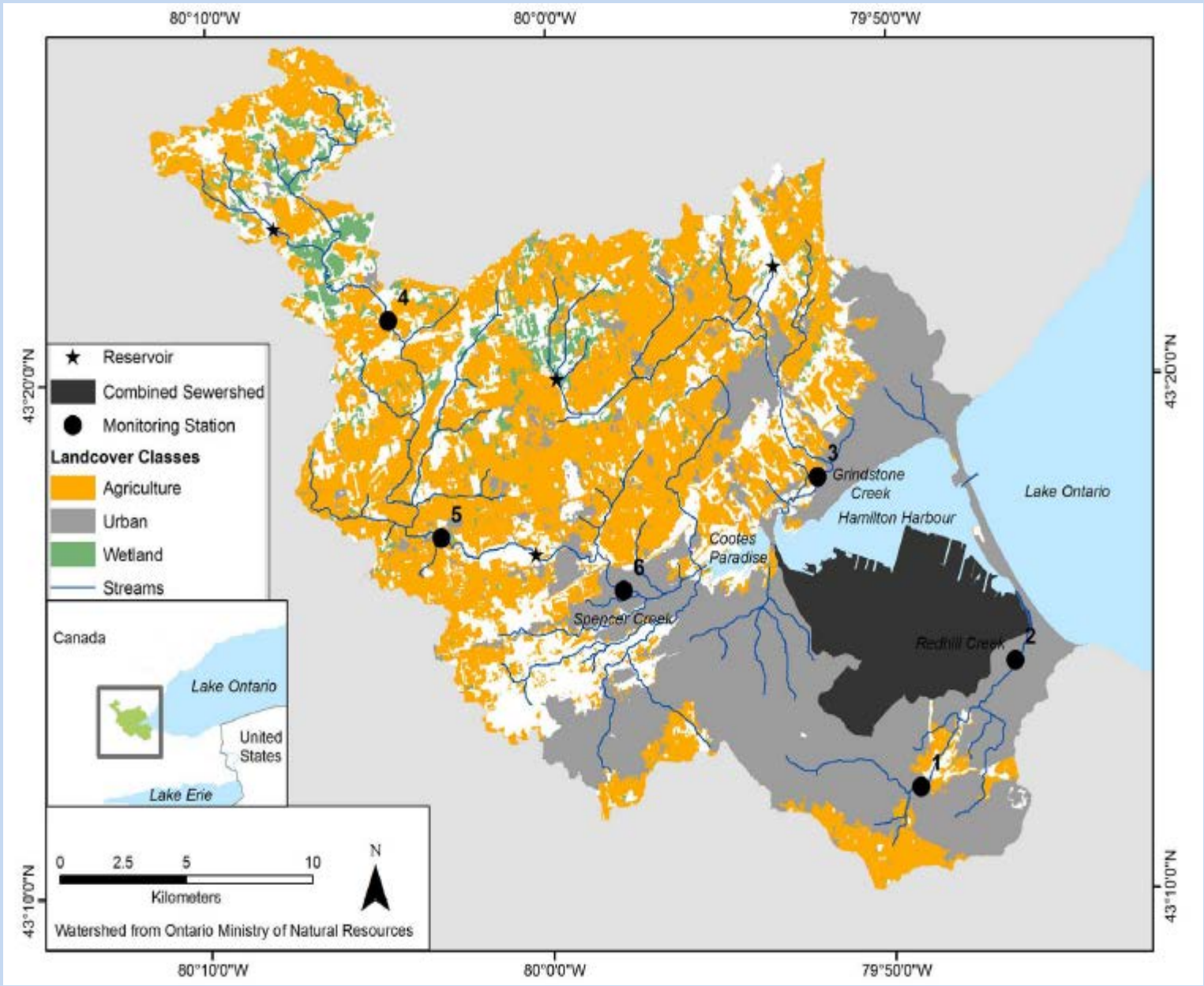
Standard deviations

Estimated contribution of each subwatershed to the total phosphorus loading in Hamilton Harbour





Space versus Time



Accommodating temporal variability

$$Y_{i,t} \sim N(Load_{i,t}, \delta^2_{i,t})$$

$$Load_{i,t} \sim N(\mu_i + W_{v,t}\gamma_v, \sigma^2)$$

$$\mu_i = \ln\left(\sum_{n=1}^{N}\sum_{j=1}^{J_i}\beta_n S_{n,j} e^{(-\alpha Z_j)} H^S_{i,j} H^R_{i,j}\right)$$

$$\sigma^{-2} \sim gamma(0.001, 0.001)$$

Observed load ~ *N*(“Error-free” load, Measurement error)

“Error-free” load ~ *N*(Predicted load, Structural error)

Predicted load = SPARROW model +∑ coefficient*meteorological variable

Accommodating temporal variability

$$Y_{i,t} \sim N(Load_{i,t}, \delta_{i,t}^2)$$

$$Load_{i,t} \sim N(\mu_i + W_{v,t}\gamma_v + v_t, \sigma^2)$$

$$\mu_i = \ln\left(\sum_{n=1}^{N}\sum_{j=1}^{J_i} \beta_n S_{n,j} e^{(-\alpha Z_j)} H_{i,j}^S H_{i,j}^R\right)$$

$$v_t | v_{-t} \sim \begin{cases} N(v_{t+1}, \psi^2) & \text{for} \quad t = 1 \\ N\left(\dfrac{v_{t-1} + v_{t+1}}{2}, \dfrac{\psi^2}{2}\right) & \text{for} \quad t = 2, \ldots, T-1 \\ N(v_{t-1}, \psi^2) & \text{for} \quad t = T \end{cases}$$

$$\sigma^{-2}, \psi^{-2} \sim gamma(0.001, 0.001)$$

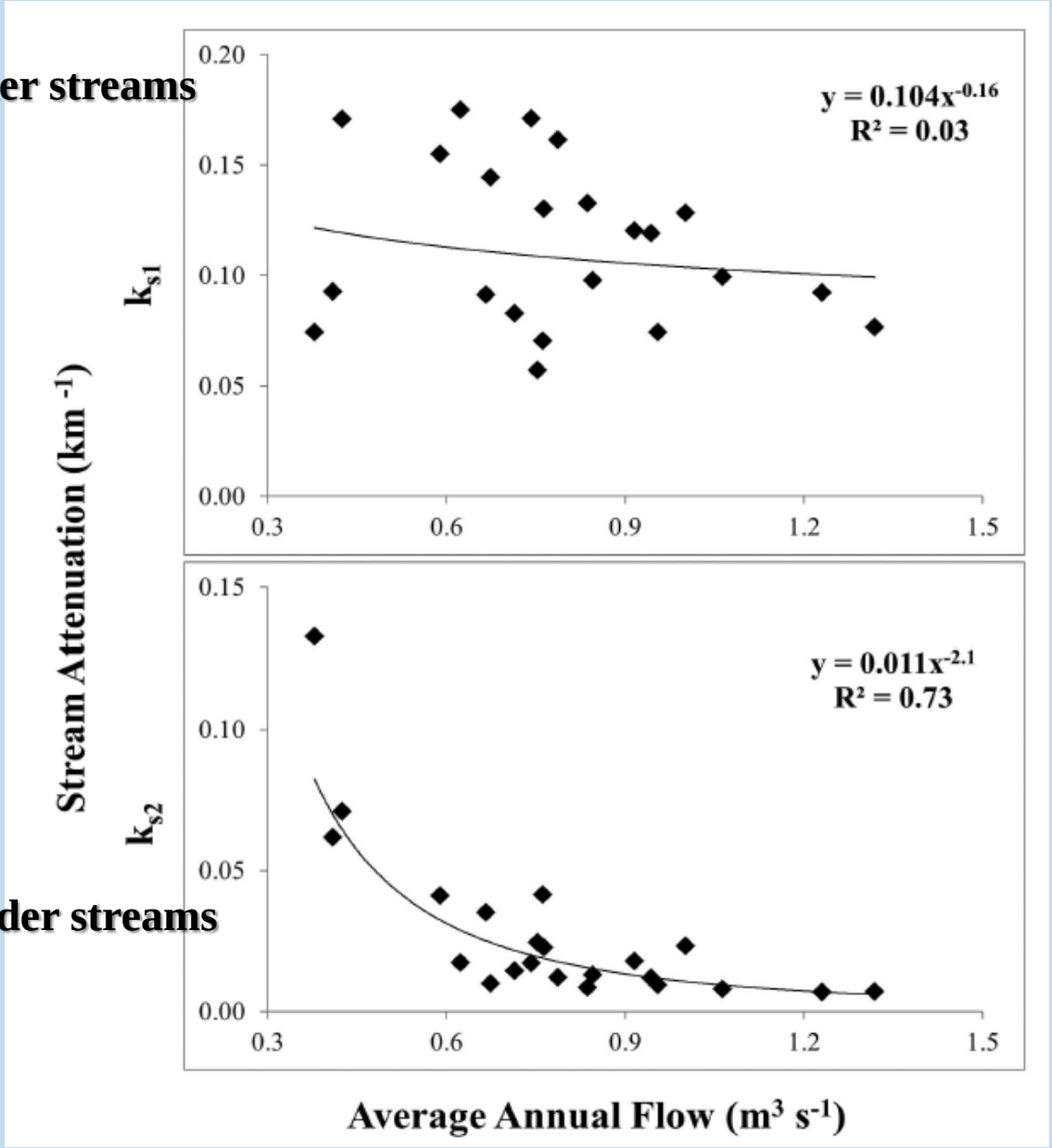
Observed load ~ *N*(“Error-free” load, Measurement error)

“Error-free” load ~ *N*(Predicted load, Structural error)

Predicted load = SPARROW model +∑ coefficient*meteorological variable + Conditional AutoRegressive (time variant error) term

Stream attenuation rates

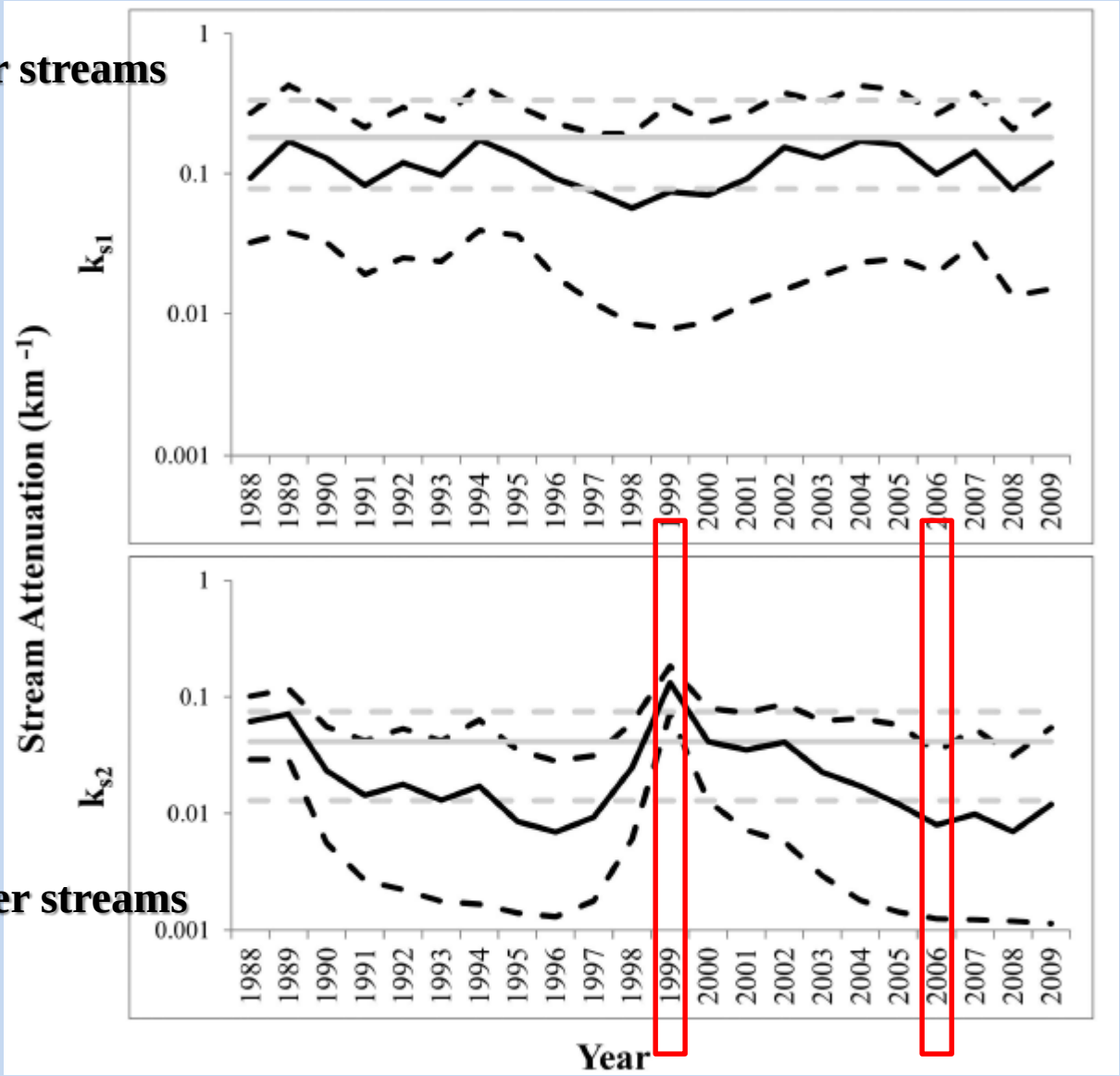
First/Second order streams



Third/higher order streams

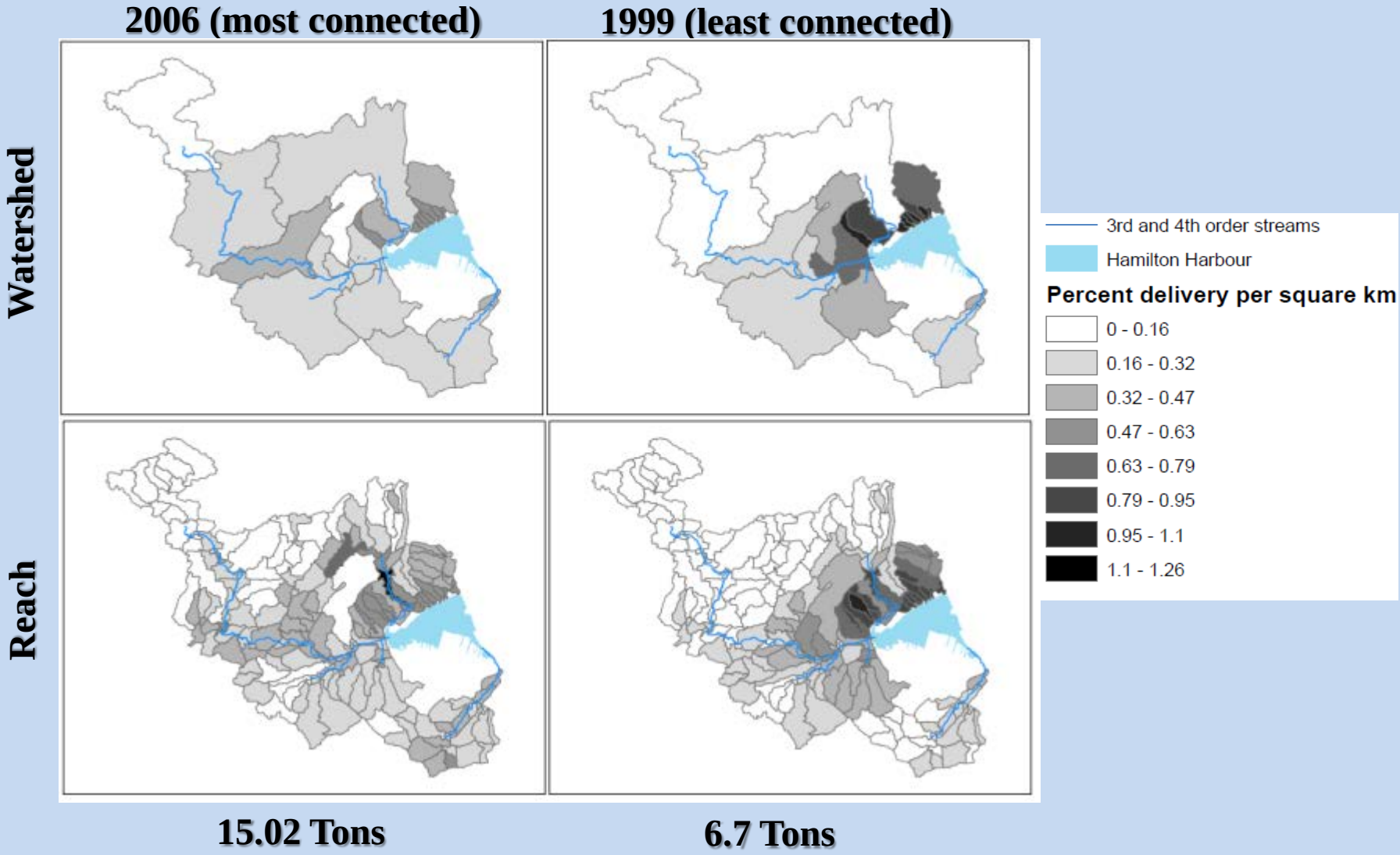
Stream attenuation time series

First/Second order streams

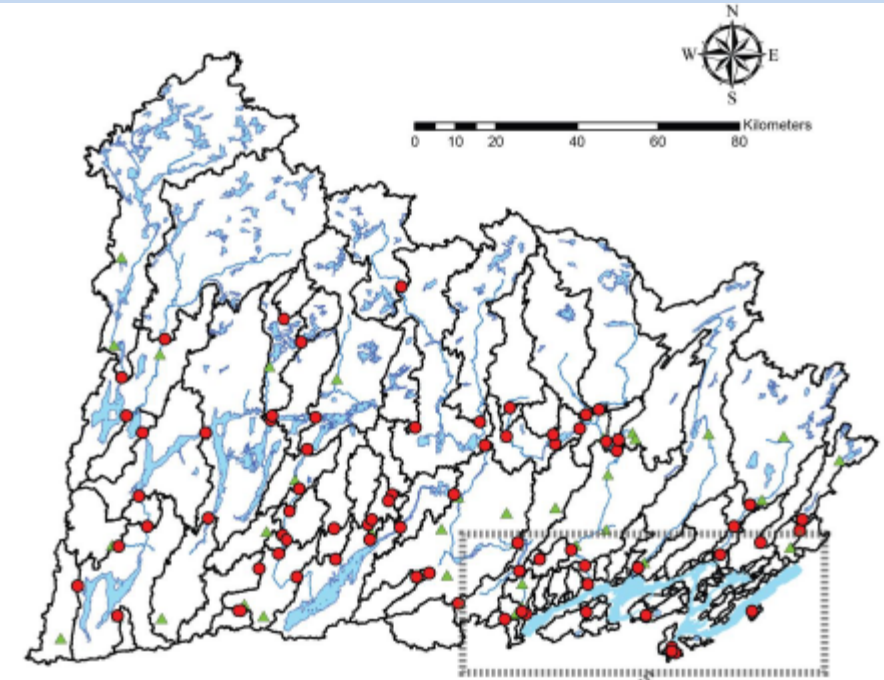


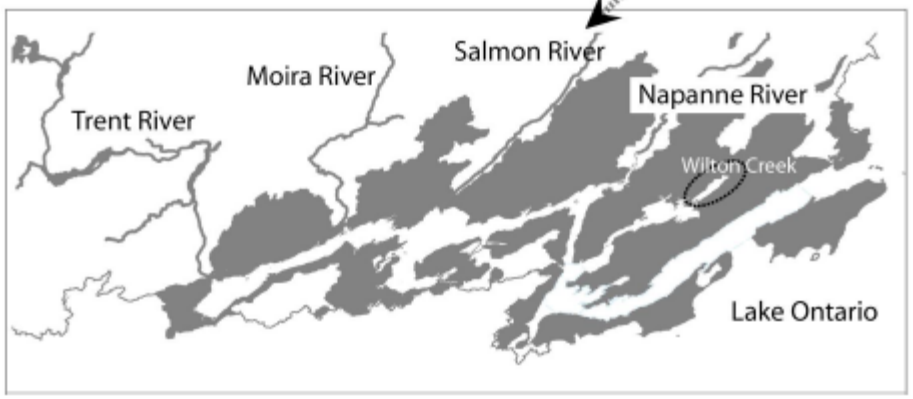
Third/higher order streams

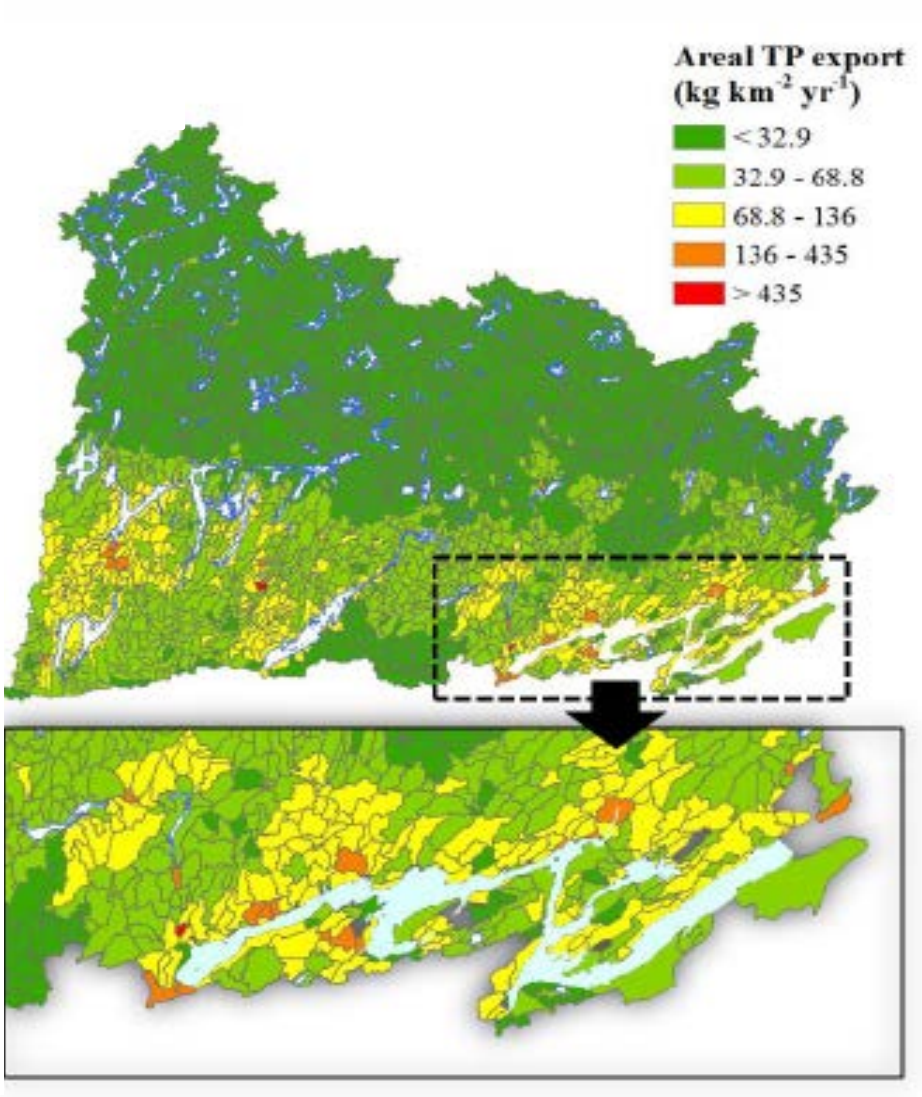
Most connected vs least connected year - TP



Determining the Optimal Model Complexity







Determining the Optimal Model Complexity

$$\overline{MAL_i} = \left(\sum_{n=1}^{N} \sum_{j=1}^{J_i} \beta_n S_{n,j} e^{(-\alpha Z_j)} H_{i,j}^{S} H_{i,j}^{R} \right)$$

- TP export coefficient (cropland)
- TP export coefficient (forest)
- TP export coefficient (pasture)
- TP export coefficient (urban area)
- TP export coefficient (septic tank)

- TP export coefficient (cotton)
- TP export coefficient (corn)
- TP export coefficient (alfalfa)
- TP export coefficient (wheat)
- TP export coefficient (fallow)

SPARROW posterior patterns

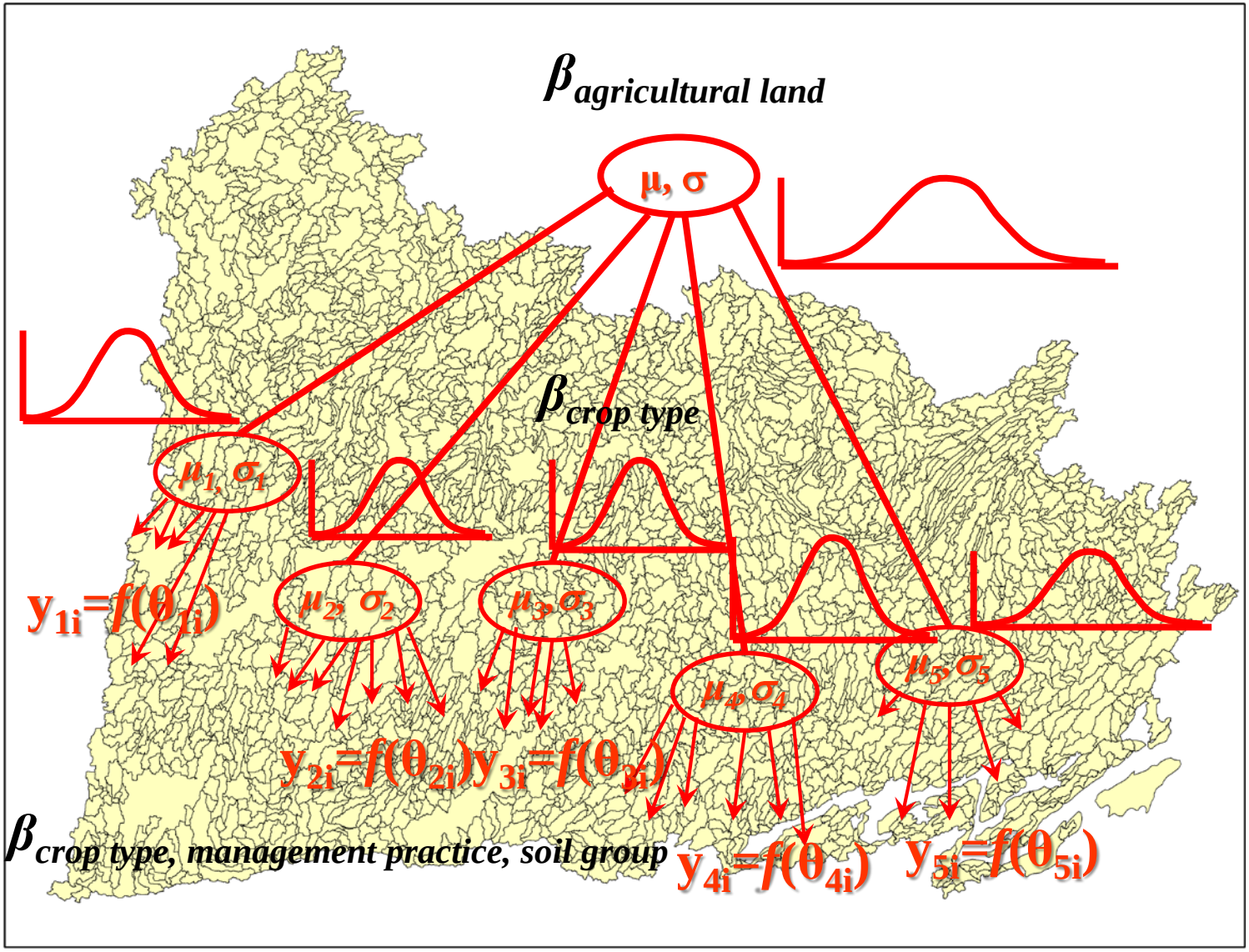
		SPARROW (Global export coeff.)					SPARROW (5 crop-based export coeff.)				
Parameter	unit	mean	sd	2.5%	median	97.5%	mean	sd	2.5%	median	97.5%
alpha	h/cm	0.190	0.065	0.064	0.187	0.320	0.210	0.060	0.097	0.210	0.332
beta[1]	ton/km^2	0.034	0.013	0.014	0.031	0.064					
wheat	ton/km^2						0.074	0.030	0.030	0.069	0.142
oat	ton/km^2						0.131	0.036	0.074	0.128	0.212
corn	ton/km^2						0.041	0.023	0.011	0.036	0.101
alfalfa	ton/km^2						0.026	0.009	0.012	0.024	0.049
fallow	ton/km^2	0.034	0.013	0.014	0.031	0.064	0.072	0.039	0.023	0.063	0.169
beta[2]	ton/km^2	0.010	0.003	0.005	0.009	0.017	0.012	0.003	0.006	0.011	0.020
beta[3]	ton/km^2	0.026	0.012	0.008	0.024	0.054	0.032	0.015	0.010	0.029	0.067
beta[4]	ton/km^2	0.119	0.082	0.025	0.098	0.331	0.123	0.086	0.025	0.100	0.352
beta[5]	ton/tank	0.001	0.000	0.000	0.001	0.002	0.001	0.001	0.000	0.001	0.003
beta.point	-	1.051	0.307	0.452	1.047	1.639	1.043	0.306	0.442	1.034	1.653
kr	m/yr	2.952	1.332	0.695	2.853	5.760	3.554	1.465	0.915	3.456	6.752
ks	km^{-1}	0.002	0.004	< 0.001	0.002	0.009	0.004	0.004	< 0.001	0.004	0.011

Two additional layers

1. **Management Practice**
   - ***no-tillage***
   - ***conservational tillage (surface)***
   - ***conventional tillage (soil)***

2. **Hydrologic Soil Group**
   - ***A: (very) rapidly drained***
   - ***B: (moderately) well drained***
   - ***C: imperfectly and poorly drained***
   - ***D: very poorly drained***

Multilevel/hierarchical model



Hierarchical SPARROW result

Wheat

Case			unit	SPARROW (5 export coeff.) mean	sd	2.5%	median	97.5%	SPARROW (45 export coeff.) mean	sd	2.5%	median	97.5%
	no	B		0.074	0.030	0.030	0.069	0.142	0.067	0.009	0.050	0.067	0.087
		C							0.111	0.028	0.067	0.108	0.175
		D							0.022	0.012	0.007	0.019	0.052
	surface	B							0.157	0.128	0.030	0.121	0.498
wheat		C	ton/km^2						0.078	0.019	0.047	0.076	0.121
		D							0.025	0.005	0.017	0.025	0.035
	soil	B							0.126	0.533	0.003	0.046	0.708
		C							0.248	0.080	0.125	0.237	0.435
		D							0.167	0.026	0.122	0.166	0.222

Hierarchical SPARROW result

Corn

Case			unit	SPARROW (5 export coeff.) mean	sd	2.5%	median	97.5%	SPARROW (45 export coeff.) mean	sd	2.5%	median	97.5%
	no	B		0.041	0.023	0.011	0.036	0.101	0.007	0.009	0.001	0.005	0.031
		C							0.113	0.007	0.100	0.113	0.128
		D							0.074	0.135	0.003	0.034	0.404
	surface	B							0.041	0.077	0.002	0.020	0.209
corn		C	ton/km^2						0.089	0.004	0.082	0.089	0.096
		D							0.079	0.158	0.003	0.035	0.429
	soil	B							0.086	0.112	0.007	0.052	0.385
		C							0.133	0.104	0.026	0.104	0.397
		D							0.077	0.145	0.003	0.035	0.413

Acknowledgements

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Ontario

MINISTRY OF RESEARCH & INNOVATION

NATIONAL WATER RESEARCH INSTITUTE

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