APPLICATION OF THE MATHEMATICAL METHODS TO INVESTIGATION OF DYNAMICAL PROPERTIES OF A CABLE

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Abstract. *The paper is devoted to the investigation of dynamical behavior of a cable under influence of various types of excitations. Such element has a low rigidity and is sensitive to dynamic effect.*

The structural scheme is a cable which ends are located at different level.

The analysis of dynamical behavior of the cable under effect of kinematical excitation which is represented by the oscillations of the upper part of tower is given. The scheme of cable is accepted such, that lower end of an inclined cable is motionless. The motion of the upper end is assumed only in horizontal direction.

The fourth-order Runge-Kutta method was realized in software. The fast Fourier transform was used for spectral analysis. Standard graphical software was adopted for presenting results of investigations.

The mathematical model of oscillations of a cable was developed by the account of the viscous damping.

The analysis of dynamical characteristics of a cable for various parameters of damping and kinematical excitation was carried out. The time series, spectral characteristics and amplitude-frequencies characteristics was obtained. The resonance amplitude for different oscillating regimes was estimated. It is noted that increasing of the coefficient of the viscous damping and decreasing of the amplitude of tower's oscillations reduces the value of the critical frequency and the resonant amplitudes.

1 INTRODUCTION

The suspended and cable bridges have a great variety of forms, geometrical and rigid characteristics. Decreasing of the own weight and rising of the flexibility of modern bridge's elements makes their more sensitive to dynamic affecting.

The dynamic behavior of cable bridges was investigated in the works [1] and [2]. Also, in-the books [1] deformation calculation of the tower was observed, and examples of bridges which had significant oscillations of the towers were given.

The oscillations of the flexible elements of structures with the account of a friction in their ends were observed in the work [3].

Given paper is devoted to the investigation of dynamical behavior of a cable under effect of kinematical excitation which is represented by the oscillations of the upper part of tower. Before, in the work [4], oscillations of a string which ends are located at same height under the kinematical excitation were investigated. The investigation of the free oscillations of the cable was observed in the work [5]. The development of cable damper with high damping rubber bearing for cable stayed bridge was shown in the work [6]. The bending of stay cables was investigated in the work [7].

2 THE BASIC PROBLEMS

We will observe the cable which ends are located at different level. The upper part of the tower is oscillating in horizontal direction.

It is necessary:

- to investigate dynamical behavior of the cable under the influence of the kinematical excitation;
- to find spectral and amplitude-frequency characteristics of the oscillations;
- to analyze the influence of parameters of a damping and tower's oscillations on dynamical behavior of the cable.

3. THE DIFFERENTIAL EQUATION OF THE CABLE'S OSCILLATIONS UNDER EFFECT OF KINEMATICAL EXCITATION

We will investigate transverse oscillations of the cable which structural scheme is presented in Fig. 1. The scheme of cable is accepted such, that lower end is motionless, and the upper end is moving at horizontal direction. The oscillations can be presented in the form of the second order equation in the partial derivatives:

$$\frac{\partial}{\partial x}\left(T_0\frac{\partial v}{\partial x}+\frac{EA}{l}\left(\frac{dy}{dx}+\frac{\partial v}{\partial x}\right)\left(\int_0^l\left[\frac{\partial è}{\partial õ}+\frac{dy}{dx}\frac{\partial v}{\partial x}+\frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2\right]dx\right)\right)=m\ddot{v}\,, \tag{1}$$

where T_0 – the maximum extension force in cross-section of the cable; l – chord length (Fig. 1); E – modulus of elasticity of cable's material; m –unit mass of cable; $y(x)$ – the equation of a sag's curve; $\nu(x,t)$ – vertical displacement of local coordinate system, u – component along x-axis of horizontal displacement of a tower.

Let's suppose, that upper part of a tower makes horizontal oscillations and their component along a x-axis was only considered.

The integration $\dfrac{\partial u}{\partial x}$ on x at constant t on all length of the cable at x from 0 up to l gives us the next equation:

$$\frac{\partial}{\partial x}\left(T_0\frac{\partial v}{\partial x}+\frac{EA}{l}\left(\frac{dy}{dx}+\frac{\partial v}{\partial x}\right)\left(\grave{e}(l,t)-\grave{e}(0,t)+\int_0^l\left[\frac{dy}{dx}\frac{\partial v}{\partial x}+\frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2\right]dx\right)\right)=m\ddot{v},$$

where $u(x,t)$ – displacement of cable's points along x-axis.

Let's express longitudinal oscillations of the upper end $u(t)$ through the horizontal oscillations of an upper part of the tower:

$$u(t)=s(t)\cos\alpha.$$

We accept the equation of the horizontal oscillations of an upper part of the tower in the form of:

$$s(t)=S\cos\omega t, \qquad (2)$$

where S – an amplitude of oscillation; ω – an oscillation frequency.

We assume that the bottom point of cable's sag is outside of it's span (on significant distance from the left end and will accept the equation sage in the form of:

$$y(x)=Y\sin\frac{\pi x}{l}, \qquad (3)$$

where Y – the maximum value of a sag.

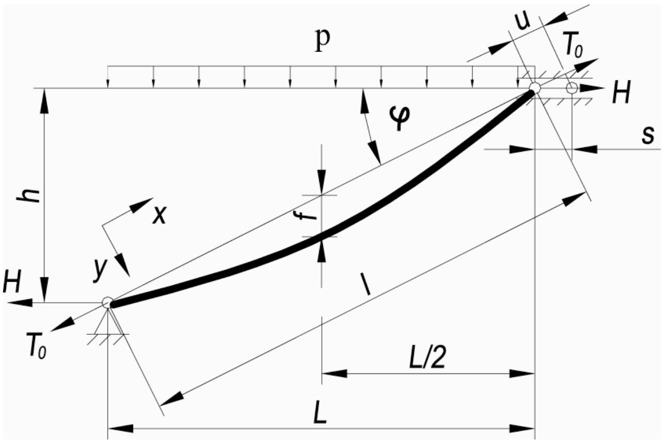

Fig.1. The structural scheme of the cable

We represent function $\nu(x,t)$ in the form of:

$$v(x,t)=\sum_{n=1}^{\infty}q_n(t)\sin\frac{\pi n x}{l}. \qquad (4)$$

By substituting the above-mentioned responses into Eq. (1) and executing the necessary transformations, we obtain the next equation where $n = 1$:

$$\ddot{q} + \alpha q + \gamma q^2 + \beta q^3 + c(q + Y) = 0\,, \tag{5}$$

where $\alpha = \left(T_0 \frac{\pi^2}{l^2} + \frac{1}{2} EAY^2 \frac{\pi^4}{l^4}\right)/m$; $\gamma = \frac{3}{4}\frac{YEA}{m}\frac{\pi^4}{l^4}$; $\beta = \frac{1}{4}\frac{EA}{m}\frac{\pi^4}{l^4}$;

$$\tilde{n} = \frac{S}{m}\cdot\cos\omega t\cdot\cos\varphi\cdot \text{ÅÀ}\frac{\pi^2}{l^3}\ .$$

Adding elastic resistance into Eq. (5) gives:

$$\ddot{q} + \varepsilon\dot{q} + \alpha q + \gamma q^2 + \beta q^3 + c(q + Y) = 0\,. \tag{6}$$

4. THE NUMERICAL MODELING

The fourth-order Runge-Kutta method adopt as the main method of the numerical integration. The integration step accept to constant and its maximum value don't exceed $1/30$ period of the oscillations of upper part of the tower.

The fast Fourier transform was used for spectral analysis of the oscillations of system. Spectral analysis is carried out for the steady oscillations. The length of transient process is accepted as $(100 \div 500)\text{Ò}$. The initial conditions on each integration step are accepted equal to the solution of the equation (6) on the previous integration step. It is used for construction of amplitude-frequency characteristics.

Standard graphical software is adopted for presenting results of integration.

5. THE INVESTIGATION OF DYNAMICAL BEHAVIOR OF THE CABLE

The analysis of dynamic behavior of the cable under effect of kinematical excitation we will consider taking system with following parameters:

- cable's span $L = 70$ m;
- slope $\varphi = 45^0$;
- diameter of the cable $d = 140$ mm;
- bending flexure in the middle of flight $f = \frac{1}{100}L$;
- the maximum value of a sag of the cable $Y = f\cos\varphi$;
- material – steel S345 ($\gamma = 7850$ kg/m^3, $E = 2{,}1\cdot 10^{10}$ kg/m^2);
- coefficient of elastic damping $\varepsilon = 0{,}1$ s^{-1}.

Cross-sectional area is $A = 0{,}0154$ m^2, a moment of inertia of cross-section is $I = 1{,}89\cdot 10^{-5}$ m^4. Load from an own weight is $p_1 = \gamma A$. Statically equivalent load distributed along the flight: $p = p_1/\cos(\varphi)$. High of the cable is $h = Ltg(\varphi)$. Distance between cable's ends in its direction is $l = L/\cos(\varphi)$. The horizontal component of extension force is $H = \frac{p\ L^2}{8f}$.

The maximum extension force:

$$T_0 = \sqrt{H^2 + \left(H\frac{h}{L} + p\ \frac{L}{2}\right)^2}\ . \tag{7}$$

Amplitude of the horizontal oscillations of an upper part of the tower is accepted equal to the maximum normative value of a deviation of an upper part of a tower from a vertical [1]:

$$S = \frac{h}{500}\,. \tag{8}$$

Amplitude-frequency characteristics of the system are resulted in Fig. 2 and Fig. 3. In Fig. 2 the responses of values of amplitude of a fundamental tone of the steady oscillations are shown at change of frequency of the oscillations from 0,5 rad/s up to 20 rad/s for three values of damping coefficient ε =0,1; 0,5 and 1,0 s^{-1} where S =0,14 m. In Fig. 3 the responses of value of amplitude of a fundamental tone of the steady oscillations from frequency of the oscillations are given for three values of an amplitude of oscillation of tower S =0,07; 0,14 0,16 m at ε =0,1 s^{-1}. The results of integration of the equation (6) are omitted on an interval of time from 0 up to 1000 s for exclusion of the influence of transient processes.

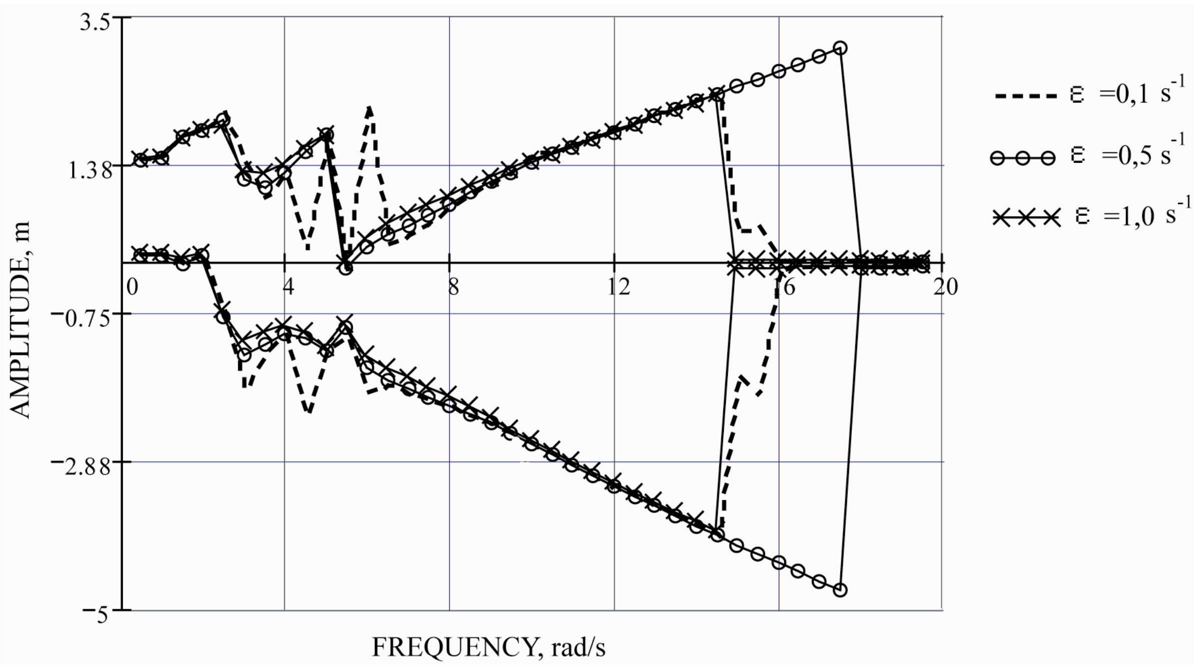

Fig.2. The influence of elastic damping on dynamical behavior of the system:
α =20,899 s^{-2}; γ =9,881 ms^{-2}; β =6,654 $m^{-2}s^{-2}$; S =0,14 m

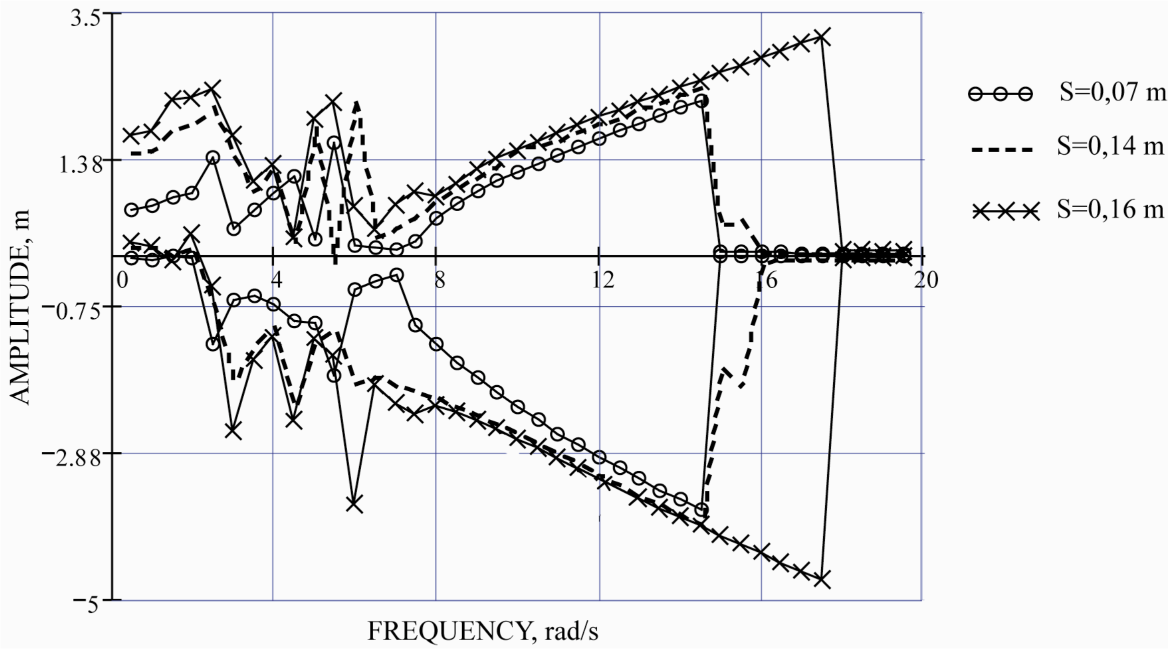

Fig.3. The influence of the amplitude of the horizontal oscillations of the tower on dynamical behavior of the system: α =20,899 s^{-2}; γ =9,881 ms^{-2}; β=6,654 $m^{-2}s^{-2}$; ε =0,1 s^{-1}.

The analysis of the amplitude-frequency characteristics shows that value of "break-down" frequency depends on damping coefficient and value of amplitude of the horizontal oscillations of the tower. Increasing of damping coefficient magnitude decreases the value of "break-down" frequency. Also, decreasing of magnitude of amplitude of the horizontal oscillations of the tower decreases the value of "break-down" frequency.

In Fig.2 "break-down" frequency is ω=14,5; 17,5 and 14,5 rad/s accordingly at ε =0,1; 0,5; 1,0 s^{-1}.

In Fig.3 "break-down" frequency is ω=14,5; 14,5 and 17,5 rad/s accordingly at S =0,07; 0,14; 0,16 m.

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