Multilevel Computation in Civil Engineering Based on Multimodel Elasto-Plastic Analysis

B.E. Melnikov, A.S. Semenov
St.-Petersburg State Polytechnical University, RUSSIA
semenov.artem@gmail.com

Abstract

Creation of hierarchical sequence of the plastic and viscoplastic models according to different levels of structure approximations is considered. Developed strategy of multimodel analysis, which consists of creation of the inelastic models library, determination of selection criteria system and caring out of multivariant sequential clarifying computations, is described. Application of the multimodel approach in numerical computations has demonstrated possibility of reliable prediction of stress-strain response under wide variety of combined nonproportional loading.

1. Introduction

At the present time the increasing requires for reliability and durability of structures and their elements with simultaneous material economy have stimulated improvement of constitutive equations for description of elasto-plastic deformation processes. This has led to the development of phenomenological modeling of complex phenomena of irreversible deformation including history-dependent and rate-dependent effects. During the last several decades many works have been devoted to the development of elasto-plastic models, in order to better predict the material behavior under combined variable thermo-mechanical loading.

The increase of accuracy of stress analysis and safety factors for complex structures with the help of modern finite-element packages (ABAQUS, ALGOR, ANSYS, COSMOS, LS-DYNA, LUSAS, MSC.MARC, MSC.NASTRAN, PERMAS and other) can be provided only by use of complex and special variants of plasticity theories, which are adequate for the considered loading conditions and based on authentic information about properties of materials. The areas of application of the various theories (models) are as a rule unknown to the users of finite-element packages at the existing variety loading conditions in machine-building designs. At the present time a universal theory of inelasticity is absent and even the most accomplished theories can not guarantee adequate description of deformation processes for arbitrary structure under wide range of loading programs.

The multilevel numerical stress analysis is one of the most effective approaches for numerical stress analysis of complex structures. Possibility to use different material models for the various levels of structure approximations can considerably reduce the time of computations without accuracy loss. Such adaptive constitutive modeling is based on creation of rational (reliable and quite simple) sequence of the elasto-plastic models according to different levels (body, element, point) of structure approximations.

2. General principles of multimodel method

At the moment there is no a universal theory of plasticity which is applicable for a wide class of materials and arbitrary paths of loading. In these conditions the multimodel approach (Melnikov and Semenov 1995, Semenov and Melnikov 1998) for the analysis of inelastic behavior of material and structures under complex loading is probably most rational. The schematic representation of multimodel analysis strategy is given in Fig. 1.

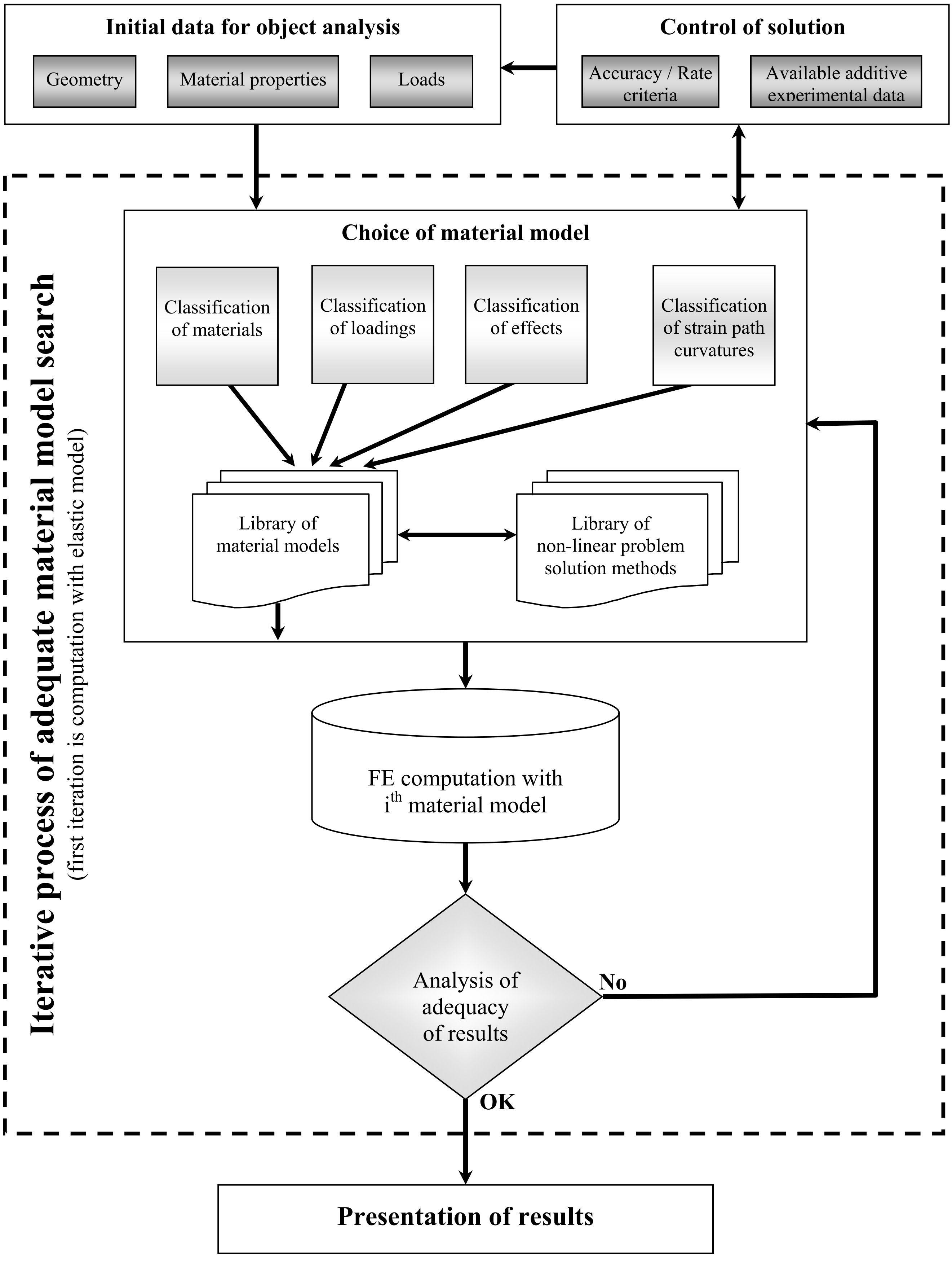


Fig. 1. Strategy of the multimodel analysis.

Basic element of considered scheme is using of several developed classifiers:

- classification of materials;
- classification of loading conditions;
- classification of special effects;
- classification of strain path curvatures.

The classifiers form preliminary hierarchical sequence of models by way of their complication. The selection of adequate model is performed on the base of iterative FE solutions with models from the generated hierarchical sequence. Control of the iterative procedure is carried out in according to user requirements to the accuracy and computation rate, and also it depends on the availability of obtaining of the additive experimental information.

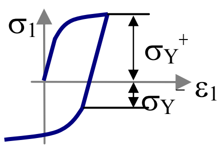
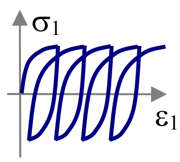
The main features of the developed multimodel approach (Semenov and Melnikov 1998) are following:

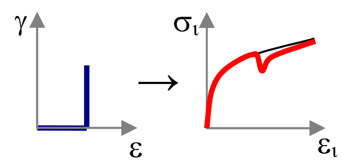
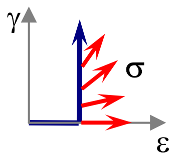
- certification of classic and modern theories of elasto-visco-plasticity with aim to determine area of application and adequacy to the special effects description;
- creation of the elasto-plastic and elasto-visco-plastic models library, providing solution of the wide spectrum of non-elastic problems;
- determination of the selection criteria system, realizing the choice of the simplest variant of theory sufficient for the correct problem solution;
- development of a material characteristics database corresponding to the basic experiments for all used models;
- development algorithms and subroutines codes for the implementation into finite-elements programs;
- caring out of multivariant sequential clarifying computations to define areas of adequate application of models and their hierarchy with positions of computational effectiveness.

3. Phenomena of inelastic behavior of metallic materials

Classification of effects (phenomena) of inelastic deformation plays one of main roles in automatic selection of material models. Inelastic behavior of metallic materials under uniaxial and multiaxial loading for the different programs demonstrates existence of large number of phenomena which are not described as a rule by many models. The systematic analysis of these phenomena is considered in (Getsov et al. 2002, Benallal et al. 1985, Tanaka et al. 1985) for the proportional and non-proportional loadings, rate-dependent and rate-independent behavior, reversible and irreversible deformation. Some important effects under non-proportional loading are given in Table 1.

Table 1. Some typical phenomena of inelastic behavior.

Phenomena	Scheme	Possible mathematical model
Bauschinger's effect (*deformation induced anisotropy*)		$\sqrt{\tfrac{3}{2}(\boldsymbol{s}-\boldsymbol{\rho})\cdot\cdot(\boldsymbol{s}-\boldsymbol{\rho})}-\sigma_Y=0$ $d\boldsymbol{\rho}=c\,d\boldsymbol{\varepsilon}^p$
Ratchetting effect (*permanent plastic strain accumulation under cyclic loading*)		$\sqrt{\tfrac{3}{2}(\boldsymbol{s}-\boldsymbol{\rho})\cdot\cdot(\boldsymbol{s}-\boldsymbol{\rho})}-\sigma_Y=0$ $d\boldsymbol{\rho}=c\,d\boldsymbol{\varepsilon}^p-b\boldsymbol{\rho}\,d\lambda$

<table>
<tr>
<td>Retardation of scalar characteristics
(as an illustration - “drop“ on the stress- strain curve corresponding to the strain path break)</td>
<td>
</td>
<td rowspan="2">$\boldsymbol{\sigma} = \boldsymbol{f}(\boldsymbol{\varepsilon}, \dot{\boldsymbol{\varepsilon}})$</td>
</tr>
<tr>
<td>Retardation of vector characteristics</td>
<td>
</td>
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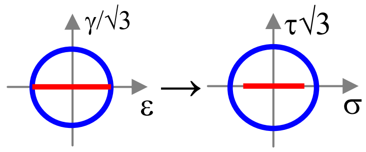
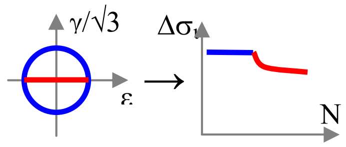
<tr>
<td>Additional hardening effect in nonproportional loading
(cyclic hardening is more strong under the nonproportional loading than that under the proportional loading)</td>
<td>
</td>
<td rowspan="3">$\dot{\sigma}_Y = \gamma[Q(A) - \sigma_Y]\dot{\varepsilon}_i^p$
$A = 1 - \frac{(\dot{\boldsymbol{\varepsilon}}^p \cdot\cdot \dot{\boldsymbol{s}})^2}{(\dot{\boldsymbol{\varepsilon}}^p \cdot\cdot \dot{\boldsymbol{\varepsilon}}^p)(\dot{\boldsymbol{s}} \cdot\cdot \dot{\boldsymbol{s}})}$</td>
</tr>
<tr>
<td>Subsequent softening effect in proportional loading
(equivalent stress range decreasing after a transition from the nonproportional loading to the proportional)</td>
<td>
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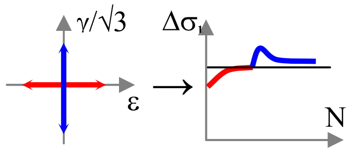
<td>Cross hardening effect
(change of a direction in the proportional cyclic loading leads to the hardening followed by the softening)</td>
<td>
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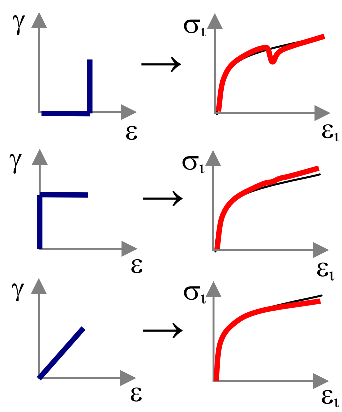
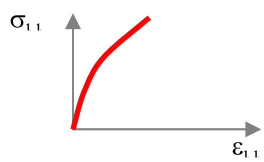
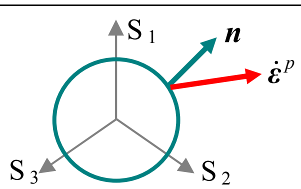
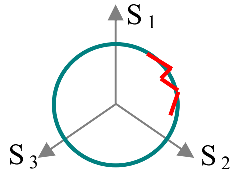
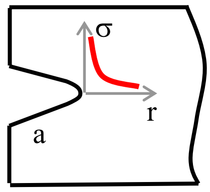
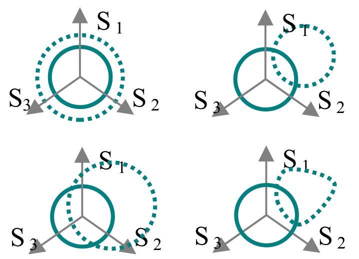
<tr>
<td>Deformation history influence on the stress state
(stress dependence on the deformation path form)</td>
<td>
</td>
<td>$\boldsymbol{\sigma} = \boldsymbol{f}(\boldsymbol{\varepsilon}, \dot{\boldsymbol{\varepsilon}})$</td>
</tr>
<tr>
<td>Hydrostatic pressure effect
(high pressure, porosity)</td>
<td>
</td>
<td>$tr\varepsilon^p \neq 0$
$\sqrt{\frac{3}{2}(\boldsymbol{s} - \boldsymbol{\rho})\cdot\cdot(\boldsymbol{s} - \boldsymbol{\rho})} - \sigma_Y(tr\boldsymbol{\sigma}) = 0$</td>
</tr>
<tr>
<td>Non-associate plastic flow
(deviation from normality)</td>
<td>
</td>
<td>$\dot{\boldsymbol{\varepsilon}}^p = \lambda \frac{\partial g}{\partial \boldsymbol{\sigma}}$
$g \neq f$</td>
</tr>
</table>

Irreversible deformation under passive loading (*plastic strain accumulation under deformation along the neutral loading surface and nearby it*)		$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\dfrac{\partial f_k^{active}}{\partial \boldsymbol{\sigma}}$ $k = 1,2,\ldots,n$
Sensitivity of the yield stress to the stress concentration (*non-local theory of plasticity*)		$\sigma_Y^* = \dfrac{\sigma_Y}{\alpha_N} + \dfrac{K_c\varphi(\alpha_N)}{\sqrt{\pi a}}$ $\varphi(1) = 0$ $\varphi(\infty) = 1$
Yield surface evolution (*expansion, translation and distortion*)		$\sqrt{(\boldsymbol{s}-\boldsymbol{\rho})\cdot\cdot{}^4\boldsymbol{Q}(\lambda)\cdot\cdot(\boldsymbol{s}-\boldsymbol{\rho})} - \sigma_Y(\lambda) = 0$

The following notations are used in Table 1: σ_Y is the yield limit, $\boldsymbol{\sigma}$ and $\boldsymbol{s}$ are the stress tensor and deviator, $\boldsymbol{\rho}$ is the micro-stress tensor deviator, $\dot{\boldsymbol{\varepsilon}}^p$ is the tensor of plastic strain rates, $\dot{\varepsilon}_i^p = \sqrt{\frac{2}{3}\dot{\boldsymbol{\varepsilon}}^p\cdot\cdot\dot{\boldsymbol{\varepsilon}}^p}$ is the equivalent plastic deformation rates, λ is the Odquist's parameter (path length in the plastic strain space), in particular for von Mises's criterion: $\lambda = \int\sqrt{\frac{2}{3}\dot{\boldsymbol{\varepsilon}}^p\cdot\cdot\dot{\boldsymbol{\varepsilon}}^p}\,dt$, g is the plastic potential, f is the yield surface, α_N is Neuber's stress concentration coefficient, K_c is the fracture toughness, operator $\cdot\cdot$ denotes a scalar product of tensors, so $\boldsymbol{A}\cdot\cdot\boldsymbol{B} = A_{ij}B_{ji}$.

4. Library of plastic and viscoplastic models

The developed library of material models represents generalized data set, including information about limitations on field application, basic experiments for material parameters determination, continuous mathematical model, discrete numerical model, computational algorithm, implementation into finite element program, recommendations about computation strategy.

4.1 Plastic models

At the present time the developed and implemented into finite element program library of rate-independent (plastic) models includes:

- *Plastic flow theories* with the various isotropic-kinematic laws of hardening. Relations of these "classical" models belong to linear tensorial equations convenient for computations.
- *Structural (rheologic) models* theories. They possess clarity of properties, thermodynamic basis, obvious creation and modification.
- *Multisurface theory* with one active surface of plastic compliance. The model provides high accuracy of the description for the complex paths of passive loading.

- *Endochronic theory* of plasticity. The equations can be applied for a wide class of materials from a metal to a soil. This theory does not use the existence of a yield surface and employs the same equations for the loading and unloading processes.

4.2 Viscoplastic models

The library of the rate-dependent (viscoplastic) models includes:

- *Technical theories of creep* (aging theory, flow theory, hardening theory). These models are convenient for the primary express analysis and are applicable for weakly variable loading. They are simplest models with least set of the necessary experimental data.
- *Elastic/viscoplastic models*. There are most popular in computations class of models. They demonstrate the viscous effects only after of static yield limit.
- *Viscoelastic/viscoplastic models*. They demonstrate the viscous effects always as before as after exceedition of yield limit.
- *Elastoviscoplastic models* (endochronic theory, nonlinear heredity theory). These models don't possess pronounced yield limit and demonstrate simultaneously elastic, viscous and plastic properties. They represent extension of viscoelasticity.
- *Structural (rheological, fraction, sublayer) models*. They allow to create and easy to modify models with wide spectrum of elastic, viscous and plastic properties combination in clarified and obvious way.

5. Unified form of the thermo-elasto-visco-plastic constitutive equations

The uniform representation of constitutive equations is actual for the creation of inelastic models library with the purpose to simplify program realization and to perform comparative analysis. The thermodynamic approach with internal state variables provides a powerful tool for representing of the constitutive equations of elasto-plasticity and elasto-visco-plasticity. All considered here models of inelastic material have been written in common quite general mathematical form. In the general case there are functional-type relations between stress and strain (Semenov 1995). These equations can be simplified for the case of quasi-linear differential evolution equations and can be presented as quasi-linear differential tensorial equation:

$$\dot{\boldsymbol{\sigma}}={}^4\boldsymbol{D}^{evp}(\boldsymbol{\varepsilon},\boldsymbol{\sigma},\boldsymbol{\chi}^{(k)},T)\cdot\cdot\,\dot{\boldsymbol{\varepsilon}}+\boldsymbol{R}^{evp}(\boldsymbol{\varepsilon},\boldsymbol{\sigma},\boldsymbol{\chi}^{(k)},T)\dot{T}+\boldsymbol{Q}^{evp}(\boldsymbol{\varepsilon},\boldsymbol{\sigma},\boldsymbol{\chi}^{(k)},T)\,, \tag{1}$$

where $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ are strain and stress tensors; $\boldsymbol{\chi}^{(k)}$ is a set of internal state variables (k=1,…,n); T is a temperature; ${}^4\boldsymbol{D}^{evp}$ is a tensor of elasto-visco-plastic moduli of 4th order; $\boldsymbol{R}^{evp}$ and $\boldsymbol{Q}^{evp}$ are 2nd order tensors describing material response to change of temperature and time. The equation (1) can be simplified in the case of rate-independent material as following (Semenov 1995):

$$\dot{\boldsymbol{\sigma}}={}^4\boldsymbol{D}^{ep}(\boldsymbol{\varepsilon},\boldsymbol{\sigma},\boldsymbol{\chi}^{(k)},T)\cdot\cdot\,\dot{\boldsymbol{\varepsilon}}+\boldsymbol{R}^{ep}(\boldsymbol{\varepsilon},\boldsymbol{\sigma},\boldsymbol{\chi}^{(k)},T)\dot{T} \tag{2}$$

The internal state variables $\boldsymbol{\chi}^{(k)}$ can be either second-order tensors or scalars. Evolution laws for these internal variables can be represented in the form:

$$\dot{\boldsymbol{\chi}}^{(k)}=\dot{\lambda}\,\boldsymbol{b}(\boldsymbol{\varepsilon},\boldsymbol{\sigma},\boldsymbol{\chi}^{(i)},T)\,, \tag{3}$$

where a multiplier λ is determined from consistency plastic condition for rate-independent behavior or from uniaxial creep-relaxation experiments for rate-sensitivity behavior.

The tensor of elasto-plastic moduli in the case of plastic flow theory can be written as:

$$
{}^4\boldsymbol{D}^{ep} = {}^4\boldsymbol{D} - {}^4\boldsymbol{D} \cdot\cdot \frac{\partial Q}{\partial \boldsymbol{\sigma}} \frac{\partial F}{\partial \boldsymbol{\sigma}} \cdot\cdot {}^4\boldsymbol{D} \left(A + \frac{\partial F}{\partial \boldsymbol{\sigma}} \cdot\cdot {}^4\boldsymbol{D} \cdot\cdot \frac{\partial Q}{\partial \boldsymbol{\sigma}} \right)^{-1}, \tag{4}
$$

where Q is a plastic potential; F is a yield function; ${}^4\boldsymbol{D}$ is a tensor of elastic moduli of 4^{th} order; A is a hardening parameter. The constitutive relation (2) with tensor of elasto-visco-plastic moduli in form (4) is usually used in standard procedures of finite-element elasto-plastic analysis.

6. Selection criteria system

The determination of the selection criteria system, based on classification of inelastic theories and their domain of advantageous applicability, is one of the main problems in multimodel analysis. Selection criteria system (see also Fig. 1) generates necessary conditions for material model on the basis of information concerning loading, available experimental data and discrete model of structure. The choice of rational model, which is the simplest among models satisfied necessary conditions, may be corrected by clarifying sequential computational experiments.

The classical examples of selection criteria are degree of plastic strains development in comparison with elastic strain and curvature (non-proportionality) of loading path. Numerous stress state analysis of elasto-plastic behavior of structures of different degree of complexity allows to formulate a new selection criterion. Suggested criterion is based on consideration of geometrical regulated levels of plastic deformation analysis. Similarly with (Semenov 1995) we have introduced in the following levels:

- Body level **B** considers the body or complex structure as a whole. "Integral" analysis corresponds to initial strength problem. In most cases zones of plasticity are local.
- Element level **E** is introduced for separate part of structure as detailed fragment of structure, area with possible defect, superelement or individual finite element. "Semi-integral" analysis is carried out in this case. In most of cases zones of plasticity can be extensive.
- Point level **P** is the basic level, related to selected points of material continuum or to a model of structure. "Local" analysis is carried out for simplest geometrical object - element of material with homogeneous stress state. The whole object is a zone of plasticity.

Finally, the criterion can be formulated by following manner. Complexity of applying theory of plasticity must correspond to the level of the structure approximation. The levels **E** and **P** with more detail description of the structure geometry and with possibility of extensive zones of plasticity demand more difficult variants of theory adequate to the loading process. Using of simple models is sufficiently at the **B** level of the investigation, when deformation of local zones of plasticity is smoothed by influence of extensive elastic region. Refined computations of first level model can be performed on the basis of obtained information at second and third levels.

7. Results of multimodel computational analysis

The wide range of mentioned above inelastic material models has been implemented into finite element programs PANTOCRATOR (Semenov 2003) (see for details www.pantocrator.narod.ru).

Comparison of the results of numerical finite element analysis and experimental data for series investigated constructions corresponding to the first level **B** (frames, pipelines, vapor producing plant, gas generator, vessel of nuclear reactor) says about relative nearness of different theories predictions. However series of computations corresponding to the second level **E** of the structures considerations (fragment of rolling mill, fastenings of vapor producing plant, various fastening knots, socket, circular ring) have shown that the considerable differences of the prediction of stress-strain state by means of different theories of plasticity were displayed for a developed zone of plasticity and complex history of loading. Set of trials according level **P** carried out on tubular specimens of 1X18H10T steel under a wide range of the combined cyclic loading, including polygonal and circular paths of deformation. In general the results different theories corresponding to the level **P** can be essentially quality differed.

The typical examples of multimodel computations corresponding to the level **P** and comparison with experimental data are shown in Fig. 2-3. Detailed description of problems is given in (Semenov 1996, Izotov 2001).

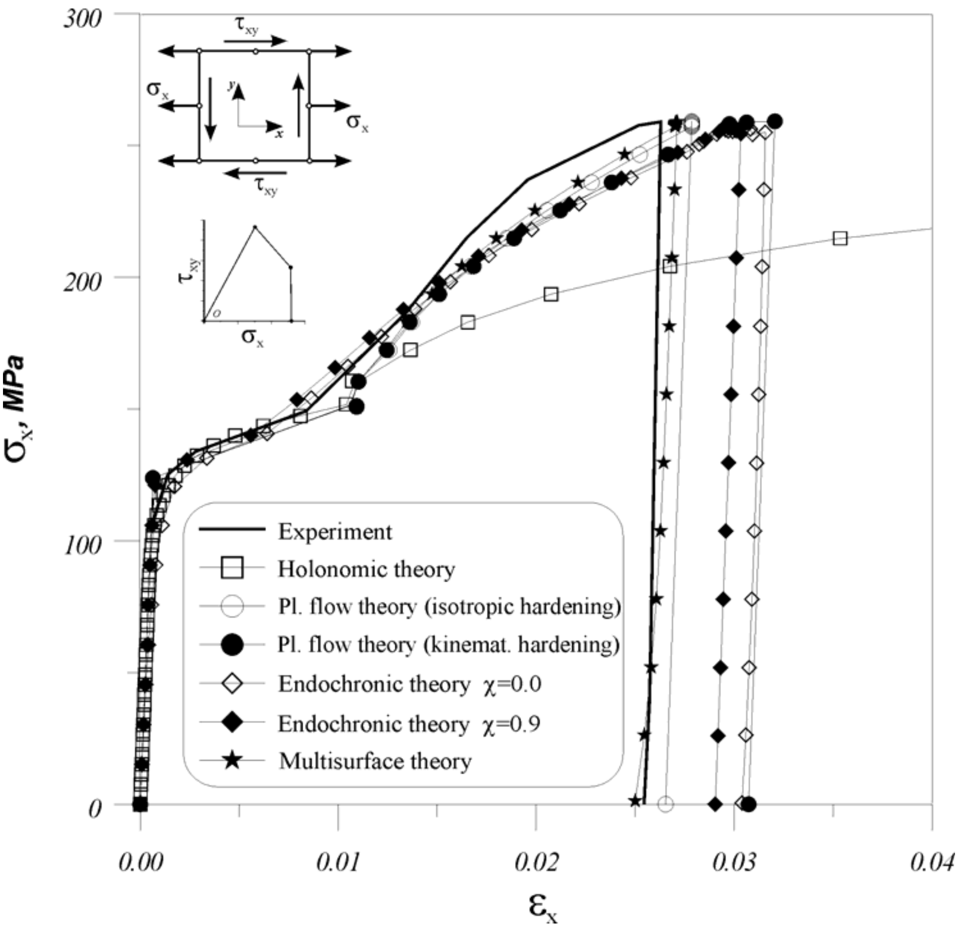
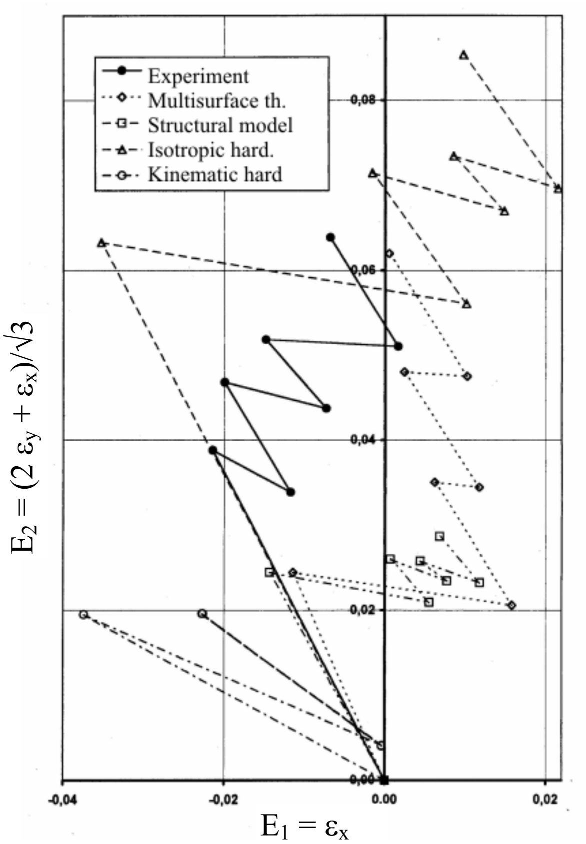
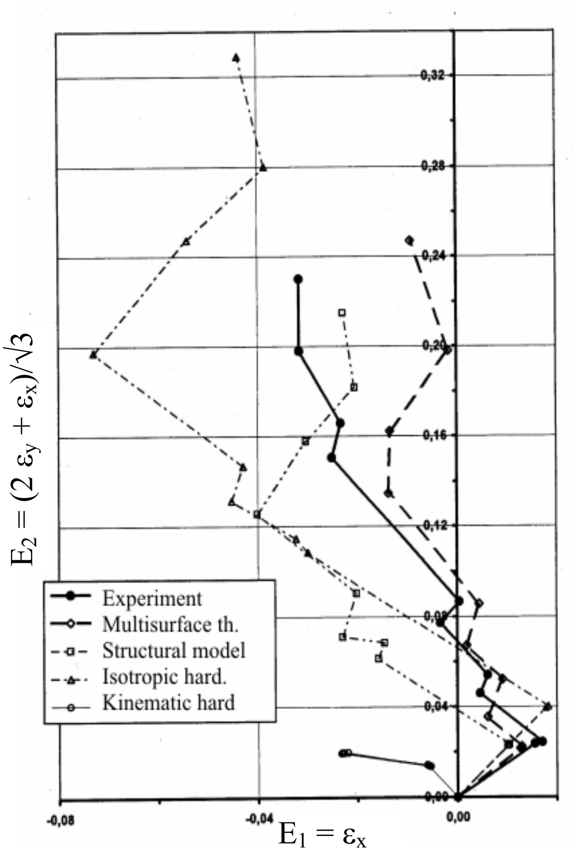


Fig. 2. The thin-walled tube under combined axial tension-torsion.



a)



b)

Fig. 3. Thin-walled tubes (steel 1X18H10T) under complex cyclic non-proportional loading**.**

Typical example of multimodel computations corresponding to the level **E** for thin circular ring being the part of more complex structure is shown in Fig. 4.

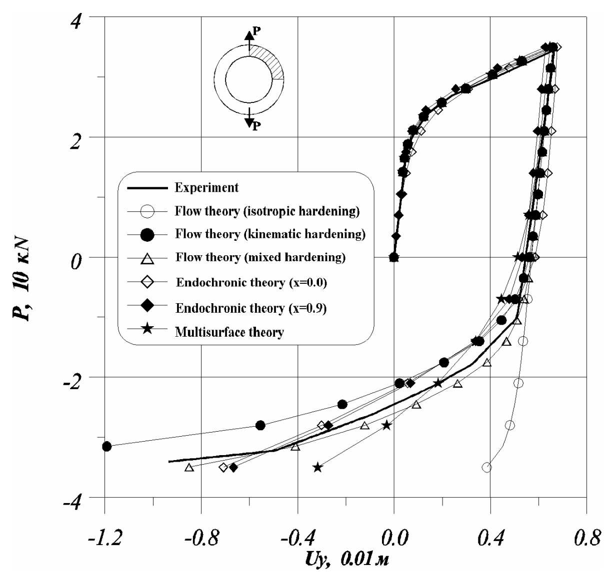
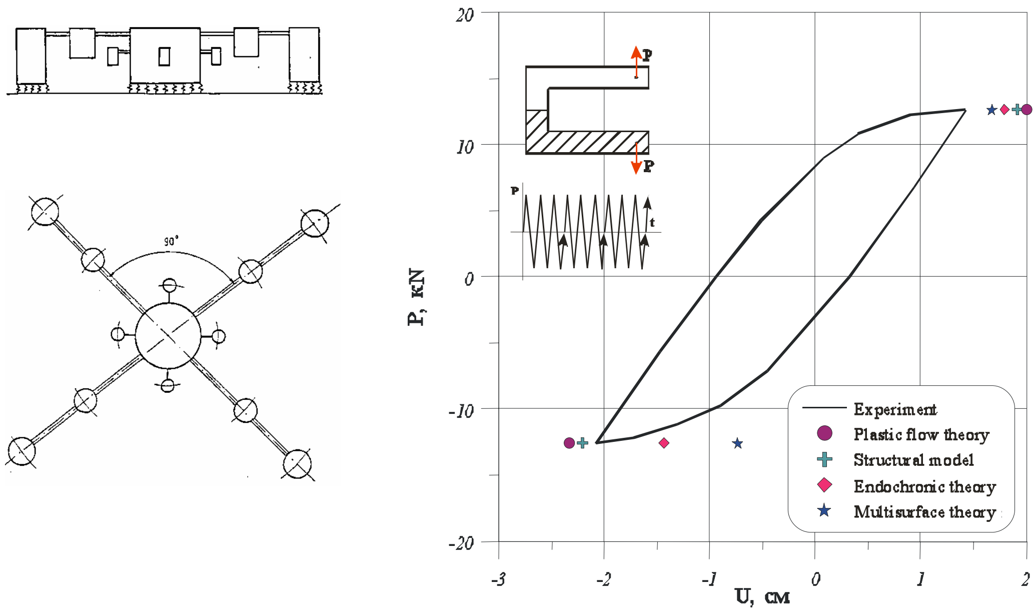


Fig. 4. Circular ring under axial tension-compression.

The stress analysis of the vapor producing plant (see Fig. 5a) is considered as an example corresponding to the level **B**. Masses approximating of the units of vapor producing plant are joined with fundament by beams and springs. The action of impact loading in the horizontal directions is set by accelerations of the fixture points.



a) b)

Fig. 5. Vapor producing plant (a) and fastening knot (b) under cyclic loading.

Clarifying computations have been carried out for the stress-strain state of the fastening knot (see Fig. 5b), which is a part of the vapor producing plant (see Fig. 5a). The hysteresis loop, corresponding 10^{th} cycle, is given in Fig. 7b. Results obtained at the level **E** (detail analysis of fastening knot) improve the results corresponding to the level **B**. The difference between predictions of different material models at the level **E** amount more 50%, while difference at the level **B** is less 10%.

The levels **P** and **E** with more detail description of the structure geometry and with possibility of extensive zones of plasticity demand the more difficult variants of theory adequate to the loading process. Using of the simple models is sufficiently at the **B** level of the investigation, when the deformation of local zones of plasticity is smoothed by influence of extensive elastic region.

Conclusions

A strategy of automatic choice of a material model has been proposed. Certainly, the method does not claim for a full completeness now. Final purpose is a creation of the automatic system for selection of models meant for the correct computations of the strain-stress state under complex loading conditions within the frame of modern finite elements systems. The application of the multimodel method can increase the effectiveness of computations and adequacy of results in the case of structure analysis under complex loading programs. We invite all authors of theories describing inelastic behavior of material to take part in a joint development of the multimodel method and expansion of material library.

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