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SPECIAL FUNCTIONS VERSUS ELEMENTARY FUNCTIONS IN HYPERCOMPLEX FUNCTION THEORY

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Abstract. *In recent years special hypercomplex Appell polynomials have been introduced by several authors and their main properties have been studied by different methods and with different objectives. Like in the classical theory of Appell polynomials, their generating function is a hypercomplex exponential function. The observation that this generalized exponential function has, for example, a close relationship with Bessel functions confirmed the practical significance of such an approach to special classes of hypercomplex differentiable functions. Its usefulness for combinatorial studies has also been investigated. Moreover, an extension of those ideas led to the construction of complete sets of hypercomplex Appell polynomial sequences. Here we show how this opens the way for a more systematic study of the relation between some classes of Special Functions and Elementary Functions in Hypercomplex Function Theory.*

1 INTRODUCTION

2 SPECIAL POLYNOMIALS AND GENERATING FUNCTIONS

3 SPECIAL FUNCTIONS VERSUS ELEMENTARY FUNCTIONS

4 APPLICATIONS: BINOMIAL IDENTITIES

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