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ENERGY RELEASE CONTROL FOR NONLINEAR MESOSCALE SIMULATIONS

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Abstract. *In nonlinear simulations the loading is, in general, applied in an incremental way. Path-following algorithms are used to trace the equilibrium path during the failure process. Standard displacement controlled solution strategies fail if snap-back phenomena occur. In this contribution, a path-following algorithm based on the dissipation of the inelastic energy is presented which allows for the simulation of snap-backs. Since the constraint is defined in terms of the internal energy, the algorithm is not restricted to continuum damage models. Furthermore, no a priori knowledge about the final damage distribution is required. The performance of the proposed algorithm is illustrated using nonlinear mesoscale simulations.*

1 INTRODUCTION

In nonlinear finite element simulations of concrete, path-following algorithms are applied to trace the equilibrium path during the failure process, which can be represented by a global load-displacement curve. Depending on the ratio between the specimen size and the size of the fracture process zone, this curve may exhibit snap-back phenomena. Path-following algorithms based on direct displacement control fail at the turning point, where the tangent of the corresponding load-displacement curve is vertical. Load-displacement-constraint methods, such as arc-length or indirect displacement control, can be used to simulate snap-backs. In these methods, the parametrized load step is coupled to the increment of selected nodal degrees of freedom using an additional constraint equation. The crack mouth opening can for example be used to control the loading. Such an approach is only applicable if the final failure process zone is known in advance. Mesoscale simulations of concrete, in which the numerical model explicitly represents the material components, are characterized by the propagation of multiple microcracks. The coalescence of these microcracks to a macroscopic crack results in a softening of the material. In general, the position of the final macroscopic fracture process zone is determined by the heterogeneous material structure on the mesoscale and therefore not known a priori. Gutiérrez [1] proposes a path-following constraint which is based on the energy dissipation during the failure process. Consequently, the constraint does not depend on the position and behavior of the fracture process zone. Since in this method the constraint is defined by the external energy, which can be calculated from the external load vector and the nodal displacements, this constraint is only applicable to simulations with damage material models. In this contribution, the energy release control is extended to general constitutive formulations. The corresponding constraint is defined in terms of the internal energy, which is calculated from stresses and strains.

2 LOAD-DISPLACEMENT-CONSTRAINT METHODS

In load-displacement-constraint methods, the external forces or the displacements at the supports after load step n are not prescribed in advance. Instead, the external load vector is parametrized by a scalar load factor μ

$$\boldsymbol{f}_{\boldsymbol{ext}}^{(n)} = \boldsymbol{f}_{\mathbf{0}} + \mu^{(n)} \hat{\boldsymbol{f}}, \tag{1}$$

where $\boldsymbol{f}_{\mathbf{0}}$ is a vector of constant loads, e.g. the dead load of the structure, and $\hat{\boldsymbol{f}}$ is a given reference load vector. The corresponding equilibrium condition reads

$$\boldsymbol{f}_{\boldsymbol{int}}^{(n)} = \boldsymbol{f}_{\boldsymbol{int}}(\boldsymbol{d}^{(n)}) = \boldsymbol{f}_{\mathbf{0}} + \mu^{(n)} \hat{\boldsymbol{f}}, \tag{2}$$

where $\boldsymbol{f}_{\boldsymbol{int}}$ is the internal load vector and $\boldsymbol{d}$ is the displacement vector. Assuming a constant load factor in each load step, standard load control is obtained. If the load factor is considered as an unknown during the iteration process, the equilibrium equation, Eq. (2), is enhanced by an additional constraint equation

$$l^{(n)} = l(\boldsymbol{d}^{(n)}, \mu^{(n)}) = 0. \tag{3}$$

Using Newton's method, the generally nonlinear system of equations given by Eqs. (2) and (3) is solved in an iterative manner for the displacement vector $\boldsymbol{d}$ and the load factor μ.

2.1 Path-following constraint based on the internal energy

In this contribution, a path-following constraint is introduced which is directly based on the dissipated (inelastic) energy. In general, the inelastic energy U_{inel} can be defined as

$$U_{inel}^{(n)} = U_{tot}^{(n)} - U_{el}^{(n)}, \tag{4}$$

where U_{tot} is the total energy and U_{el} is the elastic energy. The total energy stored in an element with volume V can be written as a function of the total energy density u_{tot} which is approximated by the trapezoidal rule

$$U_{tot}^{(n)} = \int_V u_{tot}^{(n)} \, \mathrm{d}V \approx \int_V u_{tot}^{(n-1)} + \frac{1}{2} \left(\boldsymbol{\sigma}^{(n-1)} + \boldsymbol{\sigma}^{(n)} \right) : \left(\boldsymbol{\varepsilon}^{(n)} - \boldsymbol{\varepsilon}^{(n-1)} \right) \, \mathrm{d}V, \tag{5}$$

where $\boldsymbol{\varepsilon}$ is the strain tensor and $\boldsymbol{\sigma}$ is the corresponding stress tensor. Assuming a combined damage-plasticity model, the elastic energy is given by

$$U_{el}^{(n)} = \frac{1}{2} \int_V \boldsymbol{\sigma}^{(n)} : \left(\boldsymbol{\varepsilon}^{(n)} - \boldsymbol{\varepsilon_{pl}}^{(n)} \right) \, \mathrm{d}V, \tag{6}$$

in which $\boldsymbol{\varepsilon_{pl}}$ denotes the tensor of plastic strains. It is to be noted that the plastic strains vanish if a pure damage material model is applied.

Assuming that a predefined amount of energy $\Delta\mathcal{G}$ should be dissipated in each load increment n, the corresponding constraint equation can be written as

$$U_{inel}^{(n)} - U_{inel}^{(n-1)} - \Delta\mathcal{G} = 0. \tag{7}$$

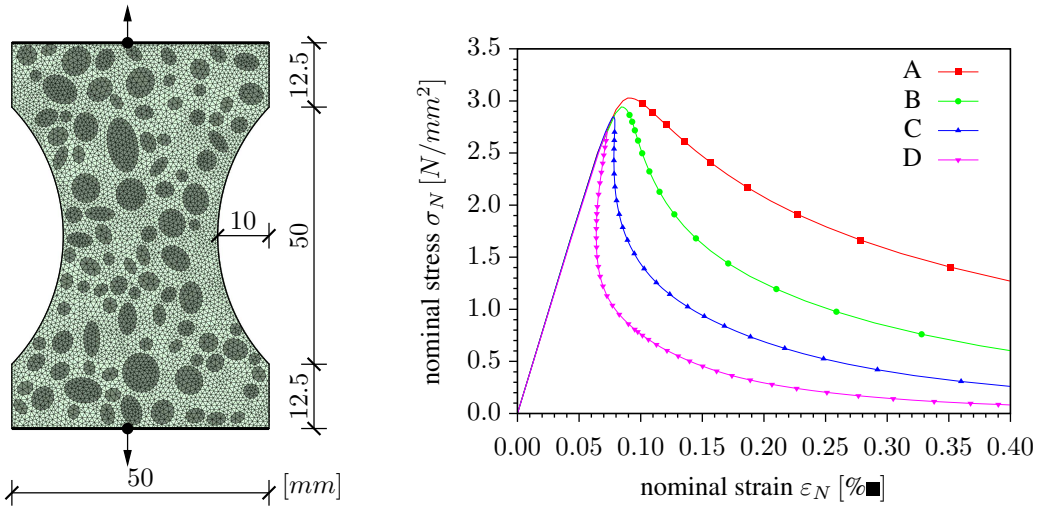
Inside the iterative solution procedure, the unknown energy dissipation is approximated by a truncated Taylor series. In the i-th iteration step of load increment n, the corresponding constraint equation reads

$$U_{inel}^{(n,i-1)} + \left[\frac{\partial U_{inel}^{(n,i-1)}}{\partial \boldsymbol{d}} \right]^T \boldsymbol{\delta d}^{(n,i)} - U_{inel}(\boldsymbol{d}^{(n-1)}) - \Delta\mathcal{G} = 0, \tag{8}$$

where $\boldsymbol{\delta d}$ is the unknown correction of the displacement vector.

Assuming nonlinear simulations with continuum damage material models, the presented approach is totally equivalent to the path-following constraint introduced by Gutiérrez [1] which is based on the external energy. Compared to this approach, a higher numerical effort is required since the derivative of the inelastic energy with respect to the displacements, which involves the computation of the tangent material matrix, must be calculated for each element and an assembling of the element values to the global vector must be performed. Due to the usage of the inelastic energy, this approach is not limited to continuum damage models. It is applicable to any material formulation for which the inelastic energy and the corresponding derivative can be defined.

The presented constraint equation can only be used if damage or plasticity is evolving during the load step. In general, this is not the case at the beginning of the loading process. As long as the structure is in the elastic regime, the simulation is performed with load control and the dissipated energy is calculated in each iteration step using Eq. (7). If damage or plasticity initiates and inelastic energy starts to dissipate, the load control is replaced by an energy release control. In order to reduce the computation time, the step size is adapted during the simulation.



(a) specimen type A

(b) nominal stress-strain curve

Figure 1: Mesoscale model for specimen type A and nominal stress-strain curves.

3 EXAMPLE

The performance of the proposed constraint is illustrated using nonlinear mesoscale simulations of the size-effect experiments of van Vliet and van Mier [2]. For a specimen of type A, Fig. 1(a) shows the three-phase mesoscale model, which consists of aggregates, mortar matrix and the interfacial transition zone (ITZ), and the corresponding finite element discretization. The evolution of microcracks inside the mortar matrix is described by a nonlocal isotropic damage model. Linear elastic behavior is assumed for the aggregates and the nonlinear behavior of the ITZ is represented by a cohesive zone model. The simulations are performed for specimens of type A, B, C and D. The specimen size is successively doubled, assuming a constant thickness of 100 mm. Consequently, a specimen of type D is scaled by a factor of 8. The diagram in Fig. 1(b), shows the nominal stress-strain curves. A clear influence of the specimen size on the maximum nominal stress can be identified. As illustrated in Fig. 1(b), snap-back phenomena observed for the specimen types C and D can be efficiently simulated with the proposed path-following algorithm. Further details on these simulations can be found in [3].

4 CONCLUSIONS

In this contribution, a path following constraint based on the energy which is dissipated during the failure process is presented which allows for an efficient simulation of snap-back phenomena. By defining the constraint in terms of the internal energy, the proposed path-following algorithm is not restricted to continuum damage models, but can be applied to any material formulation for which the inelastic energy can be defined. Since, no a priori knowledge about the final failure zone is required, this constraint is especially advantageous in nonlinear mesoscale simulations of concrete.

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