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AN AUTOMATIC MODE SELECTION STRATEGY FOR MODEL UPDATING USING THE MODAL ASSURANCE CRITERION AND MODAL STRAIN ENERGIES

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Abstract. *In the context of finite element model updating using vibration test data, natural frequencies and mode shapes are used as validation criteria. Consequently, the order of natural frequencies and mode shapes is important. As only limited spatial information is available and noise is present in the measurements, the automatic selection of the most likely numerical mode shape corresponding to a measured mode shape is a difficult task. The most common criterion to indicate corresponding mode shapes is the modal assurance criterion. Unfortunately, this criterion fails in certain cases.*

In this paper, the pure mathematical modal assurance criterion will be enhanced by additional physical information of the numerical model in terms of modal strain energies. A numerical example and a benchmark study with real measured data are presented to show the advantages of the enhanced energy based criterion in comparison to the traditional modal assurance criterion.

1 INTRODUCTION

Finite element model updating strategies, also named model validation, using modal information based on output-only measurements are still a focus of current research. These approaches try to find the modification of the finite element model that sufficiently correlates the mode shapes and natural frequencies of the numerical and experimental modal analysis. In most cases, an automatic model updating approach is required. Preliminary analyses, like global sensitivity analysis, can help to select the most sensitive parameters of the finite element model that should be considered in the updating process. All methods require a robust and stable mode pairing strategy. The measured degrees-of-freedom are in the best case only a subset of the degrees-of-freedom of the finite element model. Hence, the selection of correlated mode pairs between experimentally and numerically obtained modes has to be based on limited spatial information [9]. This is a challenge for most mode selection strategies. Due to the complexity of structures and measurement noise, an exhaustive planned measurement setup can only reduce the problems.

Each mode selection strategy is based on a criterion which indicates the correlation between two mode shapes. The numerical mode with the highest correlation with respect to a certain measured mode will be assigned. Only few, in most cases, pure mathematical correlation criteria are present in literature. One measure for the correlation of two mode shapes is the modal scale factor (MSF) [2]. This measure depends on the scaling of the two vectors and is a non-normalized indicator. A criterion suggested in [19, 16] is based on normalized modal differences (NMD) and indicates the highest correlation by zero. In case of perfect orthogonal mode shapes, the NMD yields to infinity which is not advantageous in practice. The most common approach to check the correlation between modal vectors is the modal assurance criterion (MAC). This has been introduced in [2] and discussed in several references (e.g., [1, 8]). Several other correlation measures are based on the MAC, for example, the coordinate modal assurance criterion (COMAC) [14] and ECOMAC [10]. An overview is given in [1]. The normalized modal assurance criterion or normalized cross orthogonality (NCO) [15, 7, 17] includes additional physical information of the structure by using condensed mass or stiffness matrices of the finite element model. A disadvantage is the generation of additional errors due to the condensation.

This paper presents a mode selection strategy that enhances the pure mathematical modal assurance criterion (MAC) by physical information in terms of modal strain energies of the numerically obtained mode shapes. A numerical and an experimental benchmark study are presented where the application of the modal assurance criterion (MAC) fails to find the right mode shapes. In contrast, the suggested energy based approach can assign the most likely mode shapes with limited additional numerical effort.

2 MODE ASSIGNMENT CRITERIA

2.1 Modal assurance criterion

According to [2], the modal assurance criterion (MAC) is defined as

$$\mathrm{MAC}_{ij} = \frac{\left(\hat{\mathbf{\Phi}}_i^T \hat{\mathbf{\Phi}}_j\right)^2}{\left(\hat{\mathbf{\Phi}}_i^T \hat{\mathbf{\Phi}}_i\right)\left(\hat{\mathbf{\Phi}}_j^T \hat{\mathbf{\Phi}}_j\right)} \tag{1}$$

where $\hat{\mathbf{\Phi}}_i$ is the numerical eigenvector of mode i containing only the measured degrees of freedom. $\hat{\mathbf{\Phi}}_j$ is the corresponding experimental eigenvector of mode j. The modal assurance criterion is a pure mathematical criterion to check the consistency between two eigenvectors. $\text{MAC}_{iJ} = \max_j(\text{MAC}_{ij})$ assigns the Jth numerical mode to the experimental mode i. As long as the modes are evidently separated based on the available sparse spatial information and the measurement noise is neglectable, the modes can be assigned with high reliability. Some applications can be found in [11], [13], and [6].

2.2 Energy based modal assurance criterion

In case of a mass normalized eigenvector matrix $\mathbf{\Phi}$, where the ith column corresponds to the eigenvector of the ith eigenvalue ω_i^2, equations

$$\mathbf{\Phi}^T \mathbf{M} \mathbf{\Phi} = \mathbf{I} \tag{2}$$

and

$$\mathbf{\Phi}^T \mathbf{K} \mathbf{\Phi} = \text{diag}(\omega_i^2) \tag{3}$$

hold for the mass matrix $\mathbf{M}$, stiffness matrix $\mathbf{K}$, and the identity matrix $\mathbf{I}$. With respect to Equation (3), the total modal strain energy for each mode j is ω_j^2. By separating the available degrees of freedom in n clusters, the eigenvector of mode j can be rewritten as

$$\mathbf{\Phi}_j^T = \begin{bmatrix} \mathbf{\Phi}_{j1}^T & \mathbf{\Phi}_{j2}^T & \cdots & \mathbf{\Phi}_{jn}^T \end{bmatrix}^T. \tag{4}$$

The clustered stiffness matrices $\mathbf{K}_{kl} \forall\; k, l = 1, 2 \ldots n$ are then given by

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} & \cdots & \mathbf{K}_{1n} \\ \mathbf{K}_{21} & \mathbf{K}_{22} & \cdots & \mathbf{K}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{K}_{n1} & \mathbf{K}_{n2} & \cdots & \mathbf{K}_{nn} \end{bmatrix}. \tag{5}$$

Hence, the modal strain energy

$$\text{MSE}_{jk} = \sum_{l=1}^{n} \mathbf{\Phi}_{jk}^T \mathbf{K}_{kl} \mathbf{\Phi}_{jl} \tag{6}$$

is given for mode j with respect to cluster k. Therefore, the total energy of mode j is represented by

$$\text{MSE}_j = \sum_{k=1}^{n} \sum_{l=1}^{n} \mathbf{\Phi}_{jk}^T \mathbf{K}_{kl} \mathbf{\Phi}_{jl} = \mathbf{\Phi}_j^T \mathbf{K} \mathbf{\Phi}_j = \omega_j^2 \tag{7}$$

Equations (6) and (7) yield to the relative modal strain energy of mode j with respect to cluster k

$$\Pi_{jk} = \frac{\text{MSE}_{jk}}{\text{MSE}_j} = \frac{\sum\limits_{l=1}^{n} \mathbf{\Phi}_{jk}^T \mathbf{K}_{kl} \mathbf{\Phi}_{jl}}{\mathbf{\Phi}_j^T \mathbf{K} \mathbf{\Phi}_j} \quad \text{with} \quad \text{MSE}_j \neq 0. \tag{8}$$

Therefore, Equation (1) can be extended by Equation (8) yielding an energy based modal assurance criterion for each cluster k

$$\text{EMAC}_{ijk} = \Pi_{jk} \text{MAC}_{ij}. \tag{9}$$

3 BENCHMARK STUDY: CANTILEVER TRUSS

3.1 Description of the system

The numerical example is based on a 20 degree-of-freedom cantilever beam consisting of 12 nodes and 21 truss members which has been used in [12] for a numerical model updating benchmark. The known geometry is presented in Figure 1. The cross section area, the mass density, and the Poisson's ratio of all truss members are defined to 0.03m^2, $2700\frac{\text{kg}}{\text{m}^3}$, and 0, respectively. The material is linear elastic with a Young's modulus of $7 \cdot 10^{10}\frac{\text{N}}{\text{m}^2}$.

For the numerical test example, it is assumed that the vertical modal displacements at the four measurement points (MP), indicated in Figure 1, are available for the first four vertical modes. The corresponding mode shapes are shown in Figure 2. The original and the noisy modal displacements are given in Table 1. In the following, the noise disturbed mode shapes are used as experimental mode shapes.

Table 1: Original (left) and noise disturbed (right) modal displacements

	Mode 1	Mode 2	Mode 3	Mode 4
MP1	-0.012726	0.034375	0.010705	0.028718
MP2	-0.022476	0.022356	-0.035951	0.002883
MP3	-0.032717	-0.006429	-0.014853	-0.039518
MP4	-0.042469	-0.036156	0.038418	0.027539

	Mode 1	Mode 2	Mode 3	Mode 4
MP1	-0.0117	0.0351	0.0106	0.0272
MP2	-0.0227	0.0267	-0.0361	0.0027
MP3	-0.0331	-0.0067	-0.0151	-0.0379
MP4	-0.0437	-0.0321	0.0400	0.0261

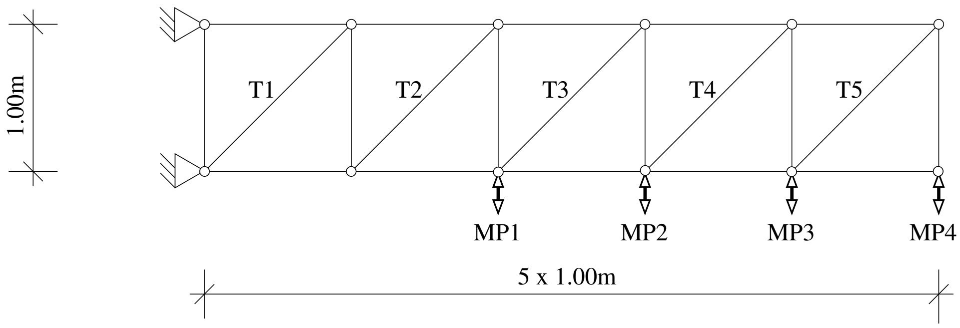


Figure 1: Geometrical description of the cantilever truss system

3.2 Application of mode assignment

It is assumed that a design of an optimization run or a sample of a sensitivity analysis leads to a small change of the Young's moduli of the diagonal elements T2 and T3 to $2.2 \cdot 10^{11}\frac{\text{N}}{\text{m}^2}$. The first nine modes of the changed system are presented in Figure 3. For reliable results, it is essential that the measured modes are assigned to the most likely numerical mode based only on the vertical modal displacements of the four measurement points. The MAC values between the numerical and noise disturbed experimental modes are presented in Figure 4. It can be observed that the MAC values between the second experimental mode and the second and third numerical modes are close to 1 which indicates an almost perfect agreement in both cases. If the original MAC is used to select the mode, the wrong 3rd numerical mode, that is mainly a longitudinal mode, will be assigned to the second experimental mode, because the MAC value is slightly larger. The results are given in Figures 5 and 6. This wrong mode assignment can

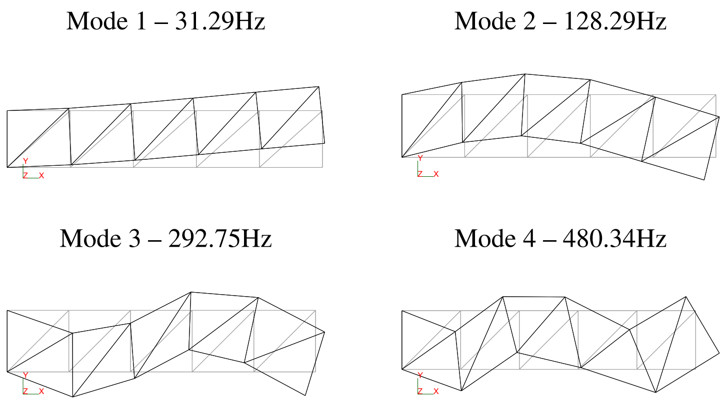


Figure 2: First four measured vertical mode shapes of the system

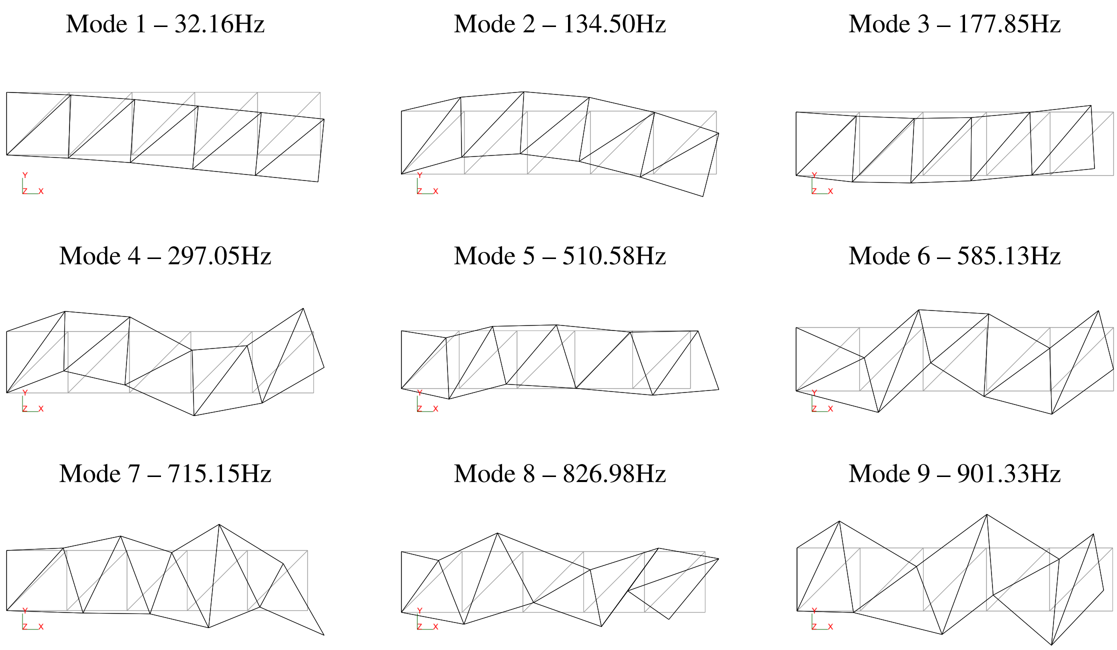
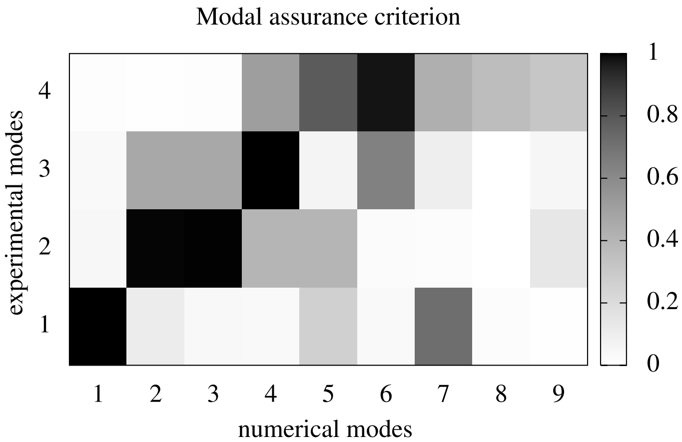


Figure 3: First nine mode shapes of the modified numerical model

cause essential problems for some investigations, for example, the sensitivity analysis or model updating.

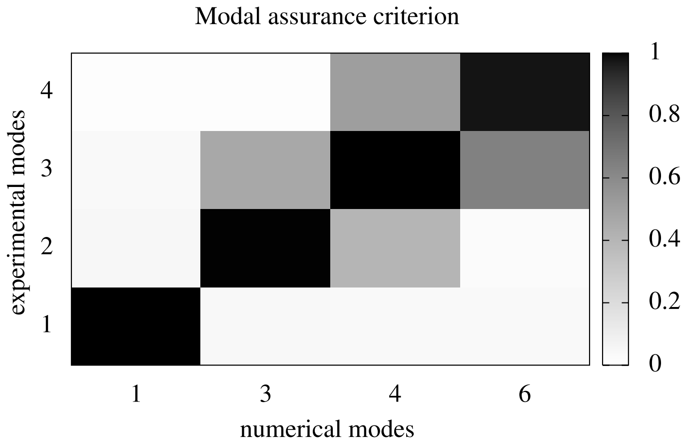
The proposed alternative approach uses the relative modal strain energy to pair the correct modes. Therefore, the total degrees-of-freedom are separated into two clusters: cluster 1 contains the vertical degrees-of-freedom and cluster 2 contains the horizontal degrees-of-freedom. The relative modal strain energies according to Equation (8) are visualized in Figure 11. By this criterion, the modes can be distinguished in primary horizontal and primary vertical modes. The EMAC according to Equation (9) is presented in Figure 7. The largest value in each row indicates the numerical mode that has to be assigned. The EMAC and the original MAC of the identified modes are shown in Figures 8 and 9, respectively. Figure 10 presents the assigned numerical mode shapes. It can be observed, that the correct second numerical mode can be assigned to the second experimental mode.

This numerical benchmark study shows that the MAC is able to pair the correct modes as long as the modes can be reliably separated based on the sparse spatial information. If the modes cannot be separated by the available information, the physical information of the modes, namely the modal strain energy, can be used to distinguish between modes with similar MAC values.



	1	2	3	4	5	6	7	8	9
1	1.00	0.08	0.03	0.03	0.19	0.02	0.57	0.01	0.00
2	0.03	0.98	0.99	0.29	0.29	0.02	0.01	0.00	0.10
3	0.03	0.34	0.34	1.00	0.04	0.50	0.07	0.00	0.04
4	0.01	0.00	0.01	0.38	0.64	0.92	0.31	0.26	0.23

Figure 4: Modal assurance criterion – numerical vs. experimental modes



	1	3	4	6
1	1.00	0.03	0.03	0.02
2	0.03	0.99	0.29	0.02
3	0.03	0.34	1.00	0.50
4	0.01	0.01	0.38	0.92

Figure 5: Modal assurance criterion – identified numerical vs. experimental modes

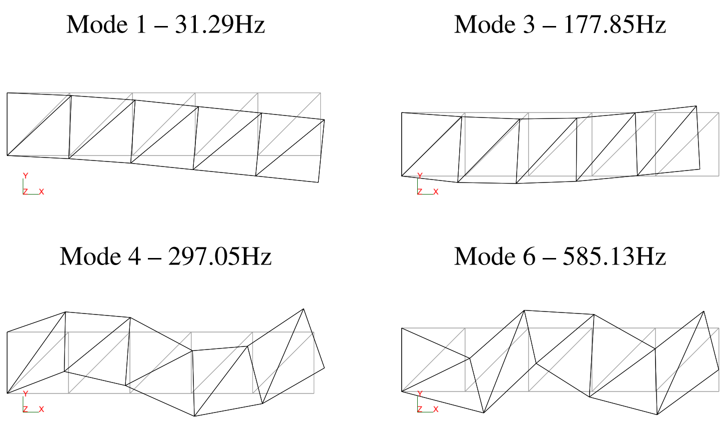
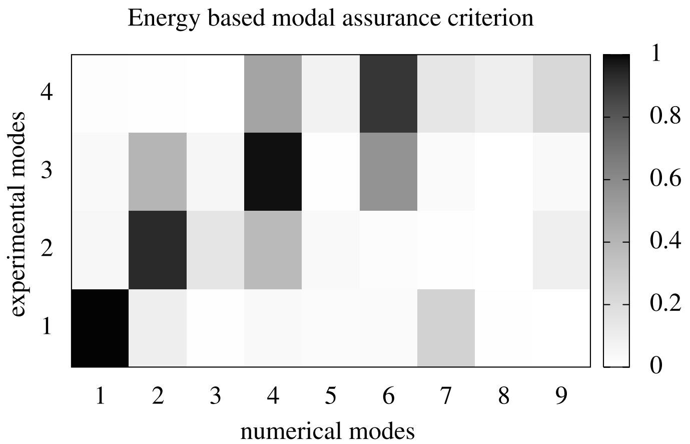
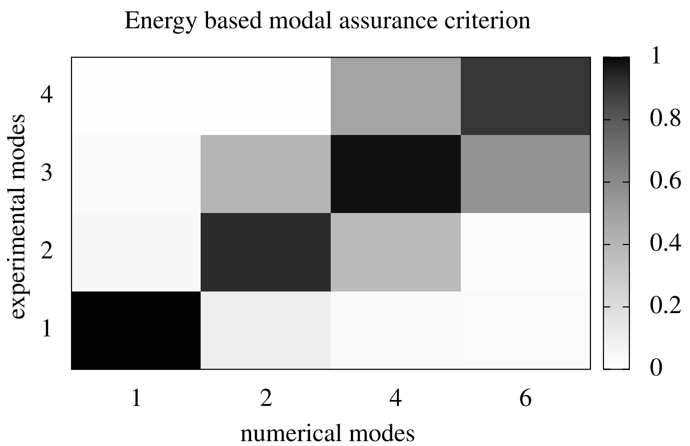


Figure 6: Identified mode shapes from numerical modal analysis using MAC



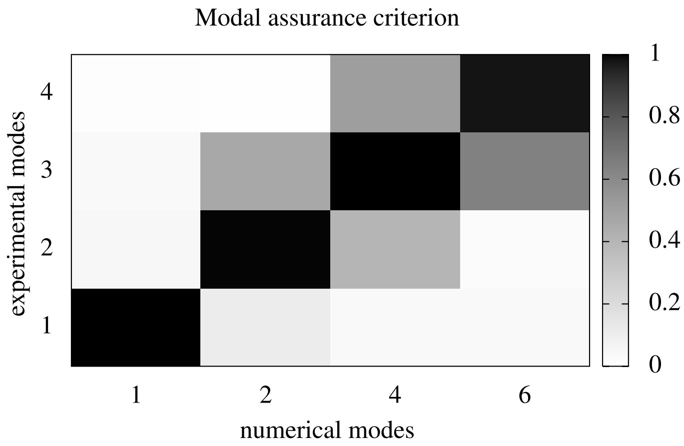
	1	2	3	4	5	6	7	8	9
1	0.99	0.07	0.00	0.02	0.01	0.02	0.18	0.00	0.00
2	0.03	0.83	0.10	0.27	0.02	0.01	0.00	0.00	0.06
3	0.03	0.30	0.03	0.94	0.00	0.42	0.02	0.00	0.02
4	0.01	0.00	0.00	0.36	0.05	0.79	0.10	0.07	0.15

Figure 7: Energy based modal assurance criterion for vertical degree-of-freedom – numerical vs. experimental modes



	1	2	4	6
1	0.99	0.07	0.02	0.02
2	0.03	0.83	0.27	0.01
3	0.03	0.29	0.94	0.42
4	0.01	0.00	0.36	0.79

Figure 8: Energy based modal assurance criterion for vertical degree-of-freedom – identified numerical vs. experimental modes



	1	2	4	6
1	1.00	0.08	0.03	0.02
2	0.03	0.98	0.29	0.02
3	0.03	0.34	1.00	0.50
4	0.01	0.00	0.38	0.92

Figure 9: Modal assurance criterion – identified numerical vs. experimental modes. Numerical modes are previously selected by the energy based modal assurance criterion (EMAC)

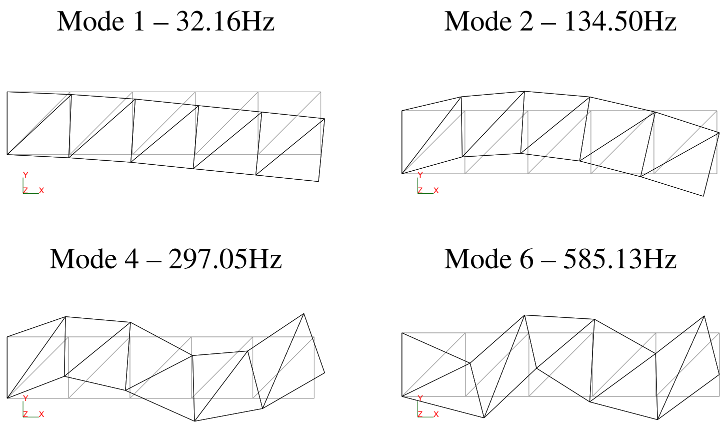


Figure 10: Identified mode shapes from numerical modal analysis using the energy based modal assurance criterion (EMAC)

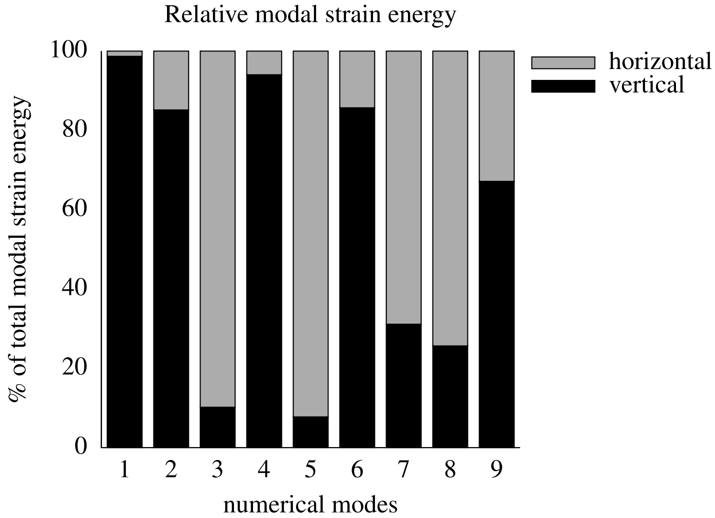


Figure 11: Relative modal strain energies for vertical and horizontal degrees-of-freedom

4 BENCHMARK STUDY: HIGH-SPEED RAILWAY BRIDGE ERFTTAL

4.1 Description of the system

The numerical model is based on a high-speed railway bridge at the line between Cologne and Brussels, which has been discussed in several references [3, 4, 5]. The filler beam bridge consists of two main substructures, each carrying one rail line. The rail is installed on ballast that is continuously distributed over both substructures and the transition zones between bridge and soil. Figures 12, 13, and 14 show the simplified bridge model and the resulting finite element model. Experimental mode shapes and natural frequencies are available from an experimental campaign which has been described in [4, 5]. The mode shapes and the natural frequencies obtained from the experimental campaign are given in Table 2. Due to the limitation in the experimental setup, the modal displacements are only available at 44 points of the bottom side of the composite slabs in vertical direction.

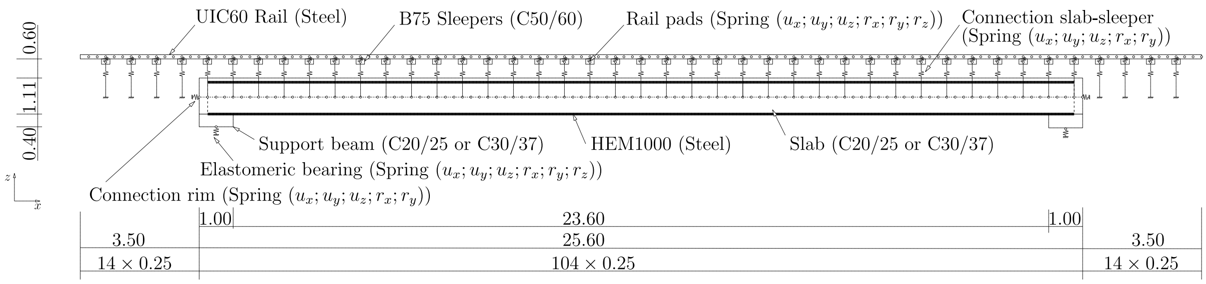


Figure 12: Longitudinal section of the simplified Erfttal bridge model.

As the model is supposed to be used for model updating, a correct mode assignment is essential. A total number of 34 uncertain material parameters has been chosen which will be varied by a stochastic sampling scheme within a global sensitivity analysis.

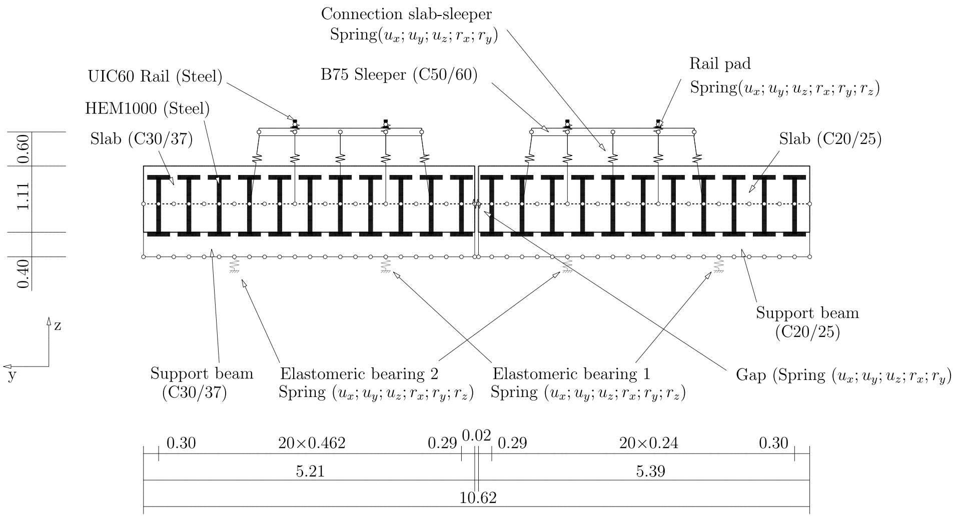


Figure 13: Cross section of the simplified Erfttal bridge model.

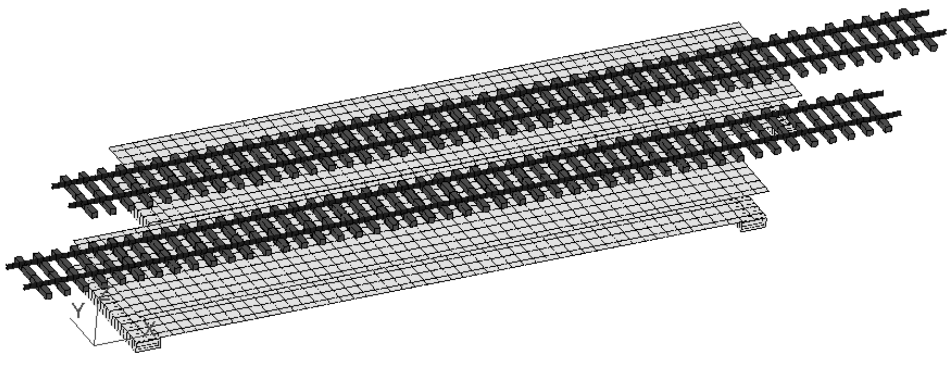


Figure 14: Finite element model of the Erfttal bridge.

4.2 Application of mode assignment

A set of 34 parameters is generated by the stochastic sampling. For this set a numerical modal analysis has been performed to extract the first 200 mode shapes and natural frequencies which represent a frequency range between 0Hz and 50Hz.

In a first approach, the original MAC will be used to assign the experimental to the corresponding numerical mode shapes. The MAC matrix for all 200 numerical and 7 experimental modes are given in Figure 15. The largest value in each row indicates the numerical mode which has to be assigned. The MAC values between the experimental and identified numerical mode shapes are arranged in a matrix, given in Figure 16.

By applying the energy based modal assurance criterion for the assignment, the EMAC and MAC matrix can be obtained, which are visualized in Figures 17 and 18, respectively. It has been assumed that the main modal strain energy is present in vertical components of the two main composite slabs.

The compared mode selection methods assign some different numerical modes. The obtained modes are compared in Table 2. In case of measured modes 3, 5, 6, and 7, it can be observed that the assignment using the original MAC fails, whereas the EMAC values constitute a more reliable result.

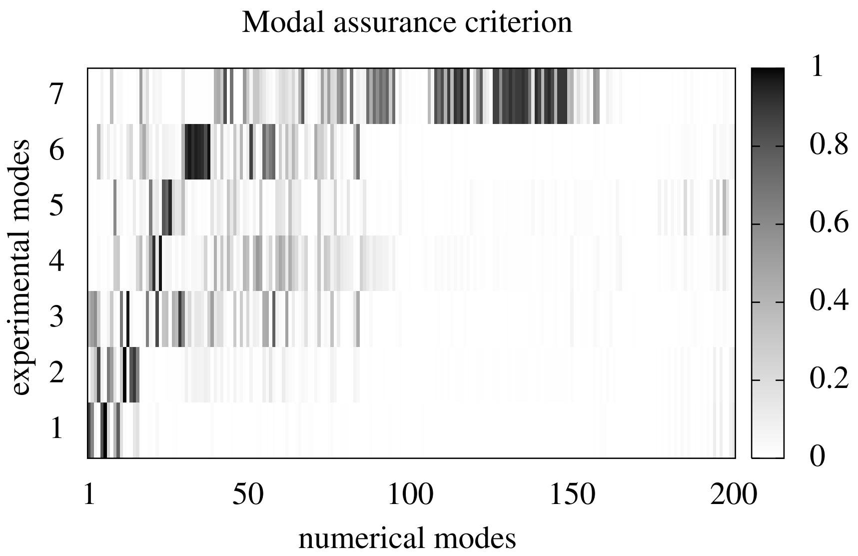


Figure 15: Modal assurance criterion (MAC) – numerical vs. experimental modes of the Erfttal bridge

4.3 Global sensitivity analysis

This example demonstrates the influence of an inappropriate mode selection algorithm by means of a global sensitivity analysis. The 34 selected, uncertain parameters of the model are varied by a stochastic sampling scheme, the latin hypercube sampling. For the resulting samples the modulus of the linear Spearman correlation coefficient [18] between each input and output parameter is assorted in a matrix. The matrix is used to assess the sensitivity of each parameter

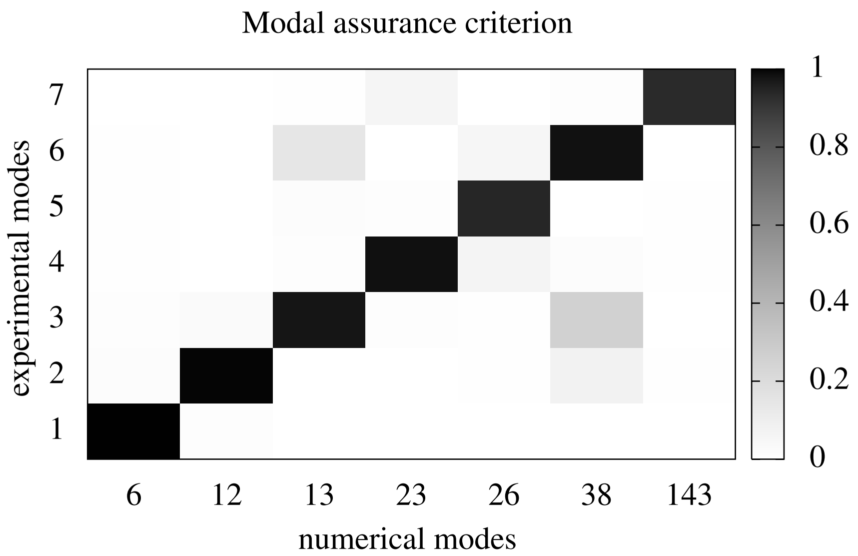


Figure 16: Modal assurance criterion – identified numerical vs. experimental modes of the Erfttal bridge

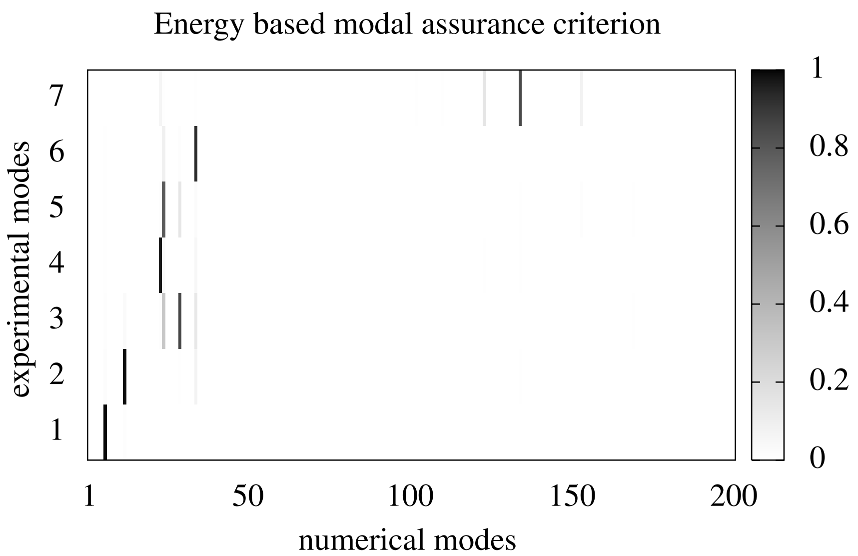


Figure 17: Energy based modal assurance criterion (EMAC) for vertical degree-of-freedom of the slab – numerical vs. experimental modes of the Erfttal bridge

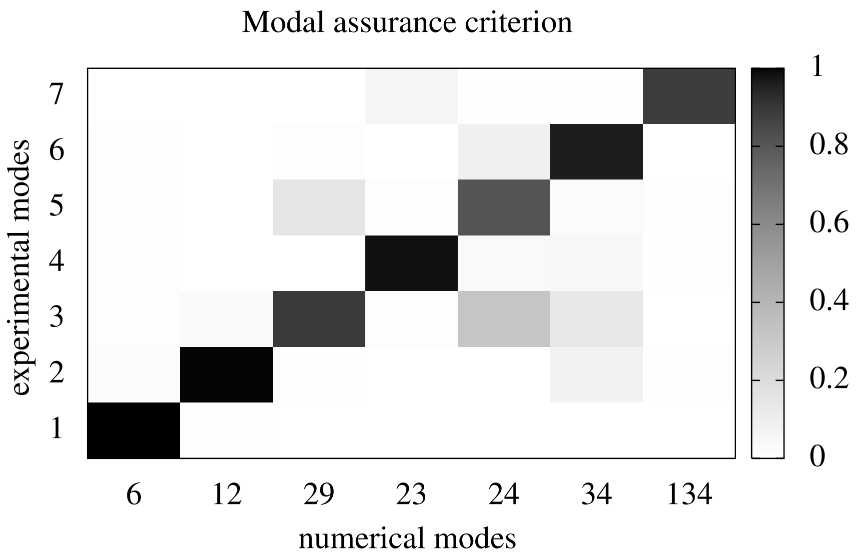


Figure 18: Modal assurance criterion – identified numerical vs. experimental modes of the Erfttal bridge. The numerical modes are selected previously by the energy based model assurance criterion.

with respect to a certain calculated modal parameter.

The assignment of numerical modes to the experimental modes is important. Figure 19 shows the result, if the traditional modal assurance criterion is used for the mode assignment. By using the same design space, but the energy based modal assurance criterion (EMAC) for the mode assignment, a different result can be obtained, as presented in Figure 20. For example, the MAC approach indicates a sensitivity of bearings, which disappears when the energy based criterion is applied. Hence, a wrong parameter set will be used for the subsequent finite element model updating.

5 SUMMARY AND CONCLUSIONS

This paper emphasises the problem of wrong mode selection by using the traditional modal assurance criterion (MAC) by means of a numerical and experimental benchmark study. An innovative criterion combines the common mathematical modal assurance criterion with additional physical information, the modal strain energies of the numerical eigenvectors. This energy based modal assurance criterion is denoted by EMAC. It has been shown that this additional information leads to a more reliable mode assignment. The consequences of a wrong mode selection have been demonstrated by a global sensitivity analysis where wrong sensitivities can lead to a wrong selection of parameters for the model updating.

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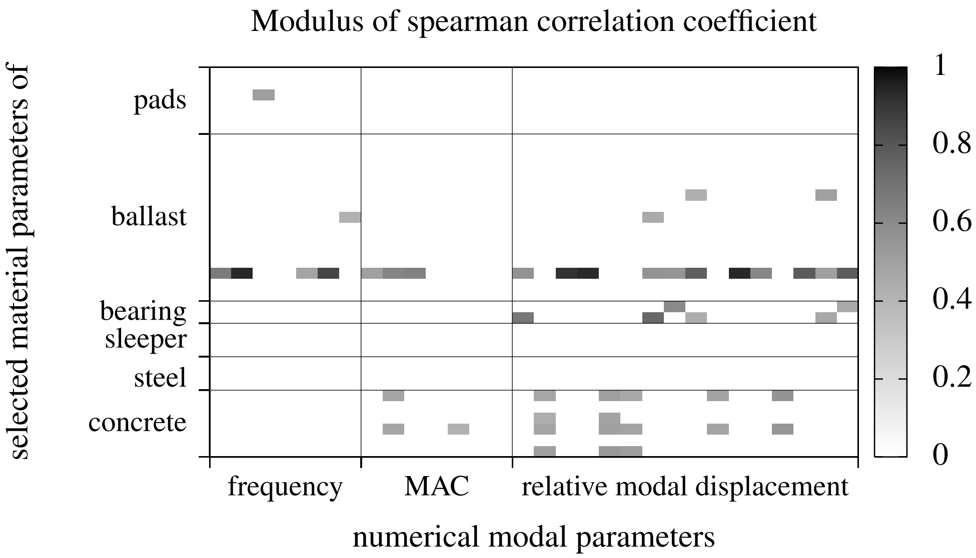


Figure 19: Modulus of the linear spearman correlation coefficient based on 750 samples using the modal assurance criterion for mode assignment. Coefficients smaller than 0.3 are set to 0.

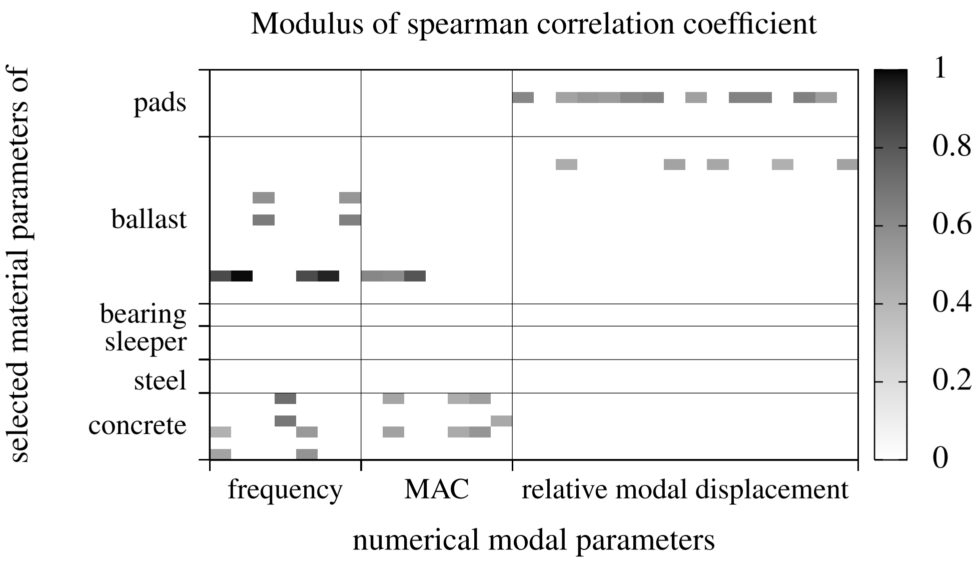
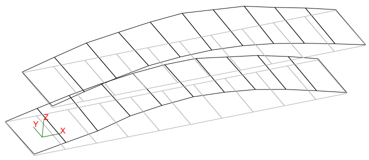
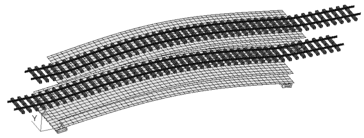
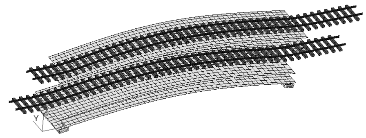
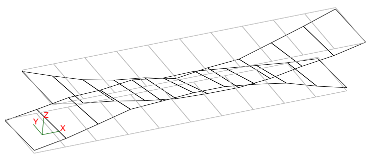
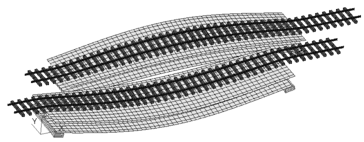
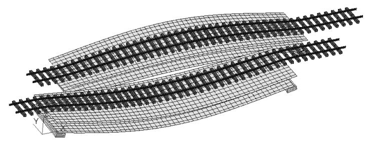
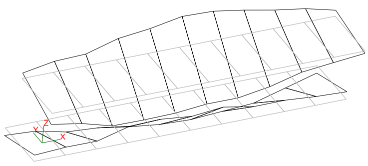
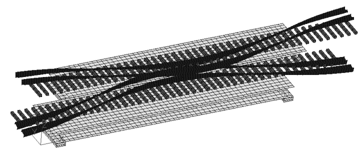
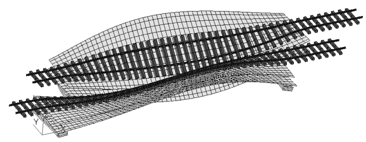
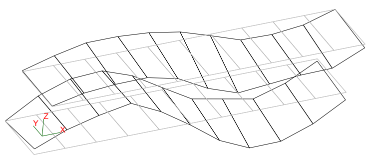
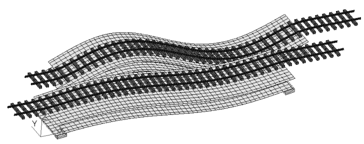
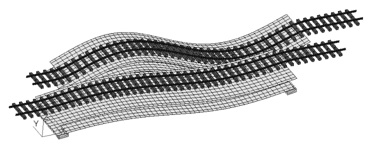
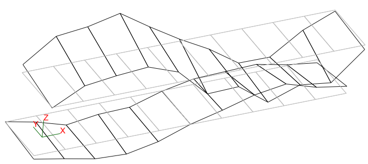
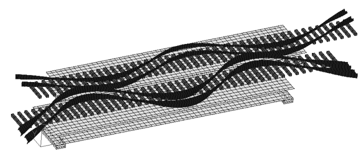
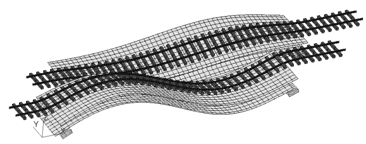
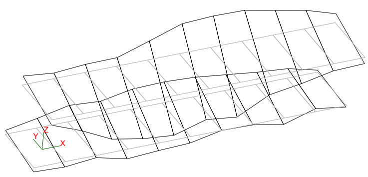
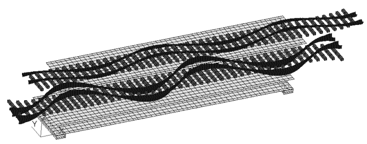
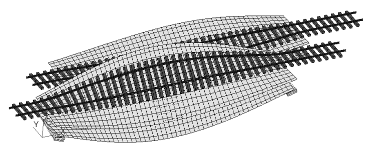
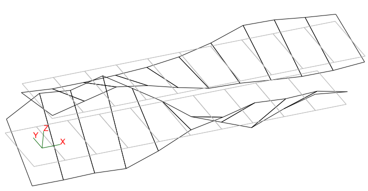
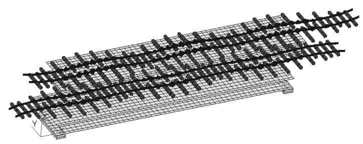
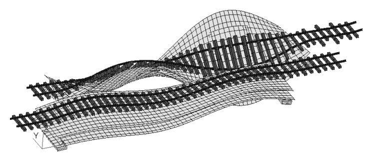


Figure 20: Modulus of the linear spearman correlation coefficient based on 750 samples using the energy based modal assurance criterion for mode assignment. Coefficients smaller than 0.3 are set to 0.

Table 2: Comparison of identified numerical mode shapes with the mode shapes obtained from measurements

	measured mode shape	numerical mode shape identified by MAC	numerical mode shape identified by EMAC
1	3.67Hz	3.67Hz	3.67Hz
			
2	5.24Hz	5.82Hz	5.82Hz
			
3	9.36Hz	7.28Hz	15.14Hz
			
4	13.17Hz	12.69Hz	12.69Hz
			
5	13.71Hz	14.38Hz	13.89Hz
			
6	15.09Hz	18.86Hz	18.38Hz
			
7	20.98Hz	31.41Hz	30.75Hz
			

opTimal Life cycle costs of high-speed rAILway bridges by enhanced monitoring systems), that is supported by the European Research Fund for Coal and Steel (RFCS).

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