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ROTATIONAL DUCTILITY OF CRACK IN STATIC AND DYNAMIC CALCULATIONS OF REINFORCED CONCRETE BAR STRUCTURES

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Abstract. *In this paper experimental studies and numerical analysis carried out on reinforced concrete beam are partially reported. They aimed to apply the rigid finite element method to calculations for reinforced concrete beams using discrete crack model. Hence rotational ductility resulting from crack occurrence had to be determined. A relationship for calculating it in static equilibrium was proposed. Laboratory experiments proved that dynamic ductility is considerably smaller. Therefore scaling of the empirical parameter was carried out. Consequently a formula for its value depending on reinforcement ratio was obtained.*

1 INTRODUCTION

Some research concerning reinforced concrete beams [1, 2] including own research [3] exhibit differences in terms of static and dynamics issues. Those differences are reflected by deflections (statics) and natural frequencies. Those quantities, however, could not be directly compared. Nonetheless, based on their values and known scheme stiffness of elements can be computed [4] (static based on deflections and dynamic based on natural frequencies). Research carried out thus far proved, those are not the same quantities.

The approach presented by authors draws on discrete crack model in rigid finite elements method [5, 6]. Hence, formulae for equivalent stiffness (both static and dynamic) could not be used. The rigid finite element method requires on the other hand implementing ductility dictated by crack occurrence. This paper discusses method of scaling relationship expressing dynamic rotational ductility based on the static one. Scaling was carried out based on own experimental studies.

2. EXPERIMENTAL STUDIES CHARACTERIZATION

Experimental studies were performed on reinforced concrete beams in half-natural scale. Each of the elements had the dimensions of 3300 mm x 250 mm x 150 mm. The cross-sections with a reinforcement are shown in figure 1.

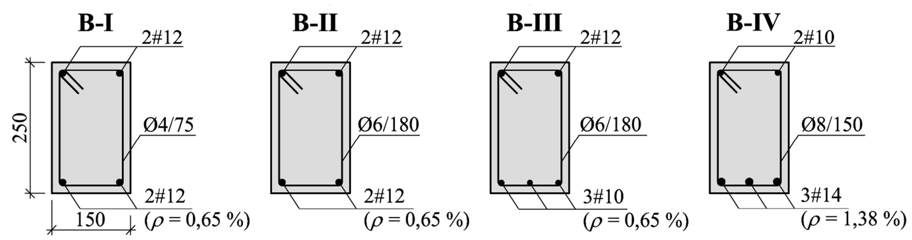

Figure 1: Investigative elements (dimensions in mm)

Series B-I, B-II, B-III had the same tensile reinforcement ratio of 0,65 %. The B-IV beams series were reinforced stronger (1,38 %). The elements were made of the C25/30 class concrete. The basic material properties are listed in table 1.

Table 1: Basic material properties

Material	Property	Series			
		B-I	B-II	B-III	B-IV
Concrete	Mean compressive strength f_{cm} [MPa]	51,7	51,2	45,0	41,1
	Mean splitting tensile strength $f_{ctm,spl}$ [MPa]	3,58	3,21	3,03	2,79
	Mean Young modulus E_{cm} [GPa]	30,3	29,6	28,5	30,0
Steel (longitudinal rebars)	Mean yield strength f_{ym} [MPa]	563	563	548	555
	Mean Young modulus E_{sm} [GPa]	202	202	200	202

The beams' deflections were registered with the inductive gauges with accuracy of 0,001 mm. The beams were loaded with concentrated force applied at the mid-span (three points bending test).

A Brüel & Kjær data acquisition and processing system was used in the dynamic measurements. The system uses the operational version of the modal analysis [7] – presently, a popular tool for nondestructive testing of engineering structures and machines. The system registers the beam's response (acceleration of certain points) on external random forces. The vibrations in the beams are caused randomly by the setup environment, and include: acoustic noise, air flow, gentle strokes in investigative element. The measurements yield basic dynamic parameters of the object investigated (eigenfrequencies, eigenforms, damping parameters). It was decided to carry out the dynamic experiments with using the suspended beam scheme. This approach is commonly used in investigating mechanisms and their characteristics [8].

Each test was preceded by the dynamic analysis of a suspended beam. Followingly, the element under tests was placed on the supports and loaded at the mid-span with a concentrated force of a given value. The beam deflection was acquired once it stabilized. Subsequently, beam was unloaded and taken from the supports for the dynamic analysis. In the next step, the beam was once again placed on the bearings and loaded with a higher force than in the previous step. The aforementioned procedure was repeated till the beam failure. When the load-bearing capacity was exhausted the modal analysis was performed in the suspended position. The detailed description of the experimental studies is included in [3].

3. STIFF FINIE ELEMENTS METHOD

3.1. General description of the method

In this method beam model consists of stiff mass discs which represent force of inertia of a structure. Discs are connected by elastic constraints (one rotation and two translation) responsible for elastic features of a structure. Movement of each mass discs is described by three general coordinates. In case of transverse vibrations which are considered in this paper, elastic constraints and general coordinates are reduced to two. Example scheme and calculation model of a beam divided into four elements are shown in figure 2. The wider description of the method is included in [9].

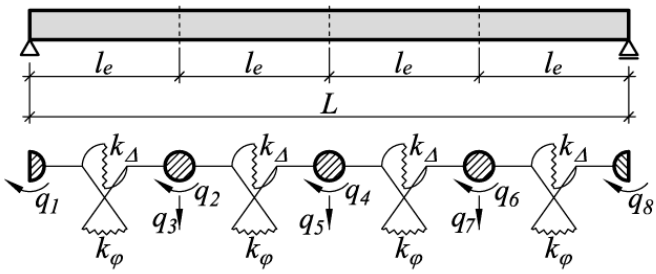

Figure 2: Scheme and numerical model of homogenous beam

The presented approach enables to include local discontinuities (among others cracks) in a discrete way [5, 6]. Adequate division into finite elements allows the introduction of cracks by means of reduction of stiff rotation constraints while calculations are performed as for the homogenous beam.

Stiffnesses of constraints k_φ, k_Δ are commuted using the element stiffness in phase I (EI_I). The stiffness of rotation constraints is reduced and has value k_φ^{cr} in the place where the cracks appear. The scheme and calculation model of the segment of beam with cracks is shown in figure 3.

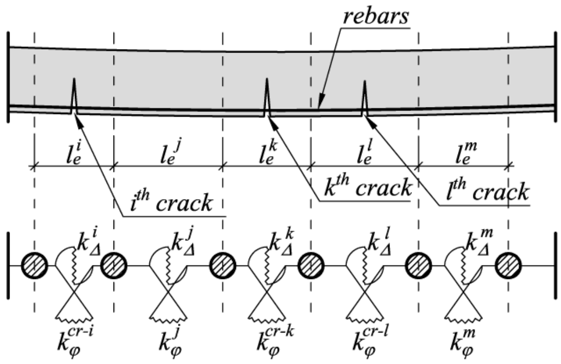

Figure 3: Scheme and numerical model of the reinforced concrete beam with cracks

3.2. Rotational ductility in static calculation

The rotational ductility resulted from crack was estimated on the basis of elementary relations of geometry and strength of materials. The scheme as in figure 4 was considered.

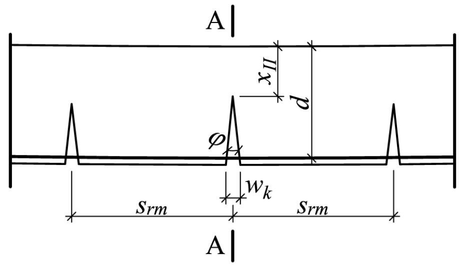

Figure 4: Considered model of beam with cracks

Forces acting in the cross-section (A-A) in the place of crack occurrence are shown in figure 5. Triangular stress distribution in compressed concrete was assumed.

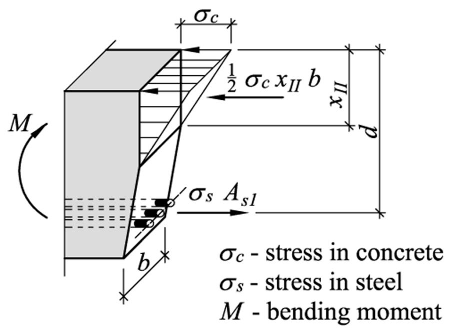

Figure 5: Forces acting in cross-section

Figures 4 and 5 enable to formulate following expression allowing to calculate the rotational susceptibility which is consequence of crack occurrence in static solution:

$$d_{\varphi,static}^{cr-i} = \frac{\psi_z\, s_{rm}}{E_s\, A_{s1}\,(d - \frac{x_{II}}{3})(d - x_{II})}, \tag{1}$$

where: ψ_z – coefficient describing violation of interaction between steel and concrete calculated according to (2), s_{rm} – average crack spacing, E_s – Young's modulus of steel, A_{s1} –

reinforcement cross-sectional area, d – useful beam height, x_{II} – height of the compressed zone in phase II.

$$\psi_z = 1.3 - s\frac{M_{cr}}{M}, \tag{2}$$

where: s – 1.1 in case of immediate loading, 0.8 in case of long-term loading, M_{cr} – cracking moment, M – maximum moment up to which the cross-section was overloaded.

3.3. Comparison with experimental results

In line with above-mentioned assumptions, proprietary program was used to compute deflections of reinforced concrete beams. Results of calculations along experimental results are shown on charts (fig. 6 – 9). Solid line traces equilibrium paths recorded during experiment, points represent analytical results for each load increment. Individual elements from different series had different colours.

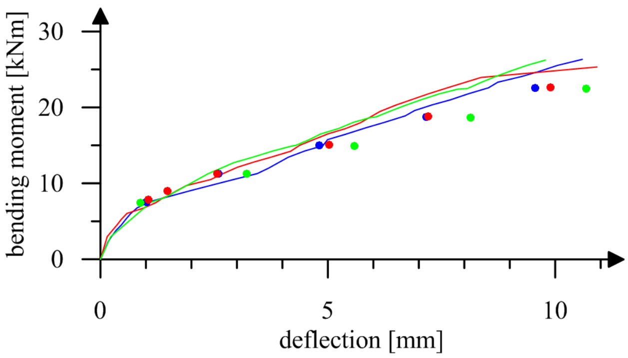

Figure 6: Deflection vs. bending moment for B-I series

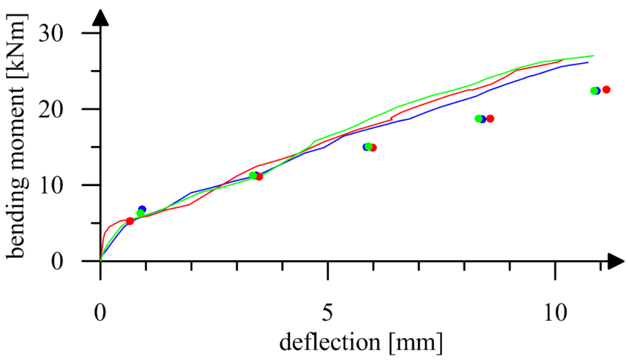

Figure 7: Deflection vs. bending moment for B-II series

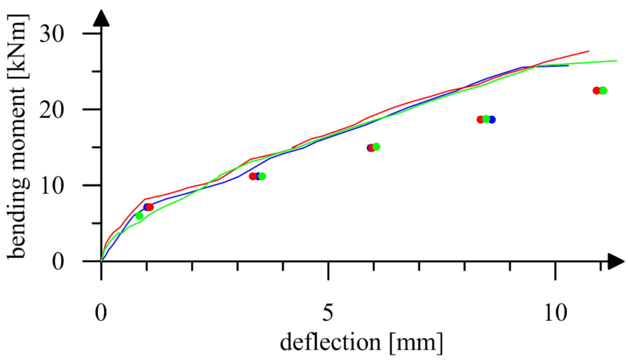

Figure 8: Deflection vs. bending moment for B-III series

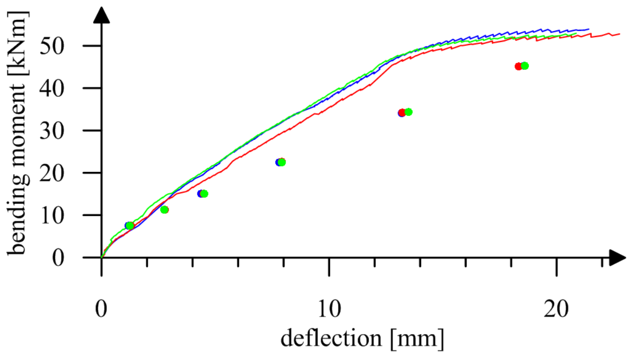

Figure 9: Deflection vs. bending moment for B-IV series

Obtained results proved highly consistent, especially up to 50% load applied to lightly reinforced beams (series B-I, B-II, B-III) and up to 30% load applied to highly reinforced beams (series B-IV). Results outside that range show greater inconsistencies (approx. 30%). Deflection could have been overestimated, because computations included every macroscopically observable during the experiment crack. Cracks occurring under heavier loads (closer to supports) were not as deep as cracks occurring under lighter loads. The proposed numerical model envisages every crack penetrating to natural axis of beam. Results of calculations have therefore confirmed correctness of the model and usability of presented method in computing static deflection of cracked reinforced concrete beams.

4. SCALING OF PARAMETER

As aforementioned in preliminary section of the paper, static and dynamic ductility of cracked reinforced concrete beams might vary. Hence it is fair to say that rotational ductility resulted from crack will differ from dynamic one. Hence:

$$d_{\varphi,dynamic}^{cr-i} = \alpha_d \cdot d_{\varphi,static}^{cr-i}, \tag{3}$$

where: α_d – empirical coefficient.

The parameter was scaled iteratively. During laboratory experiments at each load increment, crack perpendicular to element's axis were macroscopically catalogued. After cataloguing, cracks were sketched. Example sketch for B-I-1 beam is shown in figure 10. Next to each crack is given the load that caused it.

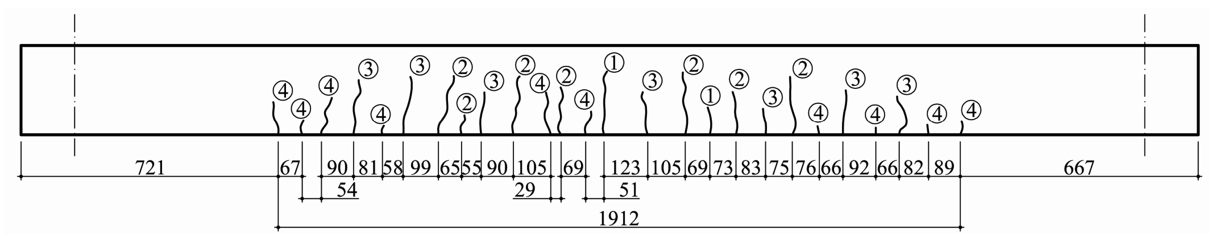

Figure 10: Sketch of crack - element B-I-1 (mm)

For each load increment, computations were then carried out using proprietary program. Each model was recalculated several times for different α_d parameters. Selected were values where inconsistencies between analytical and experimental natural frequency were the smallest. Example chart illustrating the process of scaling the B-I-1 beam is shown in figure 11.

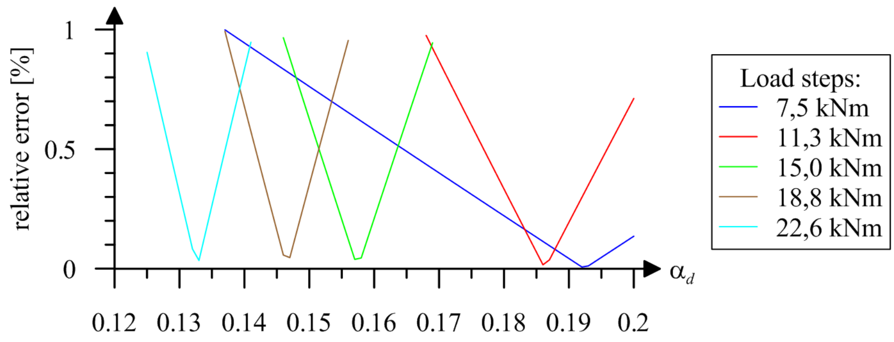

Figure 11: Relative difference depending on α_d parameter for individual load increments

Results of calculations for individual beams were analysed and function describing scaled parameter selected. Tested initial functions are shown in table 2.

Table 2: Functions describing parameter α_d

No.	Function	Relative function error [%]				
		B-I	B-II	B-III	B-I, B-II, B-III $\rho = 0{,}65$ %	B-IV $\rho = 1{,}38$ %
1	$e^{-\beta x}$	0,46	1,03	0,48	0,67	0,42
2	$\alpha \cdot e^{-\beta x}$	0,09	0,53	0,36	0,33	0,16
3	$\alpha \cdot \gamma^{-\beta x}$	0,10	0,57	0,39	-	0,17
4	$\alpha + x^{-\beta}$	**0,09**	**0,51**	**0,35**	**0,32**	**0,13**
5	$Cot(\alpha \cdot x^{\beta}) + \gamma$	0,09	0,56	0,38	0,33	0,14
6	$\alpha + e^{-\beta x}$	0,10	0,55	0,46	0,36	0,20

e – Eulerian number
α, β, γ – parameters
$x = M/M_R$ (effort level)

Model exhibiting the smallest error (function no. 4) was selected and used for further analyses. Values of parameters α and β are given in table 3.

Table 3. Parameters of used model

Parameter	B-I	B-II	B-III	B-I, B-II, B-III $\rho = 0{,}65$ %	B-IV $\rho = 1{,}38$ %
α	-0,916	-0,926	-0,947	-0,929	-0,958
β	0,0941	0,0955	0,157	0,116	0,0732

Due to negligible difference in the α parameter for both lightly and highly reinforced concrete, it was averaged for further calculations. Moreover, the β was assumed linearly variable as the function of reinforcement ratio given by the relationship:

$$\beta = -0{,}154 + 5{,}823 \cdot \rho \,. \tag{4}$$

The final formula for calculating the α_d coefficient given by:

$$\alpha_d = -0{,}944 + \left(M \big/ M_R \right)^{-0{,}154 + 5{,}823\,\rho} \,. \tag{5}$$

Figure 11 shows nomograph of the parameter for selected reinforcement ratios.

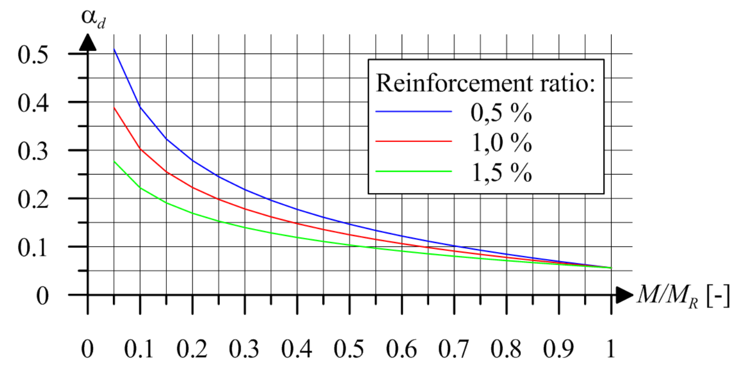

Figure 11: Nomograph of the α_d parameter for selected reinforcement ratios

5. FINAL REMARKS AND CONCLUSIONS

Carried out analyses proved differences in static and dynamic behaviour between reinforced concrete beams pertain not only to the global parameter of flexural rigidity. It was observed they also concern aspects of local description. In this paper used were the rigid finite element method and discrete crack modelling. Hence rotational ductility resulting from crack appearance had to be determined. For static issues an own relationship was used (1), derived based on elementary relationships from strength of materials. Proposed model showed promising results in terms of static calculations.

For dynamic calculations the relationship (1) was modified by introducing the empirical coefficient of α_d. Scaling was carried out based on own experimental studies. It was proved that dynamic ductility decreases relative to static ductility. It is the lower the higher the overload of element. The research has thus far proved that this difference depends also on the reinforcement ratio (fig. 11).

REFERENCES

[1] S. Jerath, M. M. Shibani: Dynamic Stiffness and Vibration of Reinforced Concrete Beams. ACI Journal, **82-18**, 196-202, 1985.

[2] K. C. Johns, M. D. Belanger: Dynamic Stiffness of Concrete Beams. ACI Journal, **78-18**, 201-205, 1981.

[3] M. Musiał: Vibrations of reinforced concrete beams with consideration of discrete crack model (in Polish) – PhD dissertation. Wroclaw University of Technology, Wroclaw, 2010.

[4] M. Musiał: Static and dynamic stiffness of reinforced concrete beams. Archives of Civil and Mechanical Engineering, **12**, 186-191, 2012.

[5] M. Kamiński, M. Musiał, A. Ubysz: Eigenfrequencies of the reinforced concrete beams – methods of calculations. Journal of Civil Engineering and Management, **17**, 278–283, 2010.

[6] M. Musiał, M. Kamiński, A. Ubysz: Free vibration frequencies the cracked reinforced concrete beams-methods of calculations. K. Gürlebeck and C. Könke eds. 18th

International Conference on the Applications of Computer Science and Mathematics in Architecture and Civil Engineering, IKM 2009, Weimar, Germany, 2009.

[7] M. Batel: Operational Modal Analysis – Another Way of Doing Modal Testing. Sound and Vibration, 22-27, August 2002.

[8] A. S. Ghods, B. Moghaddasie: Evaluating the dynamic characteristics of reinforced concrete beams. M. Motavalli ed. Fourth International Conference on FRP Composites in Civil Engineering, CICE 2008, Zurich, Switzerland, 2008.

[9] J. Langer: Dynamics of structures (in Polish). Wroclaw University of Technology Publishing House, Wroclaw, 1980.