SYSTEM IDENTIFICATION METHODS FOR GROUND MODELS IN MECHANIZED TUNNELING

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Abstract.   *Due to the complex interactions between the ground, the driving machine, the lining tube and the built environment, the accurate assignment of in-situ system parameters for numerical simulation in mechanized tunneling is always subject to tremendous difficulties. However, the more accurate these parameters are, the more applicable the responses gained from computations will be. In particular, if the entire length of the tunnel lining is examined, then, the appropriate selection of various kinds of ground parameters is accountable for the success of a tunnel project and, more importantly, will prevent potential casualties. In this context, methods of system identification for the adaptation of numerical simulation of ground models are presented. Hereby, both deterministic and probabilistic approaches are considered for typical scenarios representing notable variations or changes in the ground model.*

1 INTRODUCTION

In July 2010 a new collaborative research center (**SFB** 837) started at Ruhr-Universitt Bochum Germany, entitled *Interaction Models in Mechanized Tunneling*. The center consists of 14 subprojects and it is funded by the German Research Foundation (**DFG**). This paper is part of the work conducted in the subproject **C2** Methods of System Identification for the Adaptation of Numerical Simulation Models.

The ground model is central to computational tunneling, where a realistic ground model is crucial for predicting the distributions and magnitudes of the strains and, consequently, reducing the surface settlements caused by the TBM propagation. Based upon the information of bore holes sunken in the target area of the tunnel alignment, the ground model describes the detailed spatial distribution of the constitutive soil properties along with the geometry of the stratification. Customary bore holes, however, provide only an approximate insight into the real world geologic realities. As a consequence, only the realization of the system identification approach can result in improved and more sophisticated numerical predictions of the spatiotemporal ground behavior induced by driving the tunnel.

For system identification, a numerical simulation model is required. This model, however, represents a complex, mechanically-hydraulically coupled and a three dimensional initial boundary value problem. Also, it is characterized by various physical nonlinearities as well as various construction stages. For application in a later reference tunnel project, numerical simulation models are needed which enable the appropriate *forward computation* of typical scenarios describing relevant variations or changes in the ground model with respect to prescribed output states.
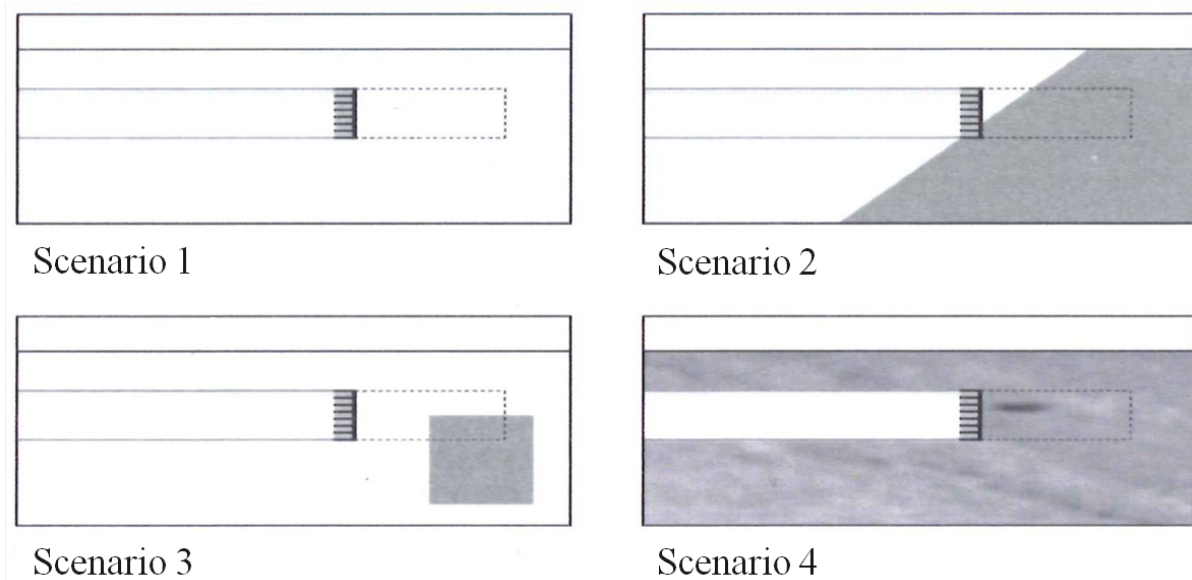

Figure 1: Subsoil scenarios for the forward computation.

According to Figure 1, the following scenarios have been defined:

- Scenario #1: advance in homogeneous soil.

- Scenario #2: advance in two homogeneous sub-soils where the parameters and inclination of the second one are unknown.

- Scenario #3: advance in a homogeneous subsoil with a cubic impediment (unknown in position, dimension and material parameters) in front of the tunnel face.

- Scenario #4: advance in an inhomogeneous subsoil with unknown parameters and spatial distribution.

2 METHODOLOGY

Methodologically, the solution of a system identification problem is based on the concatenation of observations (measurements) and computations (numerical results) using the inverse analysis procedure. By that, the defect between the measurement-based properties and the computed ones is being minimized.

2.1 2.1 Deterministic Approach

In deterministic inverse analysis, the selected numerical model is calibrated by iteratively changing a subset of its parameters until the discrepancies between the calculated/simulated responses and the observed/measured data reach a predefined minimum [1]. This procedure is illustrated in Figure 2.
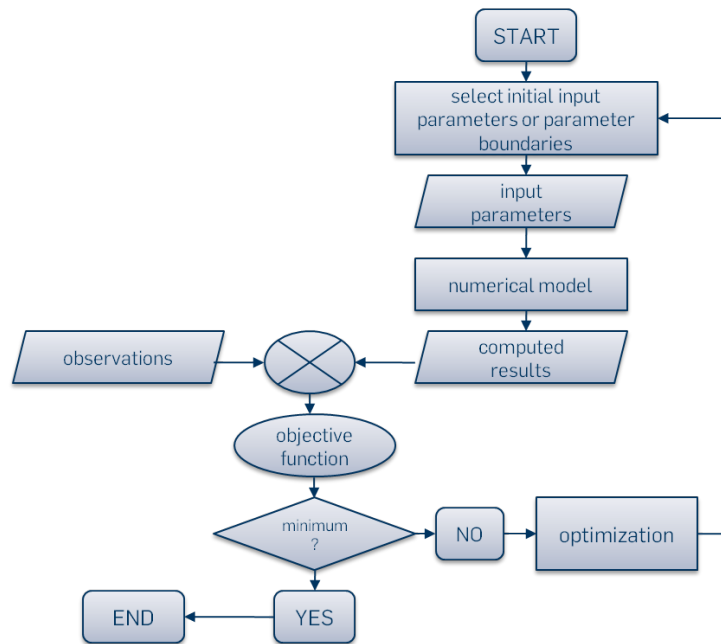

Figure 2: Scheme of inverse analysis procedure.

In the first step of the procedure, a priori estimation or guess of the unknown parameter set is given or a reasonable range of each parameter is defined. After that, the forward model (numerical simulation) is called for the initial guess of the parameters. Subsequently, the obtained numerical results are compared with the observed/measured data in field. The discrepancy between the two sets of data is quantified by an *objective function* which is to be minimized applying an *optimization algorithm*. The set of parameters which minimizes the objective function is the best estimated set of the unknown model parameters. The objective function considered in this approach is the least squares criterion:

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$$f(\mathbf{m}) = \sum_{i=1}^{N}(d_{obs}^i - g_i(\mathbf{m}))^2 \tag{1}$$

where g_i is the calculated value for the point i, $\mathbf{m}$ is the set of the model parameters, and d_{obs}^i is the observed value at the same point.

2.2 Probabilistic Approach

Customarily, the uncertainties associated with the geotechnical applications can be categorized as follows [2]:

- Natural variability (aleatory/objective uncertainty) associated with the inherent randomness of natural processes, like wind flow and geologic layers. In this case samples are taken and inferences are drawn.

- Knowledge uncertainty (epistemic/subjective uncertainty) caused by the lack of data or information about events and processes, and of understanding the physical laws, where the following subcategories may be distinguished:

  - Site characterization uncertainty.
  - Model uncertainty.
  - Parameter uncertainty.

In order to include the aforementioned uncertainties (inherent in the forward model and the measured data) and the prior knowledge about the unknown parameters (indicating the trend of the parameters), a *Probabilistic Approach* based on *Bayes Theorem* (Thomas Bayes, 1702-1761) is incorporated in the inverse problem considered here. This approach yields a solution that provides suitable uncertainty measures [3] as follows:

- The prior information of the model parameters and the uncertainties in the observed data are represented in terms of two *independent* probability density functions (PDFs), $\rho_M(\mathbf{m})$ and $\rho_D(\mathbf{D})$ respectively, with a joint PDF:

$$\rho(\mathbf{m}, \mathbf{d}) = k\rho_M(\mathbf{m})\rho_D(\mathbf{d}) \tag{2}$$

  where k is the normalization constant.

- The effect of the modeling uncertainties is mapped by a PDF referred to as the forward model probability:

$$\Theta(\mathbf{m}, \mathbf{d}) = \theta(\mathbf{d}|\mathbf{m})\mu_M(\mathbf{m}) \tag{3}$$

  where $\mu_M(\mathbf{m})$ is the homogeneous probability density over the model space M.

- Combining the prior information and the forward model probability by the *conjunction operation* gives the probabilistic solution (see Figure 3):

4

$$\begin{aligned}
\sigma(\mathbf{m},\mathbf{d}) &= k\frac{\rho(\mathbf{m},\mathbf{d})\Theta(\mathbf{m},\mathbf{d})}{\mu(\mathbf{m},\mathbf{d})} \\
&= k\frac{\rho_M(\mathbf{m})\rho_D(\mathbf{d})\theta(\mathbf{d}|\mathbf{m})\mu_M(\mathbf{m})}{\mu_M(\mathbf{m})\mu_D(\mathbf{d})} \\
&= k\frac{\rho_M(\mathbf{m})\rho_D(\mathbf{d})\theta(\mathbf{d}|\mathbf{m})}{\mu_D(\mathbf{d})}
\end{aligned} \tag{4}$$

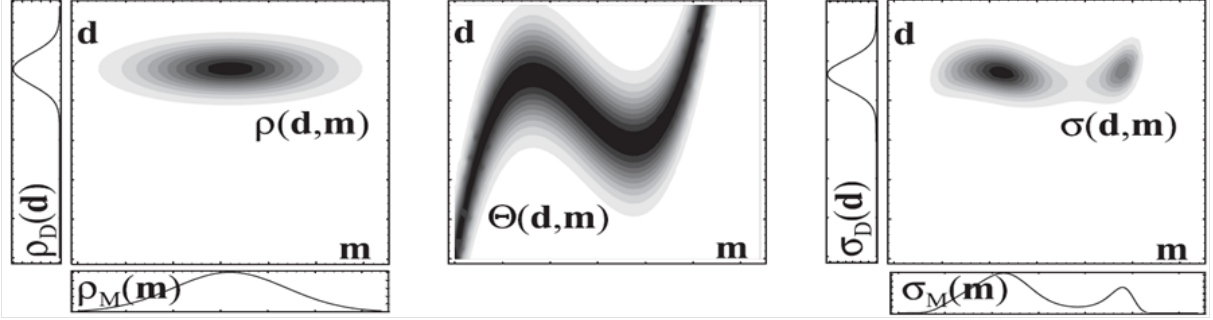where $\mu_D(\mathbf{d})$ is the homogeneous probability density over the data space D.

Figure 3: Conceptual depiction of the general probabilistic solution of an inverse problem.

- Once the posteriori probability in the $D\text{x}M$ space has been defined, the posteriori probability in the model space is given by the marginal probability density:

$$\begin{aligned}
\sigma(\mathbf{m}) &= k\int_D \sigma(\mathbf{m},\mathbf{d})d\mathbf{d} \\
&= k\int_D \frac{\rho_M(\mathbf{m})\rho_D(\mathbf{d})\theta(\mathbf{d}|\mathbf{m})}{\mu_D(\mathbf{d})}d\mathbf{d} \\
&= k\rho_M(\mathbf{m})\int_D \frac{\rho_D(\mathbf{d})\theta(\mathbf{d}|\mathbf{m})}{\mu_D(\mathbf{d})}d\mathbf{d} \\
&= k\rho_M(\mathbf{m})L(\mathbf{m})
\end{aligned} \tag{5}$$

hereby, $L(\mathbf{m})$ is the *likelihood function* which gives a measure of how good a model $\mathbf{m}$ is in explaining the data.

By assuming Gaussian PDFs for both data and model uncertainties, we have

$$\begin{aligned}
\rho_D(\mathbf{d}) &= k\exp(-0.5(\mathbf{d}-\mathbf{d}_{obs})^T\mathbf{C}_D^{-1}(\mathbf{d}-\mathbf{d}_{obs})) \tag{6} \\
\theta(\mathbf{d}|\mathbf{m}) &= k\exp(-0.5(\mathbf{d}-\mathbf{g}(\mathbf{m}))^T\mathbf{C}_M^{-1}(\mathbf{d}-\mathbf{g}(\mathbf{m}))) \tag{7}
\end{aligned}$$

and the likelihood function becomes Gaussian having a covariance matrix $\mathbf{C}_L = \mathbf{C}_D + \mathbf{C}_M$:

$$L(\mathbf{m}) = k\exp(-0.5(\mathbf{d}_{obs}-\mathbf{g}(\mathbf{m}))^T\mathbf{C}_L^{-1}(\mathbf{d}_{obs}-\mathbf{g}(\mathbf{m}))) \tag{8}$$

If no prior information about the model parameters is available and the components of the

observed and calculated data $(\mathbf{d}_{obs}, \mathbf{g}(\mathbf{m}))$ are independent as well as identically distributed with standard deviations S_D and S_M , respectively, the probabilistic solution simply becomes:

$$
\begin{aligned}
\sigma(\mathbf{m}) &= kL(\mathbf{m}) \\
&= k(S_D^2 + S_M^2)^{-N/2} \exp\left[-\frac{1}{2} \underbrace{\frac{\sum_{i=1}^{N}(d_{obs}^i - g_i(\mathbf{m}))^2}{S_D^2 + S_M^2}}_{J(\mathbf{m}, S_D, S_M)} \right]
\end{aligned}
\tag{9}
$$

where $J(\mathbf{m}, S_D, S_M)$ is the *misfit* function or *least squares cost* function.

For fixed values of S_D and S_M over the model and the data spaces, maximizing the likelihood function equals minimizing the least squares cost function. Thus, the probabilistic approach converts to the deterministic approach.

The probabilistic solution is numerically evaluated by direct sampling. Generation of realizations of the probabilistic density solution $\sigma(\mathbf{m})$ is accomplished by using *Monte Carlo Markov Chain* method along with the *Metropolis* acceptance rules [4] and [5]: Given the target probability distribution $\sigma(\mathbf{m}|\mathbf{d})$, we consider a Markov chain with a given sample $\mathbf{m}_n$. The next sample $\mathbf{m}_{n+1}$ is obtained from $\mathbf{m}_n$ as follows:

- Generate a candidate sample $\mathbf{m}$ from a *jumping* probability density function $P(\mathbf{m}^*|\mathbf{m}_n)$.

- Calculate
$$
\alpha = \min\left[1, \frac{\sigma(\mathbf{m}^*|\mathbf{d})P(\mathbf{m}_n|\mathbf{m}^*)}{\sigma(\mathbf{m}_n|\mathbf{d})P(\mathbf{m}^*|\mathbf{m}_n)} \right]
$$

- Generate a uniformly distributed sample $U \in (0.0; 1.0)$.

- If $U \leq \alpha$ accept $\mathbf{m}_{n+1} = \mathbf{m}^*$ otherwise $\mathbf{m}_{n+1} = \mathbf{m}_n$.

Repeating this sequence shows that the generated Markov Chain converges to the probability distribution function $\sigma(\mathbf{m}|\mathbf{d})$.

2.3 Sensitivity Analysis

Due to the highly nonlinear problem nature of geotechnical applications with respect to both the physical and the geometrical characteristics, the numerical simulation is normally computationally expensive. In order to make the inverse analysis efficient as well as robust, it is favorable to reduce the number of the parameters to be identified by performing a *sensitivity analysis*. This analysis evaluates the importance of each unknown model parameter with respect to the system response resulting in a decrease of the number of the forward calculations.

In this paper, a *variance based global sensitivity analysis*, which explores the space of the input parameters, has been utilized. In this analysis two different sensitivity measures have been introduced, the first index, *first order sensitivity index* [6], measures only the decoupled effect on the system response,

$$
S_i = \frac{V_{m_i}(E_{\mathbf{m}_{\sim i}}(g(\mathbf{m})|m_i))}{V(g(\mathbf{m}))}
\tag{10}
$$

where $V(g(\mathbf{m}))$ is the unconditional variance of the model output and $V_{m_i}(E_{\mathbf{m}_{\sim i}}(g(\mathbf{m})|m_i))$ is the variance of conditional expectation with $\mathbf{m}_{\sim i}$ indicating the matrix of all parameters but m_i.

In order to consider the coupling effects of the input parameters, the second index, *total effect index* [7], has been introduced,

$$S_{Ti} = 1 - \frac{V_{\mathbf{m}_{\sim i}}(E_{m_i}(g(\mathbf{m})|\mathbf{m}_{\sim i}))}{V(g(\mathbf{m}))} \tag{11}$$

where $V_{\mathbf{m}_{\sim i}}(E_{m_i}(g(\mathbf{m})|\mathbf{m}_{\sim i}))$ measures the first order effect of $\mathbf{m}_{\sim i}$ on the system response that does not include any influence corresponding to m_i.

For the estimation of the first and total sensitivity indices a numerical procedure introduced by [8] have been utilized.

2.4 Model Approximation

The forward calculation, which is a three dimensional finite element simulation, needs a significant computation time. Therefore, and due to the large number of the forward calculations that are included in the optimization process of the deterministic approach, or the sampling procedure of the probabilistic approach, or even in the global sensitivity analysis being adopted in this work, using a meta-model that substitutes the finite element simulation runs is unavoidable. For this purpose, an approximation method based on polynomial regression has been implemented.

Customarily, $g(\mathbf{m})$ the system output can be represented by the approximated value and an error ϵ

$$g(\mathbf{m}) = \hat{g}(\mathbf{m}) + \epsilon = \mathbf{p}^T(\mathbf{m})\boldsymbol{\beta} + \epsilon \tag{12}$$

where $\boldsymbol{\beta}$ is the vector of the unknown regression coefficients, and $\mathbf{p}$ is the vector of the polynomial basis functions

$$\mathbf{p}^T(\mathbf{m}) = [1 \; m_1 \; m_2 \; m_3 \, ... \, m_1^2 \; m_2^2 \; m_3^2 \, ... \, m_1 m_2 \; m_1 m_3 \, ... \, m_2 m_3 \, ...]. \tag{13}$$

The regression coefficients are estimated according to [9]:

$$\hat{\boldsymbol{\beta}} = (\mathbf{P}^T\mathbf{P})^{-1}\mathbf{P}^T\mathbf{g} \tag{14}$$

where $\mathbf{P}$ is a matrix containing the basis polynomials of the support points and $\mathbf{g}$ is the system responses of the support points.

For the assessment of the approximation quality, the Coefficient of Determination

$$R^2 = 1 - \frac{\sum_{j=1}^{n}(g_j - \hat{g}_j)^2}{\sum_{j=1}^{n}(g_j - \bar{g}_j)^2}$$

according to [10] has been introduced, where, the closer the R^2 value to one is, the better the approximation is.

3 APPLICATION

We use a three dimensional finite element model for scenario #1 of the tunnel excavation (Figure 1), using the FE-code PLAXIS 3D, version 2010. Since the geometry, the material properties, the initial and excavation conditions are in total symmetric with respect to a vertical plane parallel to the tunnel axis (X-axis), only one-half of the model needs to be analyzed (see Figure 4). The chosen slurry shield Tunnel Boring Machine TBM being 9 m long is simulated, along with the tunnel lining, by circular plate elements assuming linear elastic behavior. The ground is modeled by the *Hardening Soil Model* [11]. In Table 1, the parameters of the considered constitutive models are presented. More details about the model can be seen in [12].
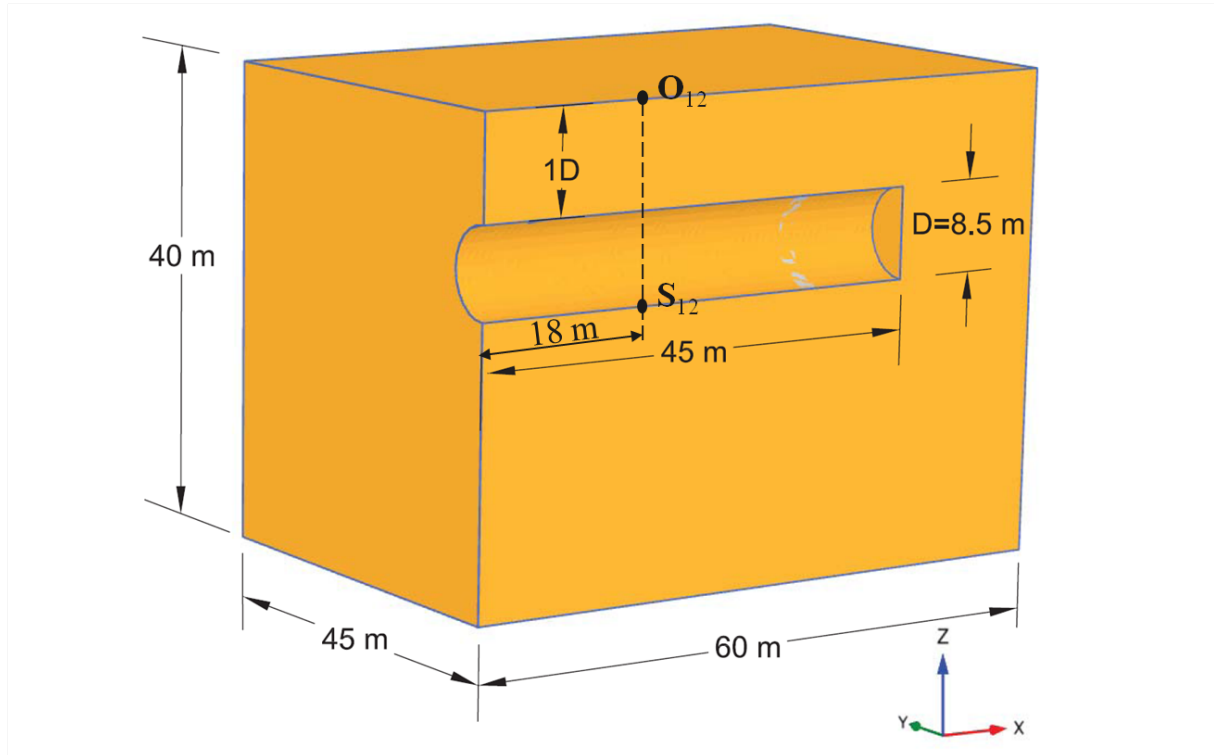

Figure 4: The geometry and properties for a three dimensional model using Plaxis 3D.

In order to identify the ground model (HS-Model) parameters, the following steps have been carried out:

- The model has been run for the parameter values stated in Table 1. Hereby, the vertical displacements at the two points O_{12} and S_{12} (see Figure 4) have been saved as observation measurements $\mathbf{d}_{obs}$ for the whole excavation phases. Subsequently, the HS-Model parameters (i.e. ϕ, c, E_{oed}^{ref}, E_{50}^{ref}, E_{ur}^{ref} with the conditions $E_{oed}^{ref} = E_{50}^{ref}$, $E_{oed}^{ref} \leq E_{ur}^{ref}/2$) are considered as unknowns that need to be estimated from the data set $\mathbf{d}_{obs}$.

- An objective function, representing the discrepancy between the observed and calculated measurements, has been adapted based on the least squares criterion (Equation 1).

- Model approximation for the system response at the observation points O_{12} and S_{12} have been carried out using polynomial regression. The coefficients of determination show a

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parameter	Soil Hardening Soil Model	Tunnel lining Linear Elastic	TBM-Shield Linear Elastic
$\phi[^\circ]$	35	-	-
$\psi[^\circ]$	5	-	-
$c[\text{kN/m}^2]$	10	-	-
$E_{50}^{ref}[\text{kN/m}^2]$	35000	-	-
$E_{oed}^{ref}[\text{kN/m}^2]$	35000	-	-
$E_{ur}^{ref}[\text{kN/m}^2]$	10^5	-	-
$P^{ref}[\text{kN/m}^2]$	100	-	-
$m[-]$	0.7	-	-
$R_f[-]$	0.9	-	-
$\nu_{ur}[-]$	0.2	-	-
$\gamma_{unsat}[\text{kN/m}^3]$	17	-	-
$\gamma_{sat}[\text{kN/m}^3]$	20	-	-
$R_{inter}[-]$	0.6	-	-
$E[\text{kN/m}^2]$	-	3.10^7	21.10^7
$\nu[-]$	-	0.1	0.3
$\gamma[\text{kN/m}^3]$	-	24	38
$d[m]$	-	0.2	0.35

Table 1: Material properties for the models used in tunnel simulation for scenario #1.

good approximation for the forward model

$$R^2_{O_{12}} = 0.98759 \quad R^2_{S_{12}} = 0.9997.$$

- A variance based sensitivity analysis has been implemented for deciding which parameters have to be identified. As a result, a parameter with small values for S_i and S_{Ti} has a negligible effect on the considered system response; and can be excluded from the identification. The analysis has shown that the four decisive parameters of the model need to be identified, Figure 5.

- The subsequent identification process has been performed following the inverse analysis procedure presented in Figure 2, where two different optimization algorithms have been applied, (*i*) Particle Swarm Optimization PSO [13], and (*ii*) Differential Evolution Algorithm [14]. In both cases, the objective function is minimized until the parameters match their real world values to a large extend (see Figure 6).

- In the probabilistic approach, we are currently still considering the simple case in which a homogeneous PDF for the a priori information about the model parameters is assumed. Also, Gaussian PDFs are used for both data and model uncertainties, according to Equation 8. The covariance matrix of the measured data (CD) depends usually on sensing devices used in recording the observations. In [15] different devices with their covariance matrices are presented that can be used in our case, as well. For the present example, the forward model is assumed to be exact and the influence of the data (measurements) uncertainty is investigated in two cases with the assumption that the observation errors are independent and identically distributed. In the first case, the standard deviation of
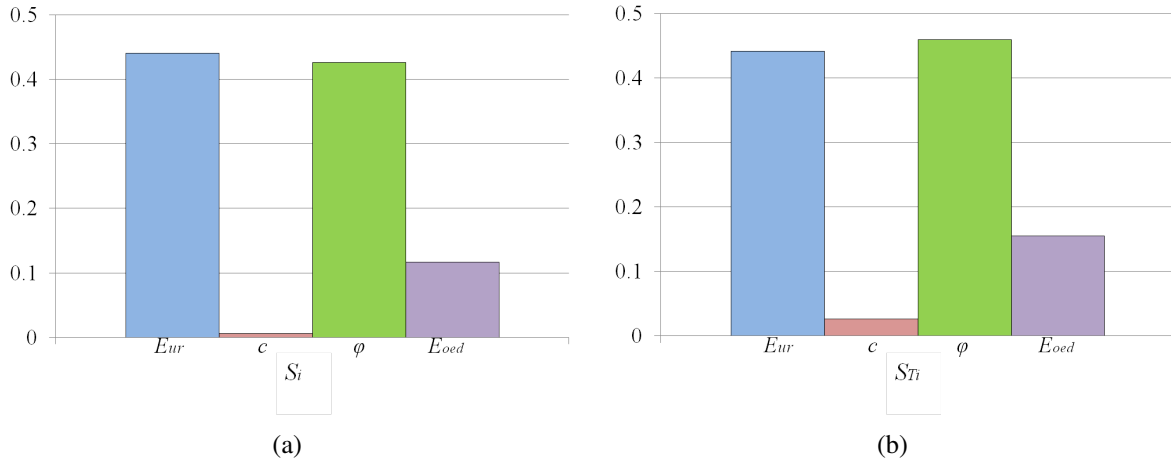
(a)

(b)

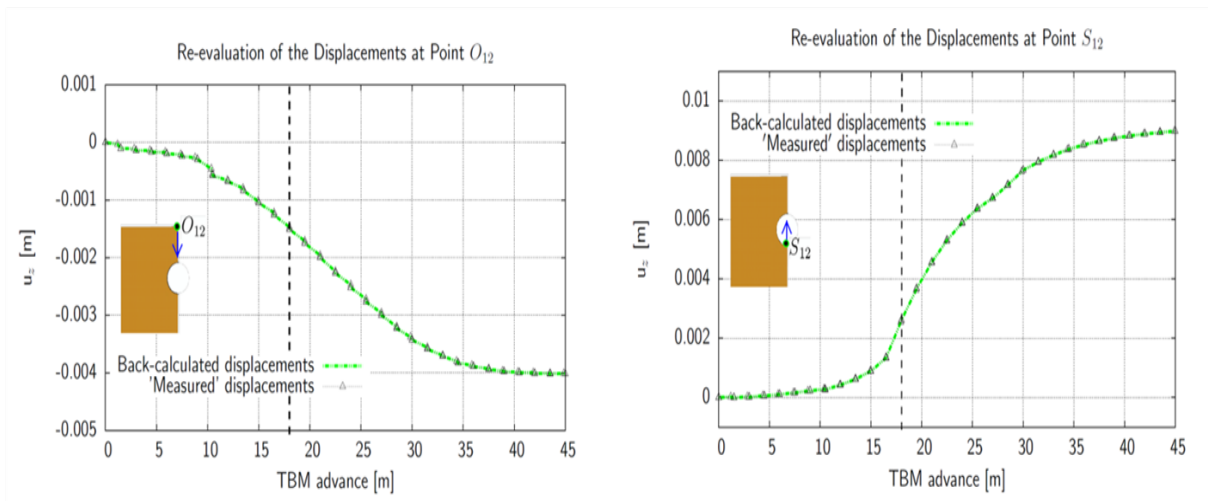Figure 5: First order and total effect sensitivity indices of the soil parameters.

Figure 6: Quality of results; forward solver = back analysis solver.

the observations error S_D is given a relatively small value in comparison to the system response at the observing points, and in the second case, a relatively higher value for S_D is considered. Using Monte Carlo Markov Chain method for sampling the probabilistic solution

$$\begin{aligned} \sigma(\mathbf{m}) &= kL(\mathbf{m}) \\ &= k(S_D^2)^{-N/2} \exp\left[-\frac{1}{2} \frac{\sum_{i=1}^{N}(d_{obs}^i - g_i(\mathbf{m}))^2}{S_D^2} \right] \end{aligned} \tag{15}$$

for both cases. Analysing the generated samples gives the results in Table 2 and Table 3.

Where, the uncertainty of the parameters increases with the data uncertainty. In addition to that, the parameters with higher sensitivity measures can be identified better than those of lower sensitivity measures.

parameter	Exact value	Mean μ	Standard deviation σ	Coefficient of Variation $CoV = \sigma/\mu$
E_{ur}^{ref}	100000	102040	12809	0.1255
c	10	310.989	2.757	0.2508
ϕ	35	333.85	2.1886	0.0655
E_{oed}^{ref}	35000	31941	5101	0.1597

Table 2: Statistical characteristics of the generated samples for the first case (relatively small data uncertainty).

parameter	Exact value	Mean μ	Standard deviation σ	Coefficient of Variation $CoV = \sigma/\mu$
E_{ur}^{ref}	100000	121065	18765	0.155
c	10	310.6	2.82	0.2665
ϕ	35	335.85	3.715	0.1036
E_{oed}^{ref}	35000	35649	7419	0.208

Table 3: Statistical characteristics of the generated samples for the second case (relatively large data uncertainty).

By drawing the relative frequency diagram of two parameters for the tow cases, the distribution of parameters within their bounds is illustrated, Figure 7.

4   CONCLUSIONS ANS OUTLOOKS

The two presented identification approaches, deterministic and probabilistic, are able to estimate the ground model parameters from observations. Where, the second approach is able to capture and quantify parameter uncertainties that result from uncertainties associated with measurement data. Different and more specific investigations are going to be considered in the probabilistic approach for representing more realistic and general cases of uncertainties. Furthermore, the captured and quantifies uncertainties are going to be utilized in the model assessment process that enables choosing the most adequate ground model for the forward model representing the tunnel excavation.
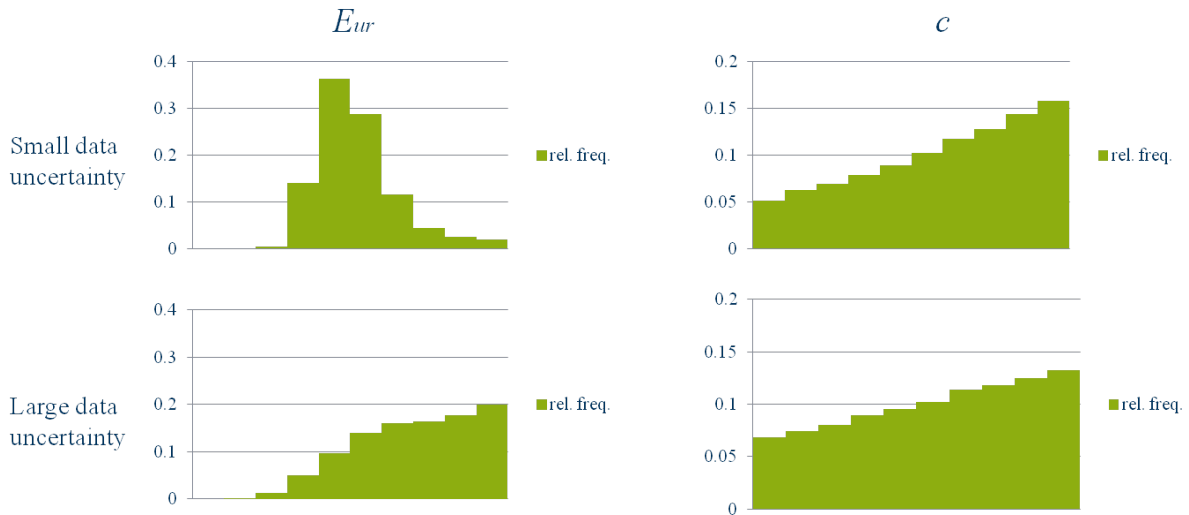
Figure 7: Relative frequencies due to data uncertainties.

5 ACKNOWLEDGEMENTS

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