19th International Conference on the Application of Computer
Science and Mathematics in Architecture and Civil Engineering
K. Gürlebeck, T. Lahmer and F. Werner (eds.)
Weimar, Germany, 04–06 July 2012

ASSESSMENT OF INTEGRAL BRIDGES USING QUANTITATIVE MODEL EVALUATION

B. Jung*, **G. Morgenthal**

Research Training Group 1462 Model Quality
Berkaer Straße 9
99423 Weimar
Germany
E-mail: bastian.jung@uni-weimar.de

Keywords: sensitivity analysis, coupled models, global model quality, integral bridge, restraint effects

Abstract. *Numerical simulations in the general field of civil engineering are common for the design process of structures and/or the assessment of existing buildings. The behaviour of these structures is analytically unknown and is approximated with numerical simulation methods like the Finite Element Method (FEM). Therefore the real structure is transferred into a global model (GM, e.g. concrete bridge) with a wide range of sub models (partial models PM, e.g. material modelling, creep). These partial models are coupled together to predict the behaviour of the observed structure (GM) under different conditions. The engineer needs to decide which models are suitable for computing realistically and efficiently the physical processes determining the structural behaviour. Theoretical knowledge along with the experience from prior design processes will influence this model selection decision. It is thus often a qualitative selection of different models.*

The goal of this paper is to present a quantitative evaluation of the global model quality according to the simulation of a bridge subject to direct loading (dead load, traffic) and indirect loading (temperature), which induce restraint effects. The model quality can be separately investigated for each partial model and also for the coupled partial models in a global structural model. Probabilistic simulations are necessary for the evaluation of these model qualities by using Uncertainty and Sensitivity Analysis. The method is applied to the simulation of a semi-integral concrete bridge with a monolithic connection between the superstructure and the piers, and elastomeric bearings at the abutments. The results show that the evaluation of global model quality is strongly dependent on the sensitivity of the considered partial models and their related quantitative prediction quality. This method is not only a relative comparison between different models, but also a quantitative representation of model quality using probabilistic simulation methods, which can support the process of model selection for numerical simulations in research and practice.

1 EVALUATION METHOD FOR GLOBAL MODEL QUALITY ASSESSMENT

Global models (GM) for numerical simulation approaches utilize different model classes (M) with subordinate partial models (PM). Material descriptions, creep, and/or shrinkage models are defined as possible M for concrete structures within this paper. Interactions and couplings of their PM are necessary for determining an appropriate structural behaviour. Therefore, the following evaluation method enables to assess the Global Model Quality. For detailed information the author recommends KEITEL et al. [11].

1.1 Sensitivity according a model class

The first step is to quantify whether the model class M has an influence on a certain target value. This is evaluated by using Sensitivity Analysis [12] which, in general, is the study of how the output of a model (Y) is related to the model input (X). By using discrete random variables for selecting the model class, the Sensitivity Study in this case is not an estimation of uncertainty, but a quantified value of the influence of the model class (X_i). The First Order Sensitivity Index is:

$$S_i = \frac{V\left(E\left(Y|X_i\right)\right)}{V\left(Y\right)}. \tag{1}$$

This index S_i illustrates the exclusive influence of model X_i. Due to interactions in complex engineering problems higher order Sensitivity Indices are needed. The Total Effect Index is defined as:

$$S_{Ti} = 1 - \frac{V\left(E\left(Y|X_{\sim i}\right)\right)}{V\left(Y\right)}. \tag{2}$$

A finite number of possible model class combinations n_{comb} are necessary for the indices:

$$n_{comb} = 2^{n_M} \tag{3}$$

with n_M random variables (model classes). A measure of the interaction between X_i and other model classes is the difference between S_i and S_{Ti}. High values of these Sensitivity Indices highlight a significant influence of this partial model class on the response of the global model. Models with values smaller than a given threshold (here: $S_{Ti} \leq 0.03$) shall be neglected for the next evaluation method step. In other words, no further investigations about their Partial Model Quality are performed.

1.2 Sensitivity according the choice in a model class

The second method step quantifies the importance of selecting a partial model from one model class. It is also based on Sensitivity Studies [11, 12]. The choice of each PM within a model class is controlled by X_i. The Total Effect Sensitivity Index indicates how this choice leads to a variation of the global model response according to a certain output value. Low values show that different partial models within the same model class give a similar contribution to the structural response value and do not significantly affect these response values. These indices are used as weighting factors for the importance of the quality of a PM in a model class.

1.3 Quality of coupled partial models

The Global Model Quality (MQ_{GM}) of coupled partial models is quantified by a path on a graph (graph theory see [3, 4, 9, 10]) with the vertex as the quality of the partial model MQ_{PM}

and the edges as the coupling quantities. A number between 0 and 1 expresses this quality. 0 signifies a poor and 1 a high MQ_{PM}. These quantitative values come from the evaluation of the PM itself, using Uncertainty, Complexity or Robustness criterias [8]. Assuming a perfect data coupling between each model classes the model quality of a global structural model is defined as [11]:

$$MQ_{GM} = \sum_{i=1}^{n_{M,red}} \frac{S_{Ti}^{MC} \cdot MQ_{PM_j}}{\sum_{i=1}^{n_{M,red}} S_{Ti}^{MC}}. \tag{4}$$

PM_j is one partial model of the model class M_i. The variable $n_{M,red}$ is the number of non-negligible partial model classes influecing the global response, determined by method step one. This Global Model Quality Evaluation method is applied to a reinforced and prestsressed semi-integral concrete bridge below.

2 APPLICATION TO SEMI-INTEGRAL CONCRETE BRIDGE

2.1 Geometry, material properties and loading

The geometry of the longitudinal and vertical direction of the bridge and the prestressing steel is shown in Fig. 1. The cross sections of the superstructure and the pier are shown in Fig. 2a and Fig. 2b and the material properties are listed in Table 1.

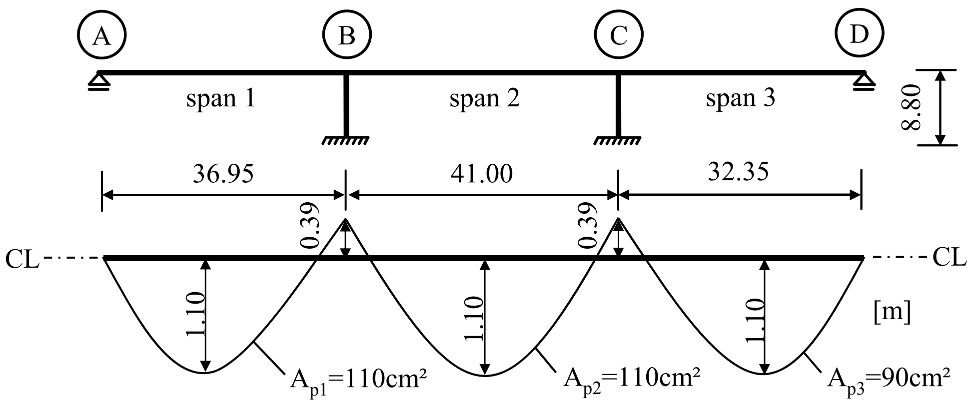
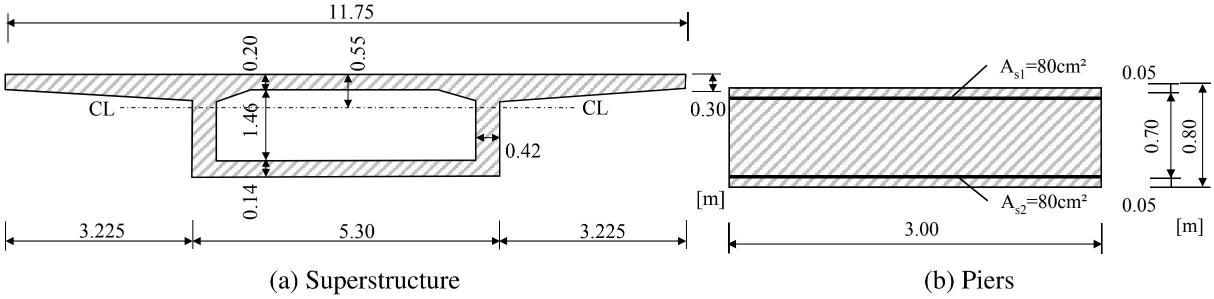


Figure 1: Bridge and prestressing geometry



(a) Superstructure

(b) Piers

Figure 2: Cross sections of the superstructure and the piers

Table 1: Material properties for the superstructure and the piers

	Unit	Superstructure	Piers
Concrete		C50/60	C35/45
CEM	-	II 52.5N	II 42.5N
E_{c0m}	[MN/m^2]	35,500	33,300
E_{cm}	[MN/m^2]	32,800	28,300
f_{cm}	[MN/m^2]	58	43
f_{ctm}	[MN/m^2]	4.1	3.2
Steel		Y1770	B500B
E_s	[MN/m^2]	190,000	200,000
f_y	[MN/m^2]	1,500	500

The commonly decoupled connection between the piers and the superstructure (differential bridge) is adjusted to a coupled semi-integral bridge. Therefore the overall structural load-deformation behaviour is affected by the interaction within the piers and superstructure, particularly in case of restraint effects. Hence, the interference between the partial models is investigated.
The structural behaviour is simulated under quasi-permanent loading [5] for 100 years of service life (see Table 2).

Table 2: Loading for quasi-permanent loading according [5]

Loading category	Loading value
dead load (G_k)	G_{k1} = 142 kN/m (superstructure) G_{k2} = 24 kN/m (pavement) G_{k3} = 6.75 kN/m (piers)
prestressing (P_k)	σ_{pk} = 1295 MN/m^2
imposed traffic (Q_{k1})	$Q_{k1,UDL}$ = 46.4 kN/m, $Q_{k1,TL}$ = 400 kN (span 1) $\Psi_{2,1}$ = 0.20
temperature load (Q_{k2})	T_0 = 10°C, T_{min} = -24°C, $T_{e,min}$ = -16° C, ΔT_N = -26K, ΔT_M = -8.8K $\Psi_{2,2}$=0.50

2.2 Considered partial models

The material description (Model Class A) for the concrete compression range is modelled with linear-elastic relation between strains and stresses. Because of the prestressing and the quasi-permanent loading, the compression stresses are smaller than $\sigma_c \leq 0.40 \cdot f_{cm}$. Therefore, linear-elastic material behaviour can be assumed. In the range of tensile concrete parts the concrete can either sustain stresses until f_{ctm} (A-1: linear-elastic material modelling) or cracking shall be considered through the application of a tension-stiffening model as $\beta_{ct} \cdot f_{ctm}$ until $\epsilon_{ct} \leq \epsilon_{sy}$ (A-2: tension-stiffening model).
In order to describe the time-dependent increase of the creep compliance two creep models (Model Class B) are investigated. These are the models according to Model Code 2010 (B-1:

MC 10 [2]) and GARDNER and LOCKMAN (B-2: GL2000 [7]).
Geometrical nonlinearities (Model Class D) can affect displacement values and section forces. The nonlinear kinematic (D-1) and the p-Δ (D-2) approaches are considered in this model class.
Restraint effects in concrete structures may occur as a result of imposed deformations such as thermal actions (Model Class E). In the standard code EN 1991 [6] specific values are stated for temperature conditions and temperature distributions. One possibility to take thermal actions on bridges into account is to assume constant temperature (ΔT_N) and linearly shift values over the cross section height (ΔT_M). Alternatively, thermal actions can also be considered by the temperature (ΔT_N) and nonlinear varying values (ΔT) over the cross section height. Combination factors for the concurrent occurrence of both temperature parts are included to account for their coincident probability. Four temperature distributions are considered as partial models in the model class temperature:

- E-1 TEMP 1 constant with linear shifting $0.35 \cdot \Delta T_N + \Delta T_M$
- E-2 TEMP 2 constant with linear shifting $\Delta T_N + 0.75 \cdot \Delta T_M$
- E-3 TEMP 3 constant with nonlinear distribution $0.35 \cdot \Delta T_N + \Delta T$
- E-4 TEMP 4 constant with nonlinear distribution $\Delta T_N + 0.75 \cdot \Delta T$

The creep $\epsilon_{c,cr}(t)$, shrinkage $\epsilon_{c,sh}(t)$ and temperature $\epsilon_{c,t}(t_0)$ strains are expressed by additional strain components of the concrete, which leads to the total strains of the concrete:

$$\epsilon_{c,tot}(t) = \epsilon_{c,el}(t) + \epsilon_{c,pl}(t) + \epsilon_{c,da}(t) + \epsilon_{c,cr}(t) + \epsilon_{c,sh}(t) + \epsilon_{c,t}(t_0) \quad (5)$$

with $\epsilon_{c,el}(t)$, $\epsilon_{c,pl}(t)$ and $\epsilon_{c,da}(t)$ as the time-dependent elastic, plastic and damage strains.

2.3 Structural response values for the quantification

In case of the first step in the evaluation method, the Sensitivity is quantified for the vertical deformations in all spans, horizontal deformations at each bridge axis, concrete compression and prestressing steel tensile stress in the superstructure, concrete and reinforcement stresses in the piers and axial and bending moment section forces at different positions. The 8 model classes lead to 256 model combinations (n_{comb} = 2^8) independent of the target values for the structural behaviour.

2.4 Sensitivity according the model class

The discrete random variables control, whether the model class is activated or deactivated. In terms of the material behaviour, either tension-stiffening or purely linear-elastic material is modelled. In terms of creep or shrinkage, either creep or shrinkage strains are computed or neglected. In terms of geometric nonlinearity, either the second order or the first order kinematic is used. Finally, in terms of temperature, either temperature strains occurring from constant and shifting parts are considered or zero. Table 3 shows the First Order and Total Effects Sensitivity Indices for a selection of target values.

The creep phenomenon increases the strains for the quasi-permanent loading for 100 years design life. The vertical displacements in the superstructure are almost exclusively sensitive to this model class. Non activated creep modelling will reduce the predicted vertical displacement

Table 3: Sensitivity indices for the model classes according target values, first row for each target value: First Order Effect S_i^M, second row for each target value: Total Effects S_{Ti}^M

Model Class	σ-ϵ super-struct. A	σ-ϵ piers A	creep super-struct. B	creep piers B	shrink. super-struct. C	shrink. piers C	geom. kine-matic D	tem-pera-ture E
Vertical dis.	0.000	0.000	**0.975**	0.001	0.000	0.012	0.000	0.000
span 1	0.000	0.000	**0.976**	0.012	0.001	0.012	0.000	0.001
Horizontal dis.	0.000	0.000	**0.076**	0.000	**0.850**	0.000	0.000	**0.074**
axis C	0.000	0.000	**0.076**	0.000	**0.850**	0.000	0.000	**0.074**
concrete stress	0.000	0.004	**0.054**	**0.265**	0.002	0.022	0.000	**0.514**
superstr. span 2	0.000	0.023	**0.098**	**0.355**	**0.072**	0.025	0.000	**0.580**
concrete stress	0.000	0.003	0.014	**0.296**	**0.548**	0.004	0.000	**0.045**
pier axis C	0.000	0.013	0.016	**0.382**	**0.620**	0.004	0.000	**0.059**
bending moment	0.000	0.003	**0.132**	**0.170**	0.001	0.023	0.000	**0.424**
right axis B	0.000	0.016	**0.316**	**0.230**	**0.047**	0.025	0.000	**0.622**
Axial force superstr.	0.000	0.002	0.006	**0.164**	**0.732**	0.001	0.000	0.047
right axis B	0.000	0.010	0.008	**0.208**	**0.770**	0.001	0.000	0.056
Bending moment	0.000	0.003	0.018	0.207	**0.663**	0.000	0.000	**0.057**
pier bottom axis C	0.000	0.016	0.020	0.252	**0.700**	0.001	0.000	**0.070**
Axial force	0.000	0.001	0.000	**0.091**	**0.631**	0.016	0.000	**0.217**
pier bottom axis C	0.000	0.004	0.024	**0.110**	**0.647**	0.016	0.000	**0.245**

significantly. The creep phenomenon regarding the vertical displacements can thus not be neglected.
In case of horizontal displacements, the major impact occurs from the shrinkage model class. Shrinkage strains must be included without any reduction factors. Temperature strains for the quasi-permanent loading are reduced by the combination factor $\Psi_{2,2} = 0.50$. This leads to:

- Shrinkage MC10 $\epsilon_{c,sh}(36510\ d) = -4.204\ e^{-4}$
- $\Delta T_N = -26$ K $\epsilon_{c,t}(36510\ d) = -26\ \text{K} \cdot 1.0\ e^{-5} \cdot 0.5 = -1.300\ e^{-4}$

and therefore to higher sensitivity of the shrinkage phenomenon according the horizontal displacement.
The difference between S_i and S_{Ti} such as the concrete stress in span 2 clarify a strong interaction between model classes ($S_{Ti} - S_i > 0.05$). The deformation behaviour of the piers and superstructure affect each other and therefore the coupling of their model classes has a strong influence on the structural response. The influence of choice of different partial models in each model class is quantified for the bolted structural response value in the first column of Table 3 (horizontal displacement at the axis c, concrete stress in the superstructure in span 2, bending moment right axis B).

2.5 Sensitivity according the model choice in a model class

The analysis of the Total Effect Sensitivity Index enables the quantification of the model choice importance (comparable as weighting factors). For example, the prognosis of the models

MC10 and GL2000 for creep and shrinkage are different, and the influence of it can be computed by Sensitivity Analysis. Table 4 shows these weighting factors, which quantify the impact of model selection according to the chosen structural response values.

Table 4: Total Effect Sensitivity Indexes S_{Ti}^{MC} for the model choice according the important model classes for different target values, * model classes with no significant influence according the target value

Model Class	σ-ϵ super-struct. A	σ-ϵ piers A	creep super-struct. B	creep piers B	shrink. super-struct. C	shrink. piers C	geom. kine-matic D	tem-pera-ture E
Horizontal dis. axis C	*	*	0.405	*	**0.496**	*	*	0.099
concrete stress superstr. span 2	*	*	0.121	0.341	0.007	*	*	**0.622**
bending moment right axis B	*	*	0.285	**0.490**	0.010	*	*	0.252

2.6 Global Model Quality

The Partial Model Quality for the creep models is analysed by uncertainty analysis including model and parameter uncertainty and is stated in [11]. The Partial Model Quality of the shrinkage models is assessed on the variation of the error of the prediction. This uncertainty is $CV_{MC10} = 0.481$ and $CV_{GL2000} = 0.433$ [1]. In relation to the lowest model uncertainty of $CV_{B3} = 0.374$ the Partial Model Quality is defined as:

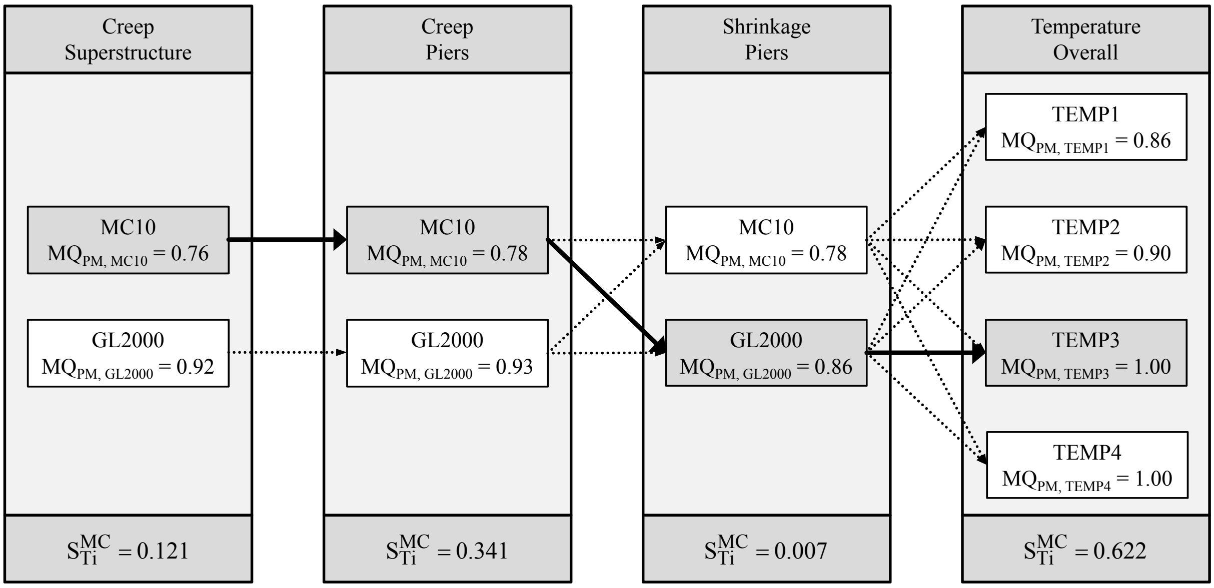
- MC10 $\quad MQ_{PM}^{MC10} = 0.374/0.481 = 0.78$
- GL2000 $\quad MQ_{PM}^{GL2000} = 0.374/0.433 = 0.86$

Linear and nonlinear temperature models [6] are quantified by their prognosis of the induced strains. The complexity of the nonlinear temperature distributions is higher in comparison to the linear approaches. It can be assumed, that their Partial Model Quality is highest (of the considered) and the linear distributions are quantified relatively by the model outputs for the concrete stress in span 2, which is selected for the next method step (Global Model Quality Evaluation).

The important model classes with their respective partial models are shown in Fig. 3. The unimportant model classes are excluded for the Global Model Quality Evaluation. The influence of the model selection in every model class is expressed by the Total Effect Sensitivity Index (bottom of Fig. 3). Partial Model Qualities mentioned above, are expressed in the vertices. The coupling (edges) is without any loss of data information.

The grey highlighted partial models express one admissible path through the graph. Because of practical reason, the structural engineer would not chooce a different creep model for the superstructure and the piers. Thats why the possible combinations is reduced ensuing from $n_{M,red}^{theoretical} = 32$ to $n_{M,red}^{practical} = 16$. The selected combination of partial models in the global

Figure 3: Global Model Quality Evaluation according the concrete stress in span 2 for the application of a semi-integral bridge, count model combination i = 7 regarding Table 5



model (see Fig. 3) will lead to the following Global Model Quality:

$$MQ_{GM} = \frac{0.12 \cdot 0.76 + 0.341 \cdot 0.78 + 0.007 \cdot 0.86 + 0.622 \cdot 1.00}{0.121 + 0.341 + 0.007 + 0.622} = 0.90 \tag{6}$$

For any other possible model combination the resulting Global Model Quality MQ_{GM} is stated in Table 5. Selecting a different Partial Model for the prediction of the creep phenomenon will mainly lead to a changed Global Model Quality. This sensitivity is forced by the high difference of the creep compliance between the MC10 and GL2000 creep model, which is expressed by the associated Partial Model Quality. The target value for this evaluation is the concrete compression strength in span 2 of the semi-integral concrete bridge. The additional strain occurring from both shrinkage models has a minor influence (very low S_{Ti}^{MC}), according this target value, in relation to the other effects. In this case, selecting a different shrinkage model, ensues an unchanged MQ_{GM}.

Table 5: Global Model Quality $MQ_{GM,i}$ for the possible model combinations, application of a semi-integral concrete concrete, target value: concrete stress in span 2, ✓... Partial Model PM_i is activated, $\ominus$ ... Partial Model PM_i is deactivated

Count i	Creep Superstr.		Creep Piers		Shrinkage Piers		Temperature Overall				$MQ_{GM,i}$
	MC10	GL2000	MC10	GL2000	MC10	GL2000	TEMP1	TEMP2	TEMP3	TEMP4	
1	✓	$\ominus$	✓	$\ominus$	✓	$\ominus$	✓	$\ominus$	$\ominus$	$\ominus$	**0.82**
2	✓	$\ominus$	✓	$\ominus$	✓	$\ominus$	$\ominus$	✓	$\ominus$	$\ominus$	0.85
3	✓	$\ominus$	✓	$\ominus$	✓	$\ominus$	$\ominus$	$\ominus$	✓	$\ominus$	0.90
4	✓	$\ominus$	✓	$\ominus$	✓	$\ominus$	$\ominus$	$\ominus$	$\ominus$	✓	0.90
5	✓	$\ominus$	✓	$\ominus$	$\ominus$	✓	✓	$\ominus$	$\ominus$	$\ominus$	**0.82**
6	✓	$\ominus$	✓	$\ominus$	$\ominus$	✓	$\ominus$	✓	$\ominus$	$\ominus$	0.85
7	✓	$\ominus$	✓	$\ominus$	$\ominus$	✓	$\ominus$	$\ominus$	✓	$\ominus$	0.90
8	✓	$\ominus$	✓	$\ominus$	$\ominus$	✓	$\ominus$	$\ominus$	$\ominus$	✓	0.90
9	$\ominus$	✓	$\ominus$	✓	✓	$\ominus$	✓	$\ominus$	$\ominus$	$\ominus$	0.89
10	$\ominus$	✓	$\ominus$	✓	✓	$\ominus$	$\ominus$	✓	$\ominus$	$\ominus$	0.91
11	$\ominus$	✓	$\ominus$	✓	✓	$\ominus$	$\ominus$	$\ominus$	✓	$\ominus$	**0.97**
12	$\ominus$	✓	$\ominus$	✓	✓	$\ominus$	$\ominus$	$\ominus$	$\ominus$	✓	**0.97**
13	$\ominus$	✓	$\ominus$	✓	$\ominus$	✓	✓	$\ominus$	$\ominus$	$\ominus$	0.89
14	$\ominus$	✓	$\ominus$	✓	$\ominus$	✓	$\ominus$	✓	$\ominus$	$\ominus$	0.91
15	$\ominus$	✓	$\ominus$	✓	$\ominus$	✓	$\ominus$	$\ominus$	✓	$\ominus$	**0.97**
16	$\ominus$	✓	$\ominus$	✓	$\ominus$	✓	$\ominus$	$\ominus$	$\ominus$	✓	**0.97**

3 CONCLUSIONS

The evaluation method for accessing the Global Model Quality for coupled partial models [11] is applied on a semi-integral bridge. Sensitivity Analyses quantify in the first step the influence of the phenomenona (model classes) like creep, shrinkage, material description, geometrical nonlinearities and temperature distributions. They depend on the structural output value (displacements, stresses, section forces). In a second step the impacts of the model choice of a partial model in the same model class are analysed. Global Model Quality is evaluated by a path through the graph of partial models whereby each possible combination of the models will lead to a changed Global Model Quality.

The structural application of a semi-integral bridge shows the applicability of the evaluation method and quantifies the important model classes and the model selection process. The Global Model Qualities are useful to compare different simulations in a quantitative manner.

REFERENCES

[1] Z. P. Bazánt, *Unbiased statistical comparison of creep and shrinkage prediction models*, ACI Material Journal (2008) No. 6, pp. 610-621.

[2] CEB-Comit Euro-International du Beton, *Structural concrete: textbook on behaviour, design and performance*, updated knowledge of the CEB/FIB Model Code 90. tech. rep. (1999) and CEP-FIP Model Code 2010 First complete draft (2010).

[3] G. Chartrand, *Introductory graph theory*, New York: Dover Paperbacks (1985)

[4] R. Diestel, *Graph theory*, Springer (2005)

[5] DIN-Fachbericht 101, *Actions on bridges*, Beuth (2009).

[6] EN 1991-1-5, *Actions on structures General actions Thermal actions*, Beuth (2010).

[7] N. Gardner, M. Lockman, *Design provisions for drying shrinkage and creep of normal-strength concrete*, ACI Mater (2001) No. 98, pp. 159-165

[8] T. Lahmer et al., *Bewertungsmethoden für Modelle des konstruktiven Ingenieurbaus*, Bautechnik Sonderheft (2011) No.6.

[9] A. Kaveh, *Structural mechanics: Graph and matrix methods*, 3rd ed. Exeter (UK): Research Studies Press, John Wiley (2004)

[10] A. Kaveh, *Optimal structural analysis*, 2nd ed. Chichester (UK): John Wiley (2006)

[11] H. Keitel, G. Karaki, T. Lahmer, S. Nikulla, V. Zabel, *Evaluation of coupled partial models in structural engineering using graph theory and sensitivity analysis*, Engineering Structures (2011) No. 33, pp. 3726-3736.

[12] A. Saltelli, M. Ratto, T. Andres, F. Campolongo, J. Cariboni, D. Gatelli, M. Saisana, S. Tarantola, *Global sensitivity analysis*, The Primer, John Wiley and Sons (2008).