59th ILMENAU SCIENTIFIC COLLOQUIUM
Technische Universität Ilmenau, 11 – 15 September 2017
URN: urn:nbn:de:gbv:ilm1-2017iwk-018:6

EFFICIENT DESCRIPTION OF THE BOUNDARY OF THE OBJECT UNDER OBSERVATION

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ABSTRACT

The efficient method for encoding the remote observation data has been considered. The description of the boundaries of the objects of interest can be used when extracting the features in the process control problem. The solution of the problem of compact presentation of the object boundary based on the static approach has been shown. In the paper, the solution of the problem of compact description of object boundaries by using uncorrelated transform coefficients is considered. The efficiency of the description is achieved by presenting the boundary as a functional series of the coefficients of expansion in the basis of eigenfunctions of the covariance matrix. The compression process is based on the function of distribution of the 2-D dispersion of the Hotelling coefficients. Application of the dispersion criterion when describing the boundary of the object under investigation makes it possible to compress the data actually without losses. In this case, the restoration error approaches zero value. The results of assessment of the efficiency of encoding the boundary image have been presented.

Index Terms **-** image, object boundary, description, efficient encoding, compression, dispersion, covariance, correlation, transform coefficients, spectral approach, eigenvectors, filtration, arrangement.

1. INTRODUCTION

The efficient description of the object boundaries becomes a challenge when solving problems related to detection or search of certain objects on images as well as recognition or identification of them. The description of the boundaries of the objects of interest can be used when extracting the features in the process control problem. Solving the problems of the continuous environment monitoring through aerospace sensing of, for example, flooded areas, etc. is vital at all times. In many applications, such as topography, analysis of medical images, etc., it is desirable to replace the set of pixels depicting the object with the description of the latter's boundary in order to reduce the volume of the data array. For example, a possible approach to solving the problem of standardization of the cytological diagnostics is based on application of the methods and technique for recognition of images presented as contours, boundaries of the cell or nucleus regions, etc. The representation in the boundary form is suitable for the cases where the following geometrical characteristics of the object are in the focus of attention: length, area, bends, contours and concavities.

The spatial data describing the boundary can have high dimensionality. It is known that the spectral approach is important for the practice of compact description of data. The purpose of this paper consists in demonstrating the method of spectral image processing reducing the excessiveness when selecting the spectral signs, detection and localization of the image objects. This paper proposes the modification of this method, which takes into account the introduction of the input data sorting procedure into the processing algorithm for the purpose of more efficient selection and filtration of the dominant values of the Hotelling transform coefficients [1]. Here the efficiency is assessed from the standpoint of data compression.

2. THEORETICAL PRINCIPLES

A distinguishing feature of many industrial and artificial images is the property of considerable correlation of the data describing these images. This feature of relatively high linear dependence / high correlation appears to the maximum extent for depicting the objects in the form of closed boundaries and contours. The presence of this property makes it possible to implement the efficient data compression actually without losses. To do this, it is necessary to perform the encoding of the spatial data by means of a linear transformation in the basis of eigenvectors of the covariance matrix of these data. The result of encoding the data with strong correlation relationship between the adjacent pixels $\boldsymbol{G}$ is the decorrelation in the region of the transformations. In this case, the essential portion of energy of the encoded data falls within a small quantity of them. In this case, the restoration error approaches zero value.
The complete decorrelation of the spatial domain data is achieved if a transposed matrix of the eigenvectors of the covariance matrix $\mathrm{cov}(\boldsymbol{G})$ of the digital image $\boldsymbol{G}$ with the dimension of $L = M \times N$ pixels is used as a transformation kernel. The two-dimensional encoding-decoding of the data pieces is determined as

$$\widehat{\boldsymbol{G}} = \boldsymbol{A}_c \boldsymbol{G} \boldsymbol{A}_r, \quad (1)$$
$$\boldsymbol{G} = \boldsymbol{A}_r \boldsymbol{G} \boldsymbol{A}_c, \quad (2)$$

где $\widehat{\boldsymbol{G}}$ – is the matrix of transformation coefficients, A_r and A_c – are the transformation kernels for rows and columns of the initial data matrix. The total number of operations of multiplications C_M, related to the computation of (1) and (2) is

$$C_M = M^2 N + N^2 M$$

The C_M value can be reduced through encoding the data with due account for the analysis of the values of the function of distribution of 2-D dispersions 2-D diag[^{2}] of Hotelling coefficients. The discrete function of distribution of dispersions is determined by the expression [2]

$$\mathrm{diag}[\ ^2] = \boldsymbol{\Lambda} = \mathrm{diag}[\mathrm{cov}(\widehat{\boldsymbol{g}}_c] \otimes \mathrm{diag}[\mathrm{cov}(\widehat{\boldsymbol{g}}_r], \quad (3)$$

where $\mathrm{diag}[\mathrm{cov}(\widehat{\boldsymbol{g}}_c)]$ and $\mathrm{diag}[\mathrm{cov}(\widehat{\boldsymbol{g}}_r)]$ are diagonal covariance matrices of the columns and rows of the matrix, respectively, $\otimes$ – is the sign of the Kronecker product of matrices. The eigenvalues λ_i of the matrices $\mathrm{cov}(\widehat{\boldsymbol{g}}_c)$ and $\mathrm{cov}(\widehat{\boldsymbol{g}}_r)$ in the region of the transformation coefficients correspond to the dispersions of the coordinates $\hat{g}_i$ of the vectors

$$\widehat{\boldsymbol{g}}_c = (\hat{g}_0 \cdots \hat{g}_{N-1})^T \text{ and } \widehat{\boldsymbol{g}}_r = (\hat{g}_0 \cdots \hat{g}_{N-1})^T.$$

The result (3) is a special-type matrix having the positive numbers $\lambda_0, \lambda_1, \ldots, \lambda_{N^2-1}$ as diagonal elements, while the other elements of this matrix are zero. For compact description and data transmission, it is sufficient to use the coefficient $\widehat{\boldsymbol{g}}_c$ and $\widehat{\boldsymbol{g}}_r$ only, which are characterized by the maximum dispersions λ_i. The lexicographic columnwise transformation of the elements of the main diagonal of the matrix (3) results in the formation of a matrix, the structure of which makes it possible to mark out the zone of the coefficients to be stored. In this case, the efficiency of encoding the image $\boldsymbol{G}$ is determined by the number of coefficients falling into this zone.

2.1. Description of the boundary

The boundary description process includes the two stages.
1. Arrangement. Let the half-tone image with the size of $L = M \times N$ be replaced with the binary one. The boundary is described by a closed discrete line $g(x, y)$ in the form of a set P of points with the space coordinates $(x_i, y_i) \in\ ^+$. The point are arranges so that the (x_i, y_i) and (x_{i+1}, y_{i+1}) are the nearest neighbours along the boundary. In practical cases, the value P is much less than L. Let the $P = P_1 P_2$ be a composite number. Then $P_1 P_2 \ll MN$. Obviously,

the respective arrangement of the pixels in the form of a matrix $\boldsymbol{P}$ with the order of $P_1 \times P_2$, mapping the boundary line only, without a set of pixels composing the whole image $\boldsymbol{G}$, will reduce considerably the computational complexity of the description. In accordance with the values of the coordinates (x, y) of the boundary points, we will determine the value of brightness of each pixel in the form of a discrete two-dimensional function $g(x, y) \in {}^{+}$. Thus, the matrix $\boldsymbol{P} = (p_{i,j})$, $p_{i,j} \in {}^{+}$ will consist of $2P$ elements mapping the boundary. By selecting the components with the value of the initial image coordinates x and y from the matrix $\boldsymbol{P}$, we will obtain two matrices $\boldsymbol{C}$ and $\boldsymbol{R}$ with the order of $D_1 \times D_2$. The values of the orders are selected based on the specified requirements of reduction of computational complexity of processing or decrease of the processor input dimensions. It is supposed that the numbers determining the order of the matrix $\boldsymbol{P}$ are composite. It is easy to implement practically, because the input dimension N of the majority of image processing algorithms are determined as $N = 2^{\nu}$, $\nu \in {}^{+}$. Representation of the matrix $\boldsymbol{P}$ in the form of two matrices with the number of components of $P_1 P_2 / 2$ makes it possible to reduce the computational efforts at the stage of computation of the eigenvectors and eigenvalues.

2. Encoding – decoding. Let us write the column vector of the matrix $\boldsymbol{C}$ in the form $\boldsymbol{c} = (c_0, c_1, \dots, c_{D_1 - 1})^T$. The direct and inverse transformation of the vectors of the matrix $\boldsymbol{C}$ in the basis of eigenvector decomposition of the data covariance matrix $\boldsymbol{C}$ are determined as

$$\widehat{\boldsymbol{C}} = \boldsymbol{A}^T \boldsymbol{C}, \qquad (4)$$
$$\boldsymbol{C} = \boldsymbol{A} \widehat{\boldsymbol{C}},$$

where $\boldsymbol{A}$ is the matrix of the eigenvectors of the covariance matrix of the order $D_1 \times D_1$. The designation $\hat{\boldsymbol{c}} = (\hat{c}_0, \hat{c}_1, \dots, \hat{c}_{D_1 - 1})^T$ describes the column vector of the matrix $\widehat{\boldsymbol{C}}$. To reduce the initial data, the matrix $\widehat{\boldsymbol{C}}$ is represented by the reduced number $d < D_1 D_2$ of coordinates. The missing $(D_1 D_2 - d)$ coordinates are replaced with the constant b_i. The finding of the constant is reduced to computation of the average values of the coordinates $\hat{c}_i$ of the vectors $\hat{\boldsymbol{c}}$. The computation of the distribution of values of the dispersions ^{2}

$$\mathrm{diag}[\sigma^2] = \mathrm{diag}\left[\mathrm{cov}(\widehat{\boldsymbol{C}})\right] = \begin{bmatrix} \lambda_0 & & \\ & \ddots & \\ & & \lambda_{D_1 - 1} \end{bmatrix} \qquad (5)$$

coordinates $\hat{c}_i$ makes it possible to get a preliminary assessment of the efficiency of encoding the boundary data and restoration of data without losses, when actually root-mean-square error is

$$\varepsilon = E\{\|\Delta \boldsymbol{c}\|^2\} = 0,$$

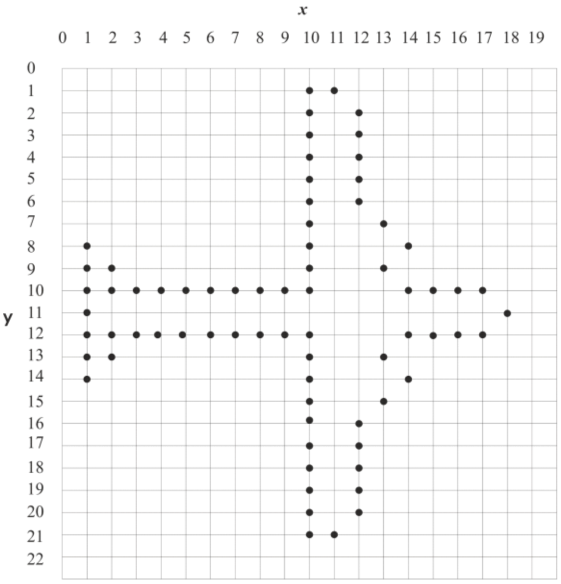
where $\Delta \boldsymbol{c} = (\boldsymbol{c} - \tilde{\boldsymbol{c}})$ – is the error vector of restoration. The encoding efficiency can be assessed using the formula

$$K = \frac{D_1 D_2 - d}{D_1 D_2}.$$

3. ASSESSMENT OF THE EFFICIENCY OF DESCRIPTION OF THE BOUNDARY

3.1. Example. *Illustration of implementation of the method of efficient encoding*

Fig. 1 and Fig. 2 show the image of the object boundary and its digital representation in the form of the matrix $\boldsymbol{G}$. The initial poin $g(x_i, y_i)$ of the boundary line has the coordinates: $x = 10, y = 1$. The arrangement of the elements of the matrix $\boldsymbol{G}$ along the axis x produces the matrix $\boldsymbol{C}$ shown in Fig. 3.



$$= \begin{pmatrix} 10 & 1 & 11 & 1 & 12 & 2 & 12 & 3 & 12 \\ 4 & 12 & 5 & 12 & 6 & 13 & 7 & 14 & 8 \\ 13 & 9 & 14 & 10 & 15 & 10 & 16 & 10 & 17 \\ 10 & 18 & 11 & 17 & 12 & 16 & 12 & 15 & 12 \\ 14 & 12 & 13 & 13 & 14 & 14 & 13 & 15 & 12 \\ 16 & 12 & 17 & 12 & 18 & 12 & 19 & 19 & 20 \\ 11 & 21 & 10 & 21 & 10 & 20 & 10 & 19 & 10 \\ 18 & 10 & 17 & 10 & 16 & 10 & 15 & 10 & 14 \\ 10 & 13 & 10 & 12 & 9 & 12 & 8 & 12 & 7 \\ 12 & 6 & 12 & 5 & 12 & 4 & 12 & 3 & 12 \\ 2 & 12 & 2 & 13 & 1 & 14 & 1 & 13 & 1 \\ 12 & 1 & 11 & 1 & 10 & 1 & 9 & 1 & 8 \\ 2 & 9 & 2 & 10 & 3 & 10 & 4 & 10 & 5 \\ 10 & 6 & 10 & 7 & 10 & 8 & 10 & 9 & 10 \\ 10 & 10 & 10 & 9 & 10 & 8 & 10 & 7 & 10 \\ 6 & 10 & 5 & 10 & 4 & 10 & 3 & 10 & 2 \end{pmatrix}$$

Fig. 1. 72-point boundary of the object object Fig. 2. Digital image $\boldsymbol{G}$ of the object boundary

$$\boldsymbol{C} = \begin{pmatrix} 10 & 12 & 14 & 16 & 16 & 14 & 12 & 10 & 10 & 10 & 7 & 3 & 1 & 1 & 3 & 7 & 10 & 10 \\ 11 & 12 & 13 & 17 & 15 & 13 & 12 & 10 & 10 & 10 & 6 & 2 & 1 & 1 & 4 & 8 & 10 & 10 \\ 12 & 12 & 14 & 18 & 14 & 12 & 12 & 10 & 10 & 9 & 5 & 2 & 1 & 2 & 5 & 9 & 10 & 10 \\ 12 & 13 & 15 & 17 & 13 & 12 & 11 & 10 & 10 & 8 & 4 & 1 & 1 & 2 & 6 & 10 & 10 & 10 \end{pmatrix}^T .$$

Fig. 3. Image of the boundary along the coordinate axis x

To assess the efficiency of encoding the image $\boldsymbol{C}$, the distribution of dispersions $[\sigma^2] = \boldsymbol{\Lambda}_{\hat{c}}$ of brightnesses of 72 pixels. Since the eigenvalues $\boldsymbol{\Lambda}_{c}$ of the covariance matrix $\mathrm{cov}(\boldsymbol{C})$ in the region of initial data coincide with those

$$\boldsymbol{\Lambda}_{\hat{c}} = \boldsymbol{A}^{\boldsymbol{T}}[\mathrm{cov}(\boldsymbol{C})]\boldsymbol{A}$$

of the covariance matrix in the region of transformations, i.e. $\boldsymbol{\Lambda}_{c} = \boldsymbol{\Lambda}_{\hat{c}}$ it is sufficient to compute $\boldsymbol{\Lambda}_{c}$ only. Fig. 4 shows the diagram of distribution of dispersions of the transformation coefficients, where $\lambda_i = 0$, for $i = 0, 1, \dots, 14$ and $\lambda_{15} = 0{,}3114$, $\lambda_{16} = 1{,}0989$, $\lambda_{17} = 10{,}8362$.

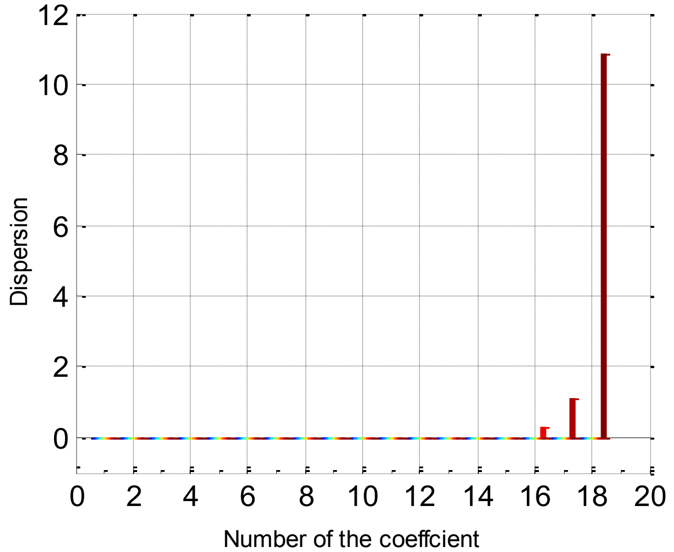


Fig. 4. Dispersions of the values of pixels of the image $\boldsymbol{C}$

The graphs of the functions of distribution of the dispersions σ^2 of the image demonstrate the filtration efficiency. As follows from the analysis of the diagram, almost the total energy of the signal is concentrated in the 3 last coordinates of the vector $\hat{\boldsymbol{c}} = \left(\hat{c}_0, \hat{c}_1, \dots, \hat{c}_{D_1-1}\right)^T$. The matrix determined by the relationship (4) is equal to

$$\boldsymbol{C} = \begin{pmatrix} \hat{\boldsymbol{c}}_1 \\ \hat{\boldsymbol{c}}_2 \\ \hat{\boldsymbol{c}}_3 \\ \hat{\boldsymbol{c}}_4 \end{pmatrix}^T = \begin{pmatrix} 14.9428 & 7.5881 & \cdots & 8.9724 & -12.7318 & -3.8227 & -4.6855 \\ 14.9428 & 7.5881 & \cdots & 8.9724 & -11.7159 & -5.3818 & -2.3323 \\ 14.9428 & 7.5881 & \cdots & 8.9724 & -12.9872 & -5.7182 & 0.5432 \\ 14.9428 & 7.5881 & \cdots & 8.9724 & -12.2473 & -3.6826 & 2.8475 \end{pmatrix}^T \quad (6)$$

The first 15 rows of the matrix (5) have the uniform structure. For illustration, the separate values for

$$\hat{c}_0 = 14{,}9428, \hat{c}_1 = 7{,}5881, \dots, \ \hat{c}_{15} = 8{,}9724$$

are shown. The obtained distribution of dispersions (5) makes it possible to determine preliminarily the encoding – decoding efficiency, at which the exact data restoration is implemented. The efficiency of encoding the boundary is determined by the number $d = 12$ of coefficients to be saved. For the example under consideration

$$K = \frac{D_1 D_2 - d}{D_1 D_2} = \frac{18 \cdot 4 - 12}{18 \cdot 4} \cong 0{,}83, \text{ or } 83\%.$$

As seen from the analysis of the diagram, there are dominant eigenvalues

$$\lambda_{17} > \lambda_{16} > \lambda_{15}.$$

Since the significance of each component $\hat{c}_i$ in performing the inverse transformation is determines by the respective eigenvalue, the value of the compression ratio K can be increased. To do this, we will compute the average value of the

$$E\{\hat{c}_{15}\} = -12{,}4206.$$

components $\hat{c}_{15}$ of the vectors $\hat{\boldsymbol{c}}$.

Besides, we will average the components $\hat{c}_{16}$ of the vectors $\hat{\boldsymbol{c}}_1{}^T$ and $\hat{\boldsymbol{c}}_4{}^T$. So we obtain

$$E\{\hat{c}_{16}\} = -3{,}7527.$$

The encoding result is the sequence of samples in the following lexicographic representation:

$$\hat{\boldsymbol{c}} = (\hat{c}_{65}, \hat{c}_{66}, \hat{c}_{68}, \hat{c}_{69}, \hat{c}_{70}, \hat{c}_{71}) =$$
$$= (-5{,}7182;\ -5{,}3818;\ -4{,}6855;\ -2{,}3323;\ 0{,}5432;\ 2.8475).$$

The compression efficiency is

$$K = \frac{72-6}{72} \cong 0{,}916, \text{ or } 92\%.$$

Thus, a data array being twelve times less than the initial one is required for describing the object boundary. The inverse transformation is implemented on the basis of use of the transformation kernel $\boldsymbol{A_d}$, which is made up of d eigenvectors of the covariance matrix $\operatorname{cov}(\boldsymbol{C})$ only. In the example, the eigenvectors of the matrix $\boldsymbol{A_d}$ correspond to the numbers λ_{17} и λ_{16}. The coordinates to be rejected are replaced with the average values of the coordinates $\hat{c}_i$ of the vectors $\hat{\boldsymbol{c}}$. So one of the inputs of the inverse transformation is the vector

$$\hat{\boldsymbol{c}}_1{}'^T = (\hat{c}_0{}', \quad \hat{c}_1{}', \dots, \hat{c}_{14}{}', \hat{c}_{15}{}', \hat{c}_{16}{}', \hat{c}_{17}{}')^T =$$
$$= (14{,}9428;\ 7{,}5881; \dots;\ 8{,}9724;\ -12{,}4206; -3{,}7527; -4{,}6855)^T.$$

As seen, the 17 average values $(\widehat{c}_0,\ \hat{c}_1, \dots, \hat{c}_{14}, \hat{c}_{15}, \hat{c}_{16})$, which have already been computed (6), and one component $\hat{c}_{17} = -4{,}6855$, which makes the major contribution to the representation $\hat{\boldsymbol{c}}_1$ participate in the description of the vector $\hat{\boldsymbol{c}}_1$. The actual compression at this processing stage corresponds to the efficiency value.

$$K = \frac{D_1 - d}{D_1} = \frac{18-1}{18} \cong 0{,}94, \text{ or } 94\%.$$

Thus, a data array being eigheen times less than the initial one is required for describing the object boundary.

After applying the inverse transformation to the (6) and performing the rounding operation, we obtain the following estimates of the coordinates of the boundary along the x axis:

$$\text{round}\,\{\tilde{c}_1\} = (10, 12, 14, 16, 16, 14, 12, 10, 10, 10, 7, 3, 1\ ,1, 3, 7, 10, 10).$$

The root-mean-square error is $\varepsilon = 0$.

Further, such computations are performed on the basis of the data describing the boundary by the coordinate y.

CONCLUSIONS

1.1. Instead of transfer of all the data describing the object boundary curve, it is sufficient to transfer the data featured by maximum dispersions in the region of transformations.

1.2. To obtain a compact description of the object boundary, it is sufficient to save the transformation coefficients with maximum dispersions, mean vectors and truncated kernel of the transformation only.

2. Application of the dispersion criterion when describing the boundary of the object under investigation makes it possible to compress the data actually without losses.

3. The preliminary computation of the function of distribution of the dispersion makes it possible to speed up the process of transmission, processing and analysis of the image, decoding of its main features and information attributes.

4. Solving the problems related to detection or seeking of certain objects on images is simplified.

5. At present, the considered method can be regarded as a practical computational procedure.

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