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DESIGN OF HIGH-PRECISION WEIGHING CELLS BASED ON STATIC ANALYSIS

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ABSTRACT

The present contribution deals with the design of a monolithic weighing cell for the realization of a further developed mass comparator for 1kg-standards. The monolithic structure with semi-circular flexure hinges is approximated by a rigid body model. The resulting equations can be used as design equations for a first layout of the mechanical system. Existing adjustment concepts for the stiffness characteristic and the sensitivity to quasi-static ground tilt are included. They are extended by the novel approach to manipulate adjustment masses on the levers of the linear guide. Based on this concept, an optimal design for the weighing cell is determined. The comparison to a geometric non-linear finite element model reveals the limits of the rigid body model. By a parameter study of the adjustment parameters in the finite element model, the stiffness and tilt sensitivity were reduced by five orders of magnitude compared to the unadjusted weighing cell.

Index Terms - electromagnetic force compensated balance, mass comparator, adjustment concept, finite element model, semi-circular flexure hinge

1. INTRODUCTION

Precision weighing technology is a research area of persisting importance for the global economy. The reference of the SI-unit of mass depends on the performance of mass comparators in the dissemination chain, [1]. Presently available mass comparators consist of a monolithic mechanical system with flexure hinges, a fixed counterweight and an electromagnetic force compensation (EMFC). The scheme of an EMFC-balance is presented in Fig. 1. The present work focuses on the monolithic mechanical structure. Actuators, sensors and the controller are strongly simplified as forces or displacement constraints. The mechanic system can be divided into two main functional groups, the linear guiding system and the transmission lever. The linear guiding system is a monolithic realization of a parallelogram linkage including parts (2),(3) and (4), see Fig. 1. The transmission lever (7) is a simple beam suspended by a flexure hinge. These subsystems are coupled by a coupling element (6).

Balances for high resolutions have a nominal load with a small weighing range restricted to a few grams. The counter weight is designed to compensate the weight force of the sample weight, except for small mass differences to the nominal load (Δm). This difference is compensated by the electromagnetic actuator of the EMFC. The required current for the electromagnetic force is proportional to Δm, [2]. The further development of present mass comparators in terms

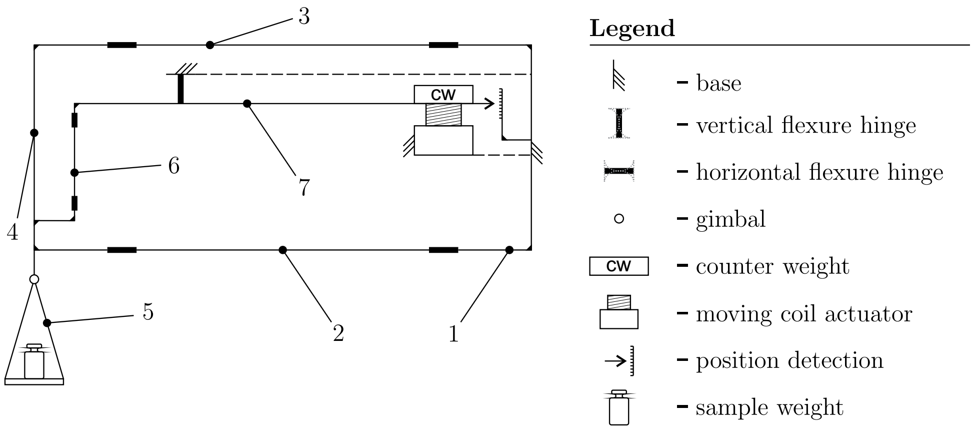

Figure 1: Monolithic EMFC-weighing cell in two dimensions with the flexure hinges depicted with thick solid lines as indication of the joint orientation. 1-base, 2-lower lever, 3-upper lever, 4-load carrier, 5-weighing pan with sample weight, 6-coupling element, 7-transmission lever.

of reduced measurement uncertainties requires a significant reduction in susceptibility to external perturbations. Ground vibrations and quasi-static ground tilt are major limiting factors, [3]. Adjustments to the weighing cell can reduce the spurious influences of ground tilt. This has already been shown in extensive studies on beam balances, both for knife edge- and flexure strip suspensions, [4], [5], [6]. In [7], a rigid body model of an EMFC-weighing cell is analyzed with the result that zero stiffness and zero tilt sensitivity can be achieved by adjustments to the centers of mass (CM) of the transmission lever and the parallelogram linkage. This possibility is outlined in [8] as well.
In the present work, a quasi-static rigid body model of the weighing cell is derived using Lagrange's equations of second kind. The results of this model are compared to a finite element model (FE model) revealing the limits of the rigid body model for monolithic weighing cells. Further, the capabilities of the adjustment concept are checked by parameter studies with the FE model. The objectives for the characteristic values are a stiffness of $C \leq 10\,\mathrm{mN\,m^{-1}}$ and a tilt sensitivity of $D \leq 2.5\,\mu\mathrm{N\,rad^{-1}}$.

2. MECHANICAL MODEL AND ADJUSTMENT CONCEPT

The mechanical model of the monolithic mechanism is simplified based on the following assumptions: The compliant mechanism has concentrated compliance - semi-circular flexure hinges. All other parts are considered as rigid bodies with lumped masses. The flexure hinges are modeled as perfect rotational joints with a fixed rotational axis and a constant rotational stiffness $\approx 17.94\,\mathrm{Nmm\,rad^{-1}}$. The stiffness is determined by a FE model of a single flexure hinge. Every flexure hinge is modeled with equal rotational stiffness. Frictional losses in the joints remain unconsidered.

The coupling element of the subsystems is modeled as a deflection dependent transmission ratio between the deflection angles q_2 (linear guide) and q_7 (transmission lever), see Fig. 2. With this constraint, the degree of freedom of the rigid body mechanism equals 1. The angle q_7 of the transmission lever is designated as independent system variable. Instead of using an additional adjustment mass, the total CM of the respective part is displaced.

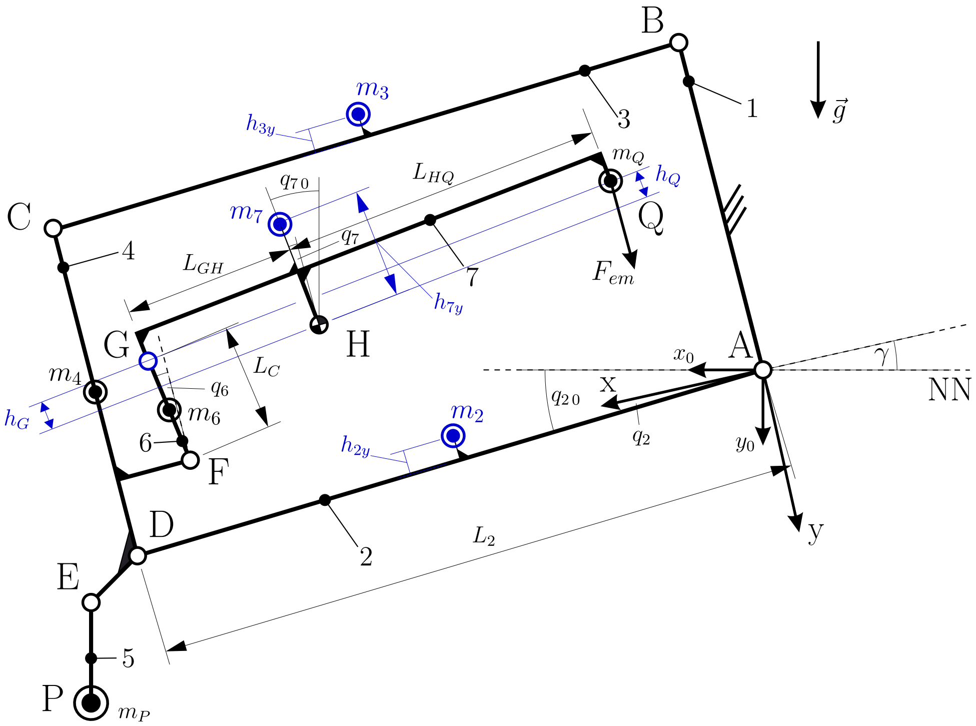

Figure 2: Rigid body model of the deflected weighing cell with an inclined base with respect to $\vec{g}$. The four adjustable parameters $h_{2y} = h_{3y}$, h_{7y}, h_G and h_Q are highlighted in blue.

2.1. Adjustment concept

Weighing cells with high resolution rely on very thin flexure hinges as rotational joints to obtain the highest possible sensitivity. The minimum thickness of the joints has a technological limit that lies in the range of 50 μm, [9]. To further enhance the sensitivity, the following adjustments can be applied to the weighing cell:

i) **transmission lever:** y-distance of center of rotation (CR) (joint H) to:
   a) CM transmission lever (m_7) $\rightarrow h_{7y}$
   b) joint G $\rightarrow h_G$
   c) joint Q $\rightarrow h_Q$

ii) **linear guide:** y-distance of center of rotation (CR) (joint A, B) to
   a) CM parallel levers (m_2, m_3) $\rightarrow h_{2y}, h_{3y}$

The quasi-static analytic model of the weighing cell includes all adjustment parameters (see Fig. 2) to derive a statement for a design with minimal stiffness and tilt sensitivity.

2.2. Static model of the weighing cell

The system equation for the static equilibrium is derived using Lagrange's equations of second kind:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j \quad \text{with} \quad L = T - U \quad \text{and} \quad j = 1, 2, ..., f$$

Here, j represents the number of the independent system variable, Q_j the generalized forces and f the degree of freedom of the mechanical system. The number of independent system variables is $f = 1$: q_7. The system is conservative, except for the force applied to the transmission lever (Q_{EMFC}):

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_7}\right) - \frac{\partial L}{\partial q_7} = Q_{EMFC} \tag{1}$$

For a quasi-static consideration, the kinetic energy T is zero and (1) simplifies to:

$$\frac{\partial U}{\partial q_7} = Q_{EMFC}$$

The potential energy U of the weighing cell is formulated based on $(\vec{e}_{x_0}, \vec{e}_{y_0}, \vec{e}_{z_0})$ with the contributions of the point masses U_m and the elastic potential of the joints U_{el}:

$$\begin{aligned}
U_m &= -m_P\vec{g}\cdot\vec{r}_P - m_Q\vec{g}\cdot\vec{r}_Q - m_{S2}\vec{g}\cdot\vec{r}_{S2} - m_{S3}\vec{g}\cdot\vec{r}_{S3} - m_{S4}\vec{g}\cdot\vec{r}_{S4} - m_{S6}\vec{g}\cdot\vec{r}_{S6} - m_{S7}\vec{g}\cdot\vec{r}_{S7} \\
U_{el} &= \frac{1}{2}c_H\,q_7^2 + \frac{1}{2}\left(c_A + c_B + c_C + c_D\right)q_2^2 + \frac{1}{2}c_G\left(q_7 - q_6\right)^2 + \frac{1}{2}c_F\,q_6^2 \\
U &= U_m + U_{el}
\end{aligned} \tag{2}$$

With the definition of the gravity vector $\vec{g}$:

$$\vec{g} = 0\,\vec{e}_x + g\,\vec{e}_y + 0\,\vec{e}_z.$$

In order to treat the system in a straightforward manner, a simplification concerning the kinematic coupling of the two subsystem is necessary. The relation is based on the assumption that the points F and G travel the same vertical distance. This is justified since the weighing cell is practically not deflected. Hence, the trigonometric functions can be replaced by their respective Maclaurin Series truncated after the second term: $\left(\sin(q_7) \approx q_7 - \frac{1}{6}q_7^3\,,\ \cos(q_7) \approx 1 - \frac{1}{2}q_7^2\right)$. With third order terms neglected, this results in (3) for the coupling of the subsystems transmission lever and linear guide:

$$i_t = \frac{L_{GH} + \frac{h_G}{2}q_7}{L_2} \approx \frac{q_2}{q_7} \tag{3}$$

The angle of the coupling element (q_6) is approximated by:

$$q_6 \approx \frac{h_G\,q_7 - \tau\,q_7^2}{L_C} \qquad \text{with} \qquad \tau := \frac{L_{GH}\,L_2 - L_{GH}^2}{2\,L_2}$$

The resulting equation from (2) and (3) is derived by ∂q_7:

$$S_{wc} := \frac{\partial U}{\partial q_7} = f(q_7, \gamma) = Q_{EMFC} \tag{4}$$

The generalized force for the electromagnetic force of the moving coil actuator F_{em} is given by:

$$Q_{EMFC} \approx F_{em}\,h_Q\,q_7 - F_{em}\,L_{HQ}.$$

The linearization of (4) leads to a rather simple equation that can be sorted according to q_7 and γ by partial differentiation. This clear structure provides a good overview about the main factors that influence the stiffness and the tilt sensitivity. Furthermore, it becomes obvious which parameters can be used for an adjustment of the weighing cell in terms of stiffness and tilt sensitivity. The optimal system configuration is fulfilled if the following conditions hold:

$$\begin{aligned} C := \frac{\partial S_{wc}}{\partial q_7} = 0 = &-\frac{L_{2x}\,h_G\,m_2\,g}{L_2} - \frac{L_{3x}\,h_G\,m_3\,g}{L_2} - h_G\,(m_P + m_4 + m_6)\,g - h_{7y}\,m_7\,g \\ &- h_{Q1}\,m_Q\,g - h_{Q2}\,F_{em} - h_{2y}\,m_2\,g\,\frac{L_{GH}^2}{L_2^2} - h_{3y}\,m_3\,g\,\frac{L_{GH}^2}{L_2^2} \\ &+ (c_A + c_B + c_C + c_D)\,\frac{L_{GH}^2}{L_2^2} + c_F\,\frac{h_G^2}{L_C^2} + c_G\left(1 - \frac{h_G}{L_C}\right)^2 + c_H \end{aligned} \tag{5}$$

$$D := \frac{\partial S_{wc}}{\partial \gamma} = 0 = -\frac{L_{GH}\,h_{2y}\,m_2\,g}{L_2} - \frac{L_{GH}\,h_{3y}\,m_3\,g}{L_2} - h_{7y}\,m_7\,g - h_Q\,m_Q\,g \tag{6}$$

$$\begin{aligned} B := S_{wc}(q_7 = 0, \gamma = 0) = 0 = &\,F_{em}\,L_{HQ} + m_Q\,g\,L_{HQ} \\ &- L_{GH}\,g\left(\frac{L_{2x}}{L_2}m_2 + \frac{L_{3x}}{L_2}m_3 + m_P + m_4 + m_6\right). \end{aligned} \tag{7}$$

The linear equation system (5) to (7) describes the relevant properties of the weighing cell, stiffness - C, tilt sensitivity - D and the equilibrium condition of the non-deflected system - B. The weighing cell is designed to comply with the solution of the equation system. Fig. 3 shows the values of h_{2y} and h_{7y} for solutions of the linear equation system $\{(5), (6), (7)\}$ in dependence of h_G. Note, that throughout this paper m_7 is zero and $h_{7y} = h_{Q1}$.

Due to manufacturing- and mounting tolerances, the manufactured and assembled weighing

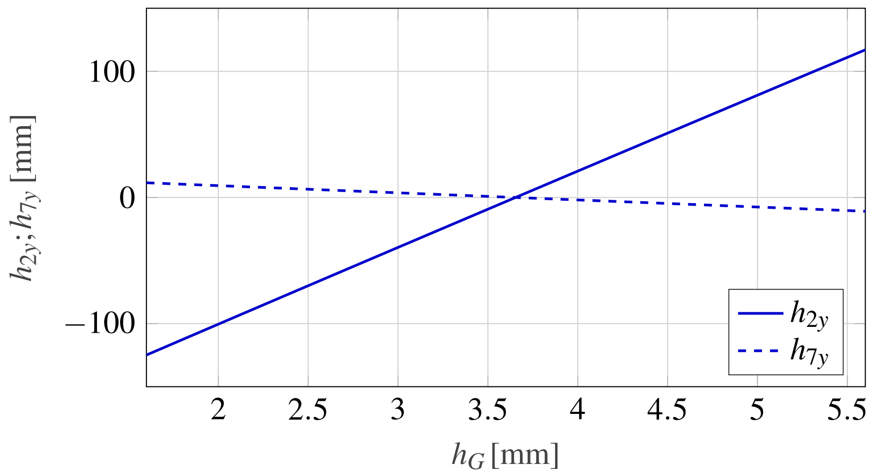

Figure 3: Dependency of the adjustment parameters for the solution of the equation system of the weighing cell for the given parameter set in Tab. 2a.

cell has to be adjusted to compensate the geometrical deviations. Additionally, parasitary deformations of the monolithic structure have to be considered.

2.3. Limits of the derived rigid body model

Investigations on flexure hinges reveal that flexures show a limited precision of rotation due to a shift of the rotational axis once they are deflected, [10]. EMFC-weighing cells are operated

Table 1: Model parameters of the weighing cell.

(a) General model parameters

parameter	value	unit
L_{GH}	18.0	[mm]
L_2	75.0	[mm]
$L_C(h_G = 0)$	40.0	[mm]
L_{2x}	37.5	[mm]
L_{3x}	37.5	[mm]
L_{HQ}	70.0	[mm]
h_{Q2}	0.0	[mm]
c	17.9377	[Nmm rad^{-1}]
m_P	1.0	[kg]
m_Q	0.257	[kg]
m_2	0.05	[kg]
m_3	0.05	[kg]
m_4, m_6, m_7	0.0	[kg]
$\lvert\vec{g}\rvert$	9.81	[m s^{-2}]

(b) FE model parameters

parameter	value	unit
L_{AB}	100.0	[mm]
R	3.0	[mm]
H	4.0	[mm]
h	0.05	[mm]
b	10.0	[mm]
E	71.0e3	[N mm^{-2}]
ν	0.33	[-]
ρ[a]	0.0	[kg m^{-3}]
Δy_Q[b]	-0.01	[mm]
γ	0.1	[°]

[a]Density in FE model set to zero for compliance with the lumped masses in the analytical model.
[b]Displacement of point Q in y-direction for stiffness determination.

very closely around the zero deflection position leading to the assumption that these effects are negligible.

Apart from the rotational precision of the flexure hinges, parasitary force components on the hinges may have a pronounced effect on the characteristic of the total mechanism. Especially, for a not perfectly aligned weighing cell ($\gamma \neq 0$) this results in an *s-shape* deformation of the hinges and an additional parasitary torque on the connected parts. This effect was considered for the development of the FB-2 equal-arm balance of BIPM, [3], [5].

Another aspect is a change in rotational stiffness of the joint due to the static axial load, [11]. This is especially relevant for the central flexure hinge that suspends the greatest amount of the total mass of the structure, including the sample weight and the counterweight.

3. COMPARISON TO FINITE ELEMENT MODEL

To account for the mentioned additional effects resulting from parasitary deformations of the monolithic weighing cell, a three dimensional, geometric non-linear FE model is set up in ANSYS®.

3.1. Finite element model

The FE model is chosen to be three-dimensional to be able to consider out-of-plane loading situations in the future. The geometrical parameters are adopted from Tab. 2a with the additional parameters in Tab. 2b. In contrast to the analytic model, the FE model is kinematically reversed. The base of the weighing cell is fixed and $\vec{g}$ is rotated about the z-axis. The sample weight (m_P), counter mass (m_Q) and the lever masses (m_2, m_3) are modeled as point masses coupled to surface nodes of the respective parts. The density of the material is set to zero to keep the FE model comparable to the analytical model. The EMFC is realized as a y-displacement constraint of

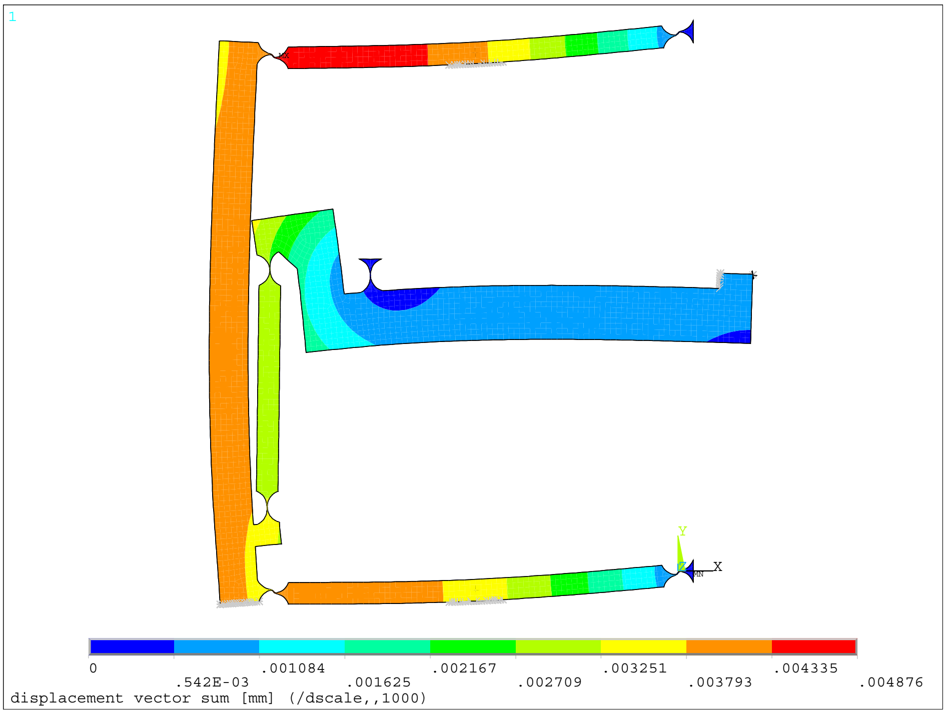

Figure 4: Parasitary deformations of the weighing cell structure presented by the false color representation of the displacement vector sum. The deformation of the structure is scaled by a factor of 1000.

nodes on the right end of the transmission lever and the required force F_{em} is the sum over the reaction forces of the constrained nodes.

Fig. 4 shows the displacement vector sum for the weight cell loaded with a mass of $m_P = 1\,\mathrm{kg}$ and a counterweight of $m_Q = 0.257\,\mathrm{kg}$. The angle between base and $\vec{g}$ is zero. Since the deformations lie in the range of micrometers, they prove to be relevant for the weighing cell design. The deflection of the transmission lever can be expected to be most critical since h_G and h_Q might deflect relative to the effective CR of joint H. The elongation of the coupling element leads to a deflection state of the linear guide. This results in an increase in sensivity to lateral force components (e.g. pan swing, see [6]).

An additional factor that influences the tilt sensitivity of flexure hinges is their lateral compliance. This has already been observed and mathematically described in [5] for a flexure strip with a constant cross section. For flexure hinges with a semi-circular contour the behavior will differ due to the pronounced change in cross sectional height. Given these additional effect, the limits for the minimization of stiffness and tilt sensitivity are investigated by parameter studies.

3.2. Behavior close to vanishing stiffness and tilt sensitivity

The parameter variation is limited to the highlighted adjustment parameters in Fig. 2. Consequently, the variations are equal to adjustments of a manufactured monolithic weighing cell. The following adjustment strategy is used:

i) adjustment of h_G to compensate the restoring forces of the flexure hinges $\rightarrow C(h_G^*) = 0$, see Fig. 5

ii) parameter variation of h_{2y} and h_{7y} with a rough grid $\rightarrow$ determine $\{c(h_{2y})|C = 0\}$ and $\{d(h_{2y})|D = 0\}$

iii) determine intersection: $c = d \rightarrow h_{2y}^*$ and h_{7y}^*

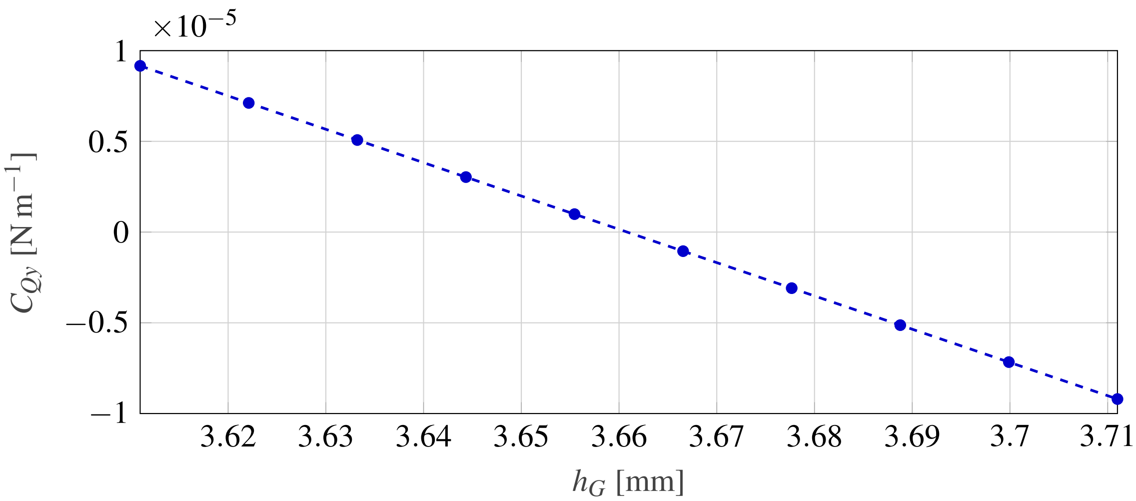

Figure 5: Relation between h_G and the stiffness of the weighing cell C_{Qy} determined at the force application point of the EMFC (point Q) in y-direction ($h_{2y} = h_{7y} = 0$).

In Fig. 5 the stiffness of the weighing cell structure (C_{Qy}) is plotted over h_G. For the zero crossing $C(h_G^*) = 0$ a value of $h_G^* = 3.661$ mm was determined. The comparison with the rigid body model in Fig. 3 reveals a difference to the FE model of $\Delta h_G^* \approx 0.042$ mm. This indicates a higher stiffness of the FE model. The determined value for $h_G^* \approx 3.661$ is used to calculate the properties of the weighing cell prior to further adjustments. The stiffness (C) is already within the aspired range but the tilt sensitivity (D) exceeds the objective, see 1$^{\text{st}}$ step in Tab. 3. By adjusting the heights of the CM of the transmission lever h_{7y} and the levers of the linear guide h_{2y}, the remaining tilt sensitivity can be reduced - with a further decrease in stiffness. For the effect of the second adjustment step compare 1$^{\text{st}}$ and 2$^{\text{nd}}$ step in Tab. 3. The final configuration of the weighing cell has close to ideal properties. This results in the statement that all parasitary deformations in the *x*-*y* plane and the resulting parasitary torques can be compensated by adjusting h_{2y}, h_{7y} and h_G. However, the results of the numeric calculations in this range of precision should be interpreted carefully. Additionally, many spurious effects like temperature gradients, dynamic effects, manufacturing tolerances, anelastic material behaviour and alignment in the *x*-*y* plane are not included in the model.

Table 3: Adjustments steps of the FE model.

parameter	unit	**unadjusted**	**1st step** $C \approx 0$	**2nd step** $C \approx 0, D \approx 0$
h_G	[mm]	0.0	3.661	3.661
$h_{2y}{}^a$	[mm]	0.0	0.0	-0.0395
$h_{7y}{}^b$	[mm]	0.0	0.0	0.0003
C_{Qy}	[N m^{-1}]	7.383	-3.140 e-4	-1.971 e-5
D_{Qy}	[N rad^{-1}]	-4.254 e-2	-1.172 e-4	2.404 e-7

[a] This adjustment parameter displaces two masses m_2, m_3 ($h_{3y} = h_{2y}$).
[b] The mass m_Q is displaced by h_{7y}.

4. CONCLUSION

In this paper, the modeling of high-precision monolithic weighing cells based on quasi-static mechanical models is discussed. A linear equation system is introduced presenting the most important mechanical properties of the weighing cell at a glance. The solution of the equation system, involving adjustable parameters, provides a foundation for a first design definition of a weighing cell based on geometry, lumped masses and joint stiffness. A comparison with a geometric non-linear FE model reveals the limitations of the linear model and stresses the need for more advanced models to refine the design. With the geometric non-linear FE model it was shown that very low values for stiffness and tilt sensitivity can be obtained by combining the adjustments. Compared to the unadjusted weighing cell, the stiffness and tilt sensitivity could be reduced by five orders of magnitude. The parasitary torques resulting from elastic deformations can thus be fully compensated by the presented adjustments for small deflections of the structure. Effects like manufacturing tolerances, out-of-plane loads and anelastic material behavior are expected to be limiting factors for the performance. These topics as well as the incorporation of further details to the mechanical models will be considered in the ongoing work. The theoretical results will be verified by experiments in the near future.

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