

## The influence of fiber undulation on the mechanical properties of FRP-laminates

*Christian Fiebig, Michael Koch*

Technische Universität Ilmenau – Fachgebiet Kunststofftechnik, Germany

### ABSTRACT

Energy consumption and mass reduction, as well as material efficiency in the machinery industry enhances the attractiveness of using fiber reinforced plastics (FRP) components in fast moving machine components. Due to the high specific stiffness and strength at minimal weight, the composite components are particularly suitable for addressing this challenge. Typically, calculation results show much better performance than manufactured components. Material properties can only be described idealistically making it necessary to understand the part design and manufacturing factors impacting on mechanical performance. The classic laminate theory (CLT) and failure criteria such as the Tsai–Wu criterion are typically used to define the performance of a laminate. However, these tools have limitations when it comes to woven fabrics, since the stress condition inside of the fabric cannot be accurately described. The undulation of the fiber is caused by the weaving and affects the mechanical properties of the fabric. There are three commonly used types of weaves for fibers: plain, twill and satin weave. Here, the plain weave is the tightest and the satin weave is the loosest fabric. It is shown, that the undulation of the roving influences the performance of the laminates. It can be seen that the properties of the laminate depend on the number of fiber crossings using the same type of fibers. In this paper, two factors,  $\chi_1$  and  $\chi_2$ , are developed. These factors were derived from a comparison of the theoretically and experimentally determined values of the tensile stiffness  $E_x$  (Young's-modulus in the longitudinal direction). It is to be assumed that the laminate with  $0^\circ/90^\circ$  unidirectional (UD) fabric has a higher young's modules due to the missing fiber crossings. In order to prove this hypothesis, a simple model is developed. This model can be used to describe the influence of the fiber's undulation on the stiffness of the laminate.

## 1. INTRODUCTION

### 1.1 General context

The presented problem deals with the influence of fiber undulation on the mechanical properties of fiber reinforced plastics (FRP) and how this influence can be integrated into the classical laminate theory (CLT). This is necessary to enhance the accuracy of the calculation of the behavior of FRP. The reliability of the simulation depends on the calculation. The fibers' undulation is caused by several phenomena. Firstly, there are no unidirectional fibers which are completely straight. Every roving is slightly wavy. Secondly, there is undulation caused by the weave of the fabrics. To identify the influence of the fiber undulation on the mechanical properties of the laminate, several specimen need to be produced and tested. For a better visibility of imperfections inside the laminate, glass fibers are used.

### 1.2 Motivation for research

Even single filaments show a slight ripple inside a fiber strand as a result of its manufacture process. This effect is intensified by bundling single fibers into rovings. The result of the undulation is a reduced young's modulus because the fibers need to be stretched under

external load before they can transfer all forces acting on them. Apart from the undulation of the single fibers and rovings and undulation caused by the crossing of rovings in woven fabrics, there is undulation caused by the shrinkage of the matrix. [1]

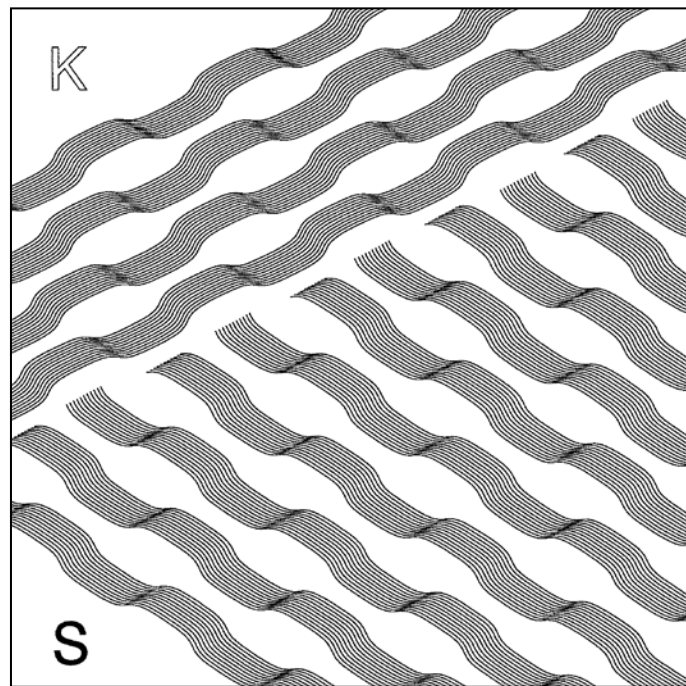


Figure 1: Undulation of fibers [2]

The effect of the undulation on the mechanical properties can be described for every single laminate by testing a specimen of this specific laminate. However, this method leads to a huge amount of tests because they need to be repeated for every single laminate configuration. Therefore, another method to describe this effect is necessary. Since the existing laminate theory is a good basis for many calculations, it is reasonable to use it and expand it by an additional factor. In contrast to metal, the determination of the failure of FRP is researched insufficiently. Although the use of simulation software facilitates the assessment of new designs, the result concerning loading conditions is still too imprecise. This leads to high safety factors and thus, to an additional mass fraction in the finished component. A factor considering the undulation of the fibers allows minor safety factors whereby the lightweight potential is exploited more effectively.

### 1.3 State of the art

#### 1.3.1 Calculation of fiber reinforced plastics

The computation of FRP is divided into the calculation of the material properties of the laminate e.g. the young's modulus and the calculation of the stresses caused by external forces and momenta.

In addition to the young's moduli  $E_1$  and  $E_2$ , there are equations for the shear modulus  $G_{12}$  and the Poisson's ratio  $\nu_{12}$ . These parameters can be described as follows [3]:

$$E_1 = \varphi \cdot E_{F1} + (1 - \varphi) \cdot E_H \quad (1)$$

$$E_2 = \frac{E_H \cdot (1 + 0,85\varphi^2)}{\varphi \cdot E_H/E_{F2} + (1 - \varphi)^{1,25}} \quad (2)$$

$$G_{12} = \frac{G_F \cdot (1 + 0,6\varphi^{0,5})}{\varphi \cdot G_H/G_F + (1 - \varphi)^{1,25}} \quad (3)$$

$$\nu_{12} = \varphi \cdot \nu_F + (1 - \varphi) \cdot \nu_H \quad (4)$$

Values for the factors for fiber volume content ( $\varphi$  – FVC), young's modulus of the fiber in the longitudinal, in the transverse direction and of the matrix ( $E_{F1}$ ,  $E_{F2}$ ,  $E_H$ ), the shear moduli of the fiber and matrix ( $G_F$ ,  $G_H$ ) and the Poisson's ratios of fiber and matrix ( $\nu_F$ ,  $\nu_H$ ) need to be determined experimentally. Additionally, the thickness of the single layer needs to be identified. The shown equations do not consider the undulation of the fibers. However, this could be done by using the CLT.

To determine the stresses inside the laminate, the CLT is used. Although a second theory, the net-theory exists, it is not described here because it is only used for an approximate calculation of the laminate. The basis of the CLT is the Hooke's plate theory, which describes the behavior of even and thin plates under load. Since FRP are an anisotropic material and in this case with plane stresses, a disc or plate model can be used. In this paper the disc model is applied because the specimens are only tested under tensile load. It is assumed that in the present case there are small strains inside the laminate and the single layer. Furthermore, there is a linear-elastic connection between stress and strain. This applies Hooke's law:

$$\sigma = E \cdot \varepsilon \quad (5)$$

In the following, the CLT is applied using the previously identified input variables. Firstly, the stiffness matrix  $Q$  of the single layers is set up:

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \quad (6)$$

Thereby, the following correlation is applied:

$$Q_{11} = \frac{E_1}{1-\nu_{12}\cdot\nu_{21}} \quad Q_{12} = Q_{21} = \frac{\nu_{21}\cdot E_1}{1-\nu_{12}\cdot\nu_{21}} \quad Q_{22} = \frac{E_2}{1-\nu_{12}\cdot\nu_{21}} \quad Q_{66} = G_{12} \quad (7)$$

In the next step, the laminate stiffness matrix  $A$  and the resulting laminate stiffness matrix is determined:

$$[A] = \sum_k \frac{t_k}{t_{Lam}} \cdot [Q] \quad (8)$$

$$[a] = [A]^{-1} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \quad (9)$$

On the basis of the foregoing, the engineering constants of the laminate can be calculated:

$$E_x = \frac{1}{a_{11}}, \quad E_y = \frac{1}{a_{22}}, \quad G_{xy} = \frac{1}{a_{33}}, \quad \nu_{xy} = \frac{a_{21}}{a_{11}}, \quad \nu_{yx} = \frac{a_{12}}{a_{22}} \quad (10)$$

Using Hooke's law, the stresses and strain of the laminate under external forces can be calculated.

$$\{\sigma\}_{lam} = \frac{1}{t_{Lam}} \cdot \{N\}, \quad \{N\} = \frac{1}{b} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \quad (11)$$

$$\{\varepsilon\}_{lam} = [a] \cdot \{\sigma\}_{lam} \quad (12)$$

In the presented experiment it is assumed that the tractive forces only act in one direction which eliminates thrusts. With the application of failure criterions the strength properties of the laminate can be determined after completion of the CLT.

### 1.3.2 Applied materials und processes

All experiments were performed using glass fiber reinforced plastics. The used fiber material is E-glass. Glass fibers are suitable for the tests, since imperfections of the material are easily visible. Thus, the quality of the produced specimen can be evaluated. Four different weaves were chosen for the experiments (fig. 2):

- unidirectional non-woven fabric (UD)
- satin weave 4/3
- twill weave 2/2
- plain weave 1/1

The four weaves show a different degree of undulation. The unidirectional layers solely have fiber undulation. The satin weave has the lowest undulation among the layers because the rovings of the warp direction merely cross them after passing four weft threads. In case of the twill weave, the passing rovings of the warp and weft direction cross after two or three uncrossed threads. The plain weave is undulated mostly since warp and weft threads are binding by turns.

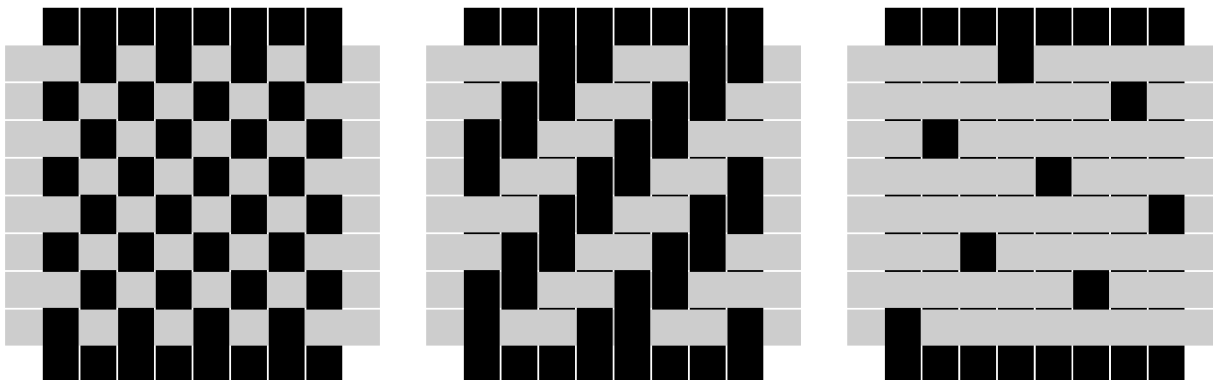


Figure 2: Plain, twill and satin weave [4]

Due to compressive stresses occurring at the crossing points [5] of the fibers, it is assumed that the following correlation applies:

$$UD < \text{satin} < \text{twill} < \text{plain} \quad (13)$$

The specimen were produced by using the RTM process. Advantages of this process are a good link of fiber and matrix and a minimal blistering in the laminate. The fiber volume

content can be achieved precisely. As a result, specimen in the same and in different weaves are reproducible. The method's parameters were kept constant to minimize the influence of the production process on the test results.

#### 1.4 Purpose

The objective of this investigation is to establish correction factors to represent the undulation effect towards the mechanical performance of a laminate and its calculation with the CLT. They ensure the consideration of the fiber's undulation while calculating fiber composites. Depending on the undulation, the mechanical properties of the test laminates need to be identified and included in the calculation equation of the young's modulus of the single layer. Using the re-calculated material data, the stress and strain is calculated by means of the CLT.

## 2. INITIAL DATA AND METHODS

### 2.1 Concept

The implementation of correction factors is done in two stages. It is based on the computation of the specific values of the laminate by using the CLT. Initially, the young's modulus of the test laminate needs to be determined using the CLT. Material data of fibers and matrix as well as the identified fiber volume content form the raw data. The next step is a comparison of the measured and calculated values of  $E_x$  for the  $0^\circ/90^\circ$  - UD fabric. The value of  $\chi_1$  is derived from the ratio of calculated to measured young's modulus  $E_x$  [6]:

$$\chi_1 = \frac{E_{X,UD,cal}}{E_{X,UD,mes}} \quad (13)$$

This factor  $\chi_1$  in combination with the value  $\chi_2$ , which depends on the undulation, can be integrated into the rules of mixing according to Puck [2]:

$$E_1 = \chi_1 \cdot \chi_2 \cdot \varphi \cdot E_{f1} + (1 - \varphi) \cdot E_m \quad (14)$$

$$E_2 = \frac{E_m \cdot (1 + 0,85 \cdot \varphi^2)}{\varphi \cdot \left( \frac{E_h}{\chi_1 \cdot \chi_2 \cdot E_{f2}} \right) + (1 - \varphi)^{1,25}} \quad (15)$$

In this process the value of  $\chi_2$  needs to be adjusted empirically until the calculated young's modulus matches the measured one.  $\chi_2$  indicates the deviation depending on the undulation of the calculated to the ideally measured young's modulus of the single layer.

The method results in two factors which only vary the young's modulus of the fiber for a single layer. Since undulation is a pure effect of the fiber, the young's modulus is the leading contribution.

The measured values are compared to the calculated results. In order to compensate deviations of the values, the factors  $\chi_1$  and  $\chi_2$  were varied.  $\chi_1$  describes the difference between the calculation model and the values of the UD fabric. To obtain  $\chi_2$ , the properties of the  $0^\circ/90^\circ$  UD fabric are compared to the values of the laminates with the woven fabric [6].

### 2.2 Basic data

The basic characteristics of the used fiber material are presented in table 1. While choosing the material, it was emphasized to use material with a similar weight per unit area.

tissue	UD	plain	twill	satin
fiber material	glass			
grammage $m_{fg}$ [g/m <sup>2</sup> ]	440	420	390	390
young's modulus $E_f$ [GPa]	72			
weave	-	1/1	2/2	4/3

Table 1: Properties of the applied fibers

Table 2 shows the characteristics of the injection resin which consists of epoxy resin and the associated hardener.

resin	density $\rho_r$ [g/cm <sup>3</sup> ]	young's modulus $E_r$ [GPa]	tensile strength $R_r$ [MPa]	elongation at break $A_r$ [%]
EP	1,1 – 1,25	2,8 – 3,6	70 – 90	6 – 8

Table 2: Properties of the resin used

Following parameters were chosen for the RTM process [6]:

- mold temperature:  $T_{mol} = 60^\circ\text{C}$
- injection temperature :  $T_{inj} = 23^\circ\text{C}$
- injection pressure:  $P_{inj} = 4 \text{ bar}$
- mold pressure:  $P_{press} = 120 \text{ bar}$
- vacuum pressure:  $P_{vac} = 0,95 \text{ bar}$

In each case, ten test plates with the dimensions 350x350 mm are produced out of ten layers of the woven fabric. All used laminates have different thickness. This difference results from the slightly differing weights per unit area of the chosen fabric. Density, fiber volume content, and the young's modulus in the longitudinal direction were determined for all specimens. Since  $E_x = E_y$  and the fabrics are woven regularly in  $0^\circ/90^\circ$ -direction, only the determining of the young's modulus  $E_x$  is necessary.

$E_x$  was determined with a 30 kN tensile testing machine following DIN EN ISO 527-4 [7]. The tested specimens comply with the "type 2"-specimen of this DIN standard. The distance between the jaws is  $s = 150 \text{ mm}$  and the test velocity is  $v = 2 \text{ mm/min}$ .

### 3. RESULTS

The examination of the sample sheets resulted in the measured values shown in table 3. They are the basis for a further calculation of laminate properties using CLT.

	thickness $t$ [mm]	FVC $\phi_{lam,mes}$ [%]	density $\rho_{lam}$ [g/cm <sup>3</sup> ]
UD	2,719	64,57	2,066
satin	2,570	53,46	1,733
twill	2,197	59,08	1,854
plain	2,315	62,95	1,956

Table 3: Measured properties of the specimen [6]

Contrary to the expectations, the measurements indicate that the undulation has a positive effect on the young's modulus  $E_x$  (fig. 3). The values of woven fabric's  $E_x$  increase by 21 to 36% compared to  $E_x$  of the UD fabric. In contrast, there are only slight differences between

the woven fabrics  $E_x$ . The laminate made with the twill weave has the highest  $E_x$ , followed by the satin and the plain weave. It is assumed that the higher stiffness of the woven fabric laminates benefits from a stronger support between the fibers. UD fabrics need a certain strain in order to transmit forces to all fibers because they are not perfectly straight but always wavy. Thereby, a reduced young's modulus may result. In comparison to the young's modulus, a tight weaving leads to a reduced strength. Hence, the laminate with the plain weave fails first, followed by the twain and satin weave. The UD fabric has the highest strength. The higher compressive stresses between the fibers might be a reason for the lower strength of the woven fabric. These stresses lead to damaged fibers which consequences in failure.

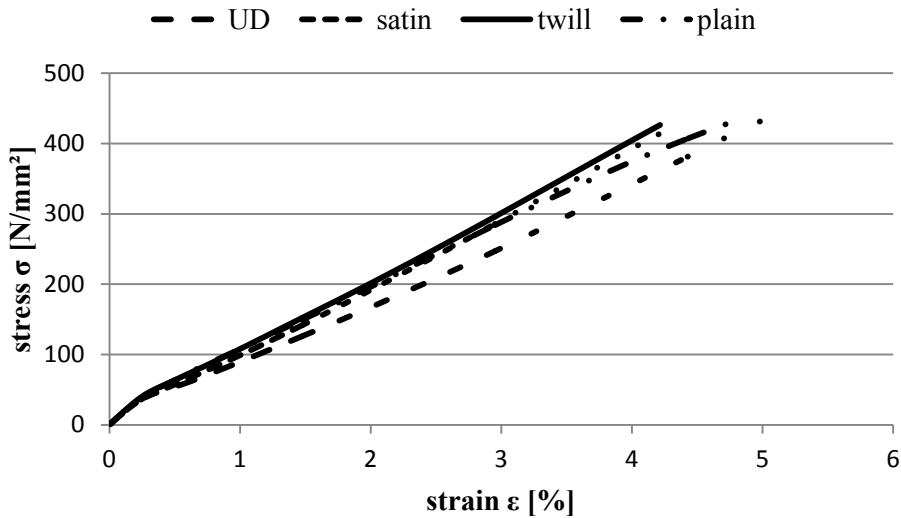


Figure 3: Stress-strain plot of RTM components from different fabric

With the recorded measurement curves the following young's moduli  $E_{x,mes}$  were determined for the tested laminates (fig 4. – in comparison to the calculated  $E_{x,cal}$ ).

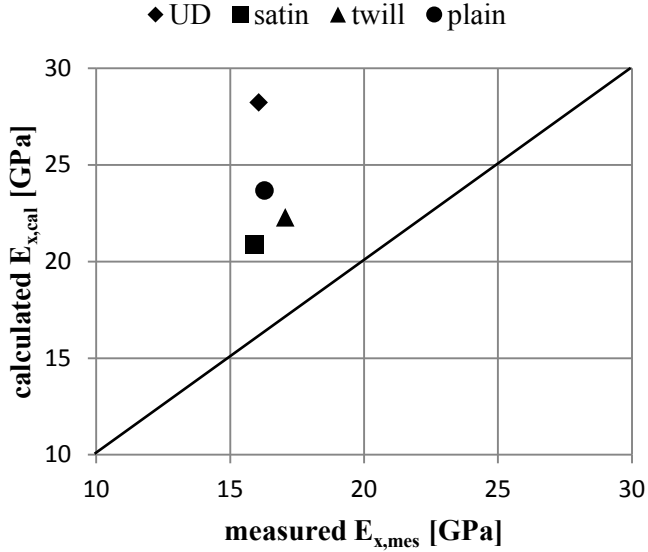


Figure 4: Measured and calculated values of  $E_x$

The results of calculation and measurement lead to a factor  $\chi_1$  (tab. 4). Subsequently, the value of  $\chi_2$  in the calculation of  $E_x$  is introduced and varied until the measured values

correspond to the calculated values for the laminate. The starting value for  $\chi_2$  is fixed at 1 and will be increased or decreased accordingly. The result of definition of the two parameters is shown in Table 4 and leads to a predictability of undulation on the mechanical properties calculated on this basis with the CLT and the derived correction parameters.

	$E_{x,cal}$ [GPa]	$E_{x,mes}$ [GPa]	factor $\chi_1$	undulation factor $\chi_2$
UD	28,22	16,06	0,5693	1,000
satin	20,87	15,91		1,3572
twill	22,28	17,07		1,3617
plain	23,67	16,29		1,2180

Table 4: Determined values of  $\chi_1$  and  $\chi_2$  [6]

#### 4. DISCUSSION AND CONCLUSION

It is shown that the undulation has an impact on stiffness of the laminate due to the fact that the fibers both support and impact on each other. Depending on the characteristics of the undulation, deviating patterns are identified. In summary, it is demonstrated that achievable component properties are a function of the undulation of the fibers. Corrective factors are applied to increase the accuracy of a pre-calculation of the mechanical performance, thus to increase the reliability of design and further support a weight reduction.

It is recommended that further work continues to deal with the influence of undulation on stiffness properties. Although tensile tests showed that the plain weave is highly impacted and damaged, followed by twill, satin und UD fabric, it was only analyzed by visual means because the used testing machine could not prove a failure of the specimen. Further studies will examine to what extent the identified reduction factors of the stiffness are applicable on other types of fibers and weights per unit area that will allow to more generically derive the correction factors for undulation into the CLT.

#### REFERENCES

- [1] H. Schürmann, *Konstruieren mit Faser-Kunststoff-Verbunden*, Springer, Berlin, 2005.
- [2] A. Puck, *Festigkeitsanalyse von Faser-Matrix-Laminaten: Modelle für die Praxis*, Hanser, München, 1996.
- [3] W. Michaeli and M. Wegener, *Einführung in die Technologie der Faserverbundwerkstoffe*, Hanser, München, 1989.
- [4] A. Rudolph and M. Groß, *R&G Handbuch*, Edition 06/2009, Waldenbuch, 2009.
- [5] F. T. Peirce, *The geometry of cloth structure*, *Journal of the Textile Institute Transactions*, Volume 28, Issue 3, 1937.
- [6] M. Barth, *Einfluss der Faser-Ondulation auf die Festigkeitseigenschaften bei Faserverbundbauteilen*, bachelor thesis, TU Ilmenau, 2013.
- [7] DIN EN ISO 527-4, *Kunststoffe – Bestimmung der Zugeigenschaften – Teil 4: Prüfbedingungen für isotrop und anisotrop faserverstärkte Kunststoffverbundwerkstoffe*, Deutsches Institut für Normung, 1997.

#### CONTACTS

Dipl.-Ing. Christian Fiebig  
Prof. Dr.-Ing. Michael Koch

[christian.fiebig@tu-ilmenau.de](mailto:christian.fiebig@tu-ilmenau.de)  
[michael.koch@tu-ilmenau.de](mailto:michael.koch@tu-ilmenau.de)