A DESCRIPTION OF THE DYNAMICS OF A FOUR-WHEEL MECANUM MOBILE SYSTEM AS A BASIS FOR A PLATFORM CONCEPT FOR SPECIAL PURPOSE VEHICLES FOR DISABLED PERSONS

Abdelrahman, M.[1]; Zeidis, I.[2]; Bondarev, O.[2]; Adamov, B.[3]; Becker, F.[2]; Zimmermann, K.[2]

1 Automotive Engineering Department - Faculty of Engineering - Mattaria - Helwan University - Cairo - Egypt
2 Technical Mechanics Group - Department of Mechanical Engineering - Technische Universität Ilmenau – Germany
3 Department Theoretical Mechanics and Mechatronics - Moscow Power Engineering Institute (Technical University) - Moscow – Russia

ABSTRACT

A four wheeled Mecanum vehicle has been analyzed kinematically and dynamically in this paper, that to understand and to determine the influence of the vehicle parameters and model them analytically. The principle of the nonholonomic mechanics has been used to introduce an exact dynamic model. The major focus in the work, presented in this paper, is on the 4WD 100mm Mecanum Wheel robot kit 10011 - Nexus Robot. Also, a set of simple and complex motion trajectories have been presented. The simulation results for the analytical models of the four wheeled Mecanum vehicle are presented and discussed briefly. A future view on the modern research trend lines in the field of omnidirectional mobile robots, in the next few years, is presented at the end of the paper.

Index Terms – Mecanum wheel, Kinematics; Dynamics; Modeling; Locomotion system

1. INTRODUCTION

Nowadays the demand on the transportation systems for the handicapped increases rapidly. The transportation systems developers have assigned their capabilities and researches for developing new mobile systems to facilitate the life of the disabled persons. However, the varieties in the products of the handicapped in the local and international markets, the handicapped suffer from difficulties in the maneuverability and overcoming the obstacles of the used products.

During the last decades, different configurations have been introduced and one from the successful configuration was the Mecanum wheel [1][2]. The Mecanum wheel is considered as an omnidirectional wheel, where the external circumference of the wheel consists of a set of rollers. The rollers are distributed equally on the circumference and inclined by angle 45° to the main rotation axis of the wheel. In general, each roller can rotate freely around its own rotation axis and simultaneously around the main rotation axis of the wheel which is connected to the driving motor. The driving force is transported from the motor to a hub and carrying plates and reaches the contact surface through the contact point between the rollers and the surface.

The Mecanum wheel is considered as a special case from the omnidirectional wheels, which can be used in different applications, see Fig. 1. The presented system in this paper is used mainly as a base for wheelchair for the disabled persons.

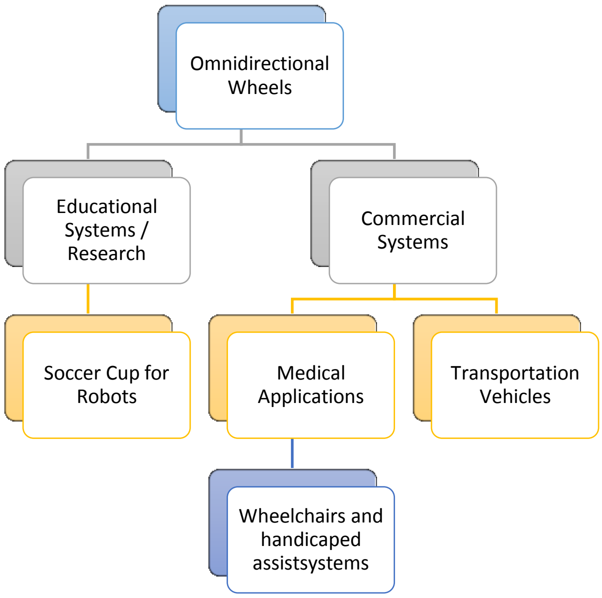

Fig. 1 Hierarchy for Omnidirectional wheels Applications

The disabled persons can use such system in the transportation in all directions within the living places. The presented system is promising and solves many problems, which are facing the handicapped during the transporting. One from the disadvantages of the four wheeled Mecanum vehicle (4WMV), the effect of shifting the center of mass on the resulted motion trajectory, which will be discussed in this paper. Also, an initial idea for solution for such problem is presented here.

2. KINEMATICS OF THE 4WMV

For the four-wheel Mecanum vehicle, it is assumed that the vehicle rotate about its center of mass C by angle ψ. The width of the vehicle is $2l$ and the distance between the vehicle center of mass C to the center of mass of the front axle O_1 is the distance ρ_1 and the distance between C and the center of mass of the rear axle O_2 is ρ_2, see Fig. 2. $\vec{e}_1, \vec{e}_2, \vec{e}_3$ are the unit vectors of the inertial coordinate system in the direction of the main x-, y- and z-axes in the motion diagram. $\vec{E}_1, \vec{E}_2, \vec{E}_3$ are the unit vectors of the body-fixed coordinates in the directions of the main x-, y- and z-axes but relative to the moving body. These unity vectors are used to describe the movement of the vehicle center and the rotation centers of the wheels in their respective side views. $\underset{i}{\vec{E}}_{1`}$, $\underset{i}{\vec{E}}_{2`}$ and $\underset{i}{\vec{E}}_{3`}$ are unit vectors, located in the center of the rollers, where i represents the wheel number from 1 to 4. The unit vectors in this case are inclined by an angle of 45° or -45° corresponding to the type of wheel. The unit vectors point in the direction of the main x-, y-and z-axes.

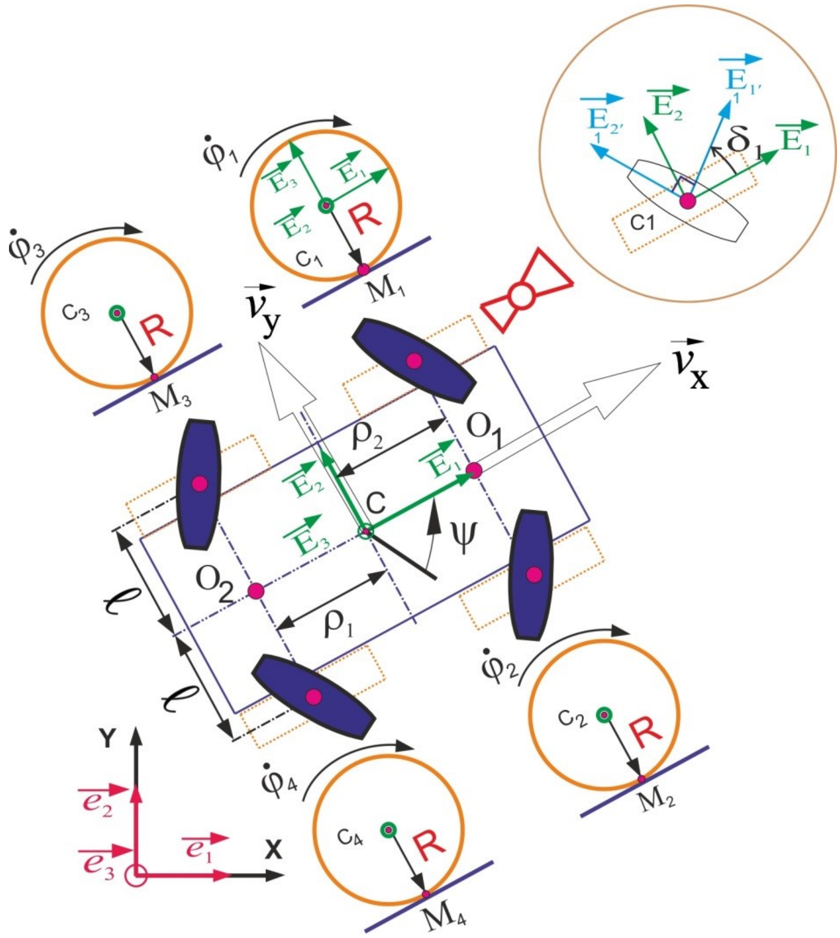

Fig. 2 Kinematic analysis of the 4WMV

This study does not take into account any slip between the rollers of the Mecanum wheels and the surface during motion. This means that all applied moments from the electric motors will be converted into traction force between the wheels and the surface.

Based on EULER's equation, it follows that

$$\dot{\vec{r}}_{M_k} = \dot{\vec{r}}_{C_k} + \vec{\omega}_k \times \overrightarrow{C_k M_k} \,, \tag{1}$$

where

k number of the wheel 1,2,3,4 ,
$\dot{\vec{r}}_{M_k}$ velocity of the roller contact point (M) for wheel k ,
$\dot{\vec{r}}_{C_k}$ velocity for the wheel center of rotation (C) for wheel k ,
$\vec{\omega}_k$ angular velocity of the wheel k ,
$\overrightarrow{C_k M_k}$ position vector from the center of the rotation (C) to the roller contact point (M) for wheel k.

The kinematic constraints for the rollers contact points are the following [7][8]

$$\begin{gathered} -(\dot{x}_C \cos\psi + \dot{y}_C \sin\psi) + (-\dot{x}_C \sin\psi + \dot{y}_C \cos\psi) \\ + l\dot{\psi} + \rho_2\dot{\psi} + R\dot{\varphi}_1 = 0 \,, \end{gathered} \tag{2}$$

$$(\dot{x}_C \cos\psi + \dot{y}_C \sin\psi) + (-\dot{x}_C \sin\psi + \dot{y}_C \cos\psi) + \dot{\psi}(l + \rho_2) - R\dot{\varphi}_2 = 0\,, \tag{3}$$

$$(\dot{x}_C \cos\psi + \dot{y}_C \sin\psi) + (-\dot{x}_C \sin\psi + \dot{y}_C \cos\psi) - \dot{\psi}(l + \rho_1) - R\dot{\varphi}_3 = 0\,, \tag{4}$$

$$-(\dot{x}_C \cos\psi + \dot{y}_C \sin\psi) + (-\dot{x}_C \sin\psi + \dot{y}_C \cos\psi) - (l + \rho_1)\dot{\psi} + R\dot{\varphi}_4 = 0\,. \tag{5}$$

By assuming that $v_x = \dot{x}_C \cos\psi + \dot{y}_C \sin\psi$ and $v_y = -\dot{x}_C \sin\psi + \dot{y}_C \cos\psi$, results the following form of the kinematic constraints for the wheels according to their order respectively.

$$v_x - v_y - (l + \rho_2)\,\dot{\psi} = R\dot{\varphi}_1\,, \tag{6}$$
$$v_x + v_y + \dot{\psi}(l + \rho_2) = R\dot{\varphi}_2\,, \tag{7}$$
$$v_x + v_y - \dot{\psi}(l + \rho_1) = R\dot{\varphi}_3\,, \tag{8}$$
$$v_x - v_y + (l + \rho_1)\,\dot{\psi} = R\dot{\varphi}_4\,. \tag{9}$$

The previous kinematic constraints are nonholonomic constraints which describes the kinematics of the discussed system during the motion on the plane without slippage or losses in the kinetic energy. They cannot be integrated with respect to the time [5].

3. DYNAMICS OF THE 4WMV

The LAGRANGE equation with multipliers are used here for introducing a dynamic model for the 4WMV. The LAGRANGE equation with multipliers states that

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}^a}\right) - \frac{\partial T}{\partial q^a} = Q_a + \lambda_b f_a^b \; (a = 1,2,\dots,n; b = 1,2,\dots,r)\,, \tag{10}$$

where
T is the general kinetic energy of the system and $T = T_{vehicle} + T_{wheels}$,
q^a is the generalized coordinate ,
$\dot{q}^a$ is the first time derivative of the generalized coordinates (velocity) ,
n is the number of generalized coordinates, in this case $n = 7$ since $\underbrace{q^1 = x_C\,,\; q^2 = y_C,\; q^3 = \psi}_{vehicle}$ and $\underbrace{q^4 = \varphi_1,\; q^5 = \varphi_2,\; q^6 = \varphi_3,\; q^7 = \varphi_4}_{wheels}$,
r is the number of nonholonomic kinematic constraints of the system, here $r = 4$,
Q_a are the generalized forces (in this case driving moments) ,
λ_b are the multipliers in the LAGRANGE equation ,
f_a^b represents the coefficients of the coordinates for the motion constraints, in this case $a = 1,\dots,7$ and $b = 1,\dots,4$.

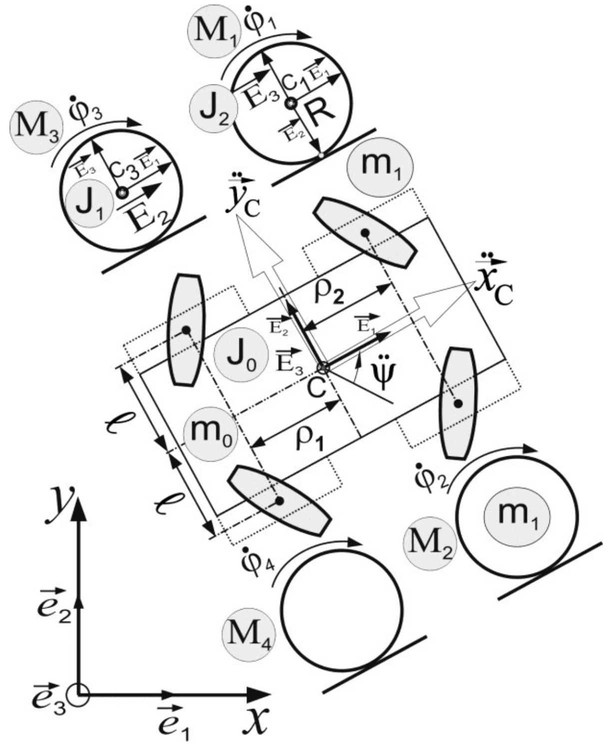

Fig. 3 Diagram for the four-wheel Mecanum vehicle – dynamic analysis

Fig. 3 shows the vehicle parameters, which has been taken in consideration while studying the dynamics of the 4WMV. The position of the center of mass has been shifted to be near to the front or the rear axles of the vehicle and the resulted trajectory has been presented. The following second order differential equations represent the introduced dynamic model in the explicit form.

$$
\begin{aligned}
\ddot{x}_C = &- \frac{2m_1R^2}{A}(\rho_1-\rho_2)\cos\psi\,\dot{\psi}^2 + \frac{2J_1m_0\alpha}{J\cdot A}(\rho_1-\rho_2)^2\sin 2\psi\,\dot{x}_C\dot{\psi} \\
&+ \frac{4J_1}{A}\left[\frac{m_0\alpha}{J}(\rho_1-\rho_2)^2\sin^2\psi - 1\right]\dot{y}_C\dot{\psi} \\
&- \frac{2\alpha}{J\cdot R}(\rho_1-\rho_2)[l(M_2-M_1+M_4-M_3)+\rho_2(M_2-M_1) \\
&+ \rho_1(M_4-M_3) - 2\alpha(\rho_1-\rho_2)(M_1-M_2-M_3+M_4)]\sin\psi \\
&+ \frac{R}{A}[(M_1+M_2+M_3+M_4)\cos\psi \\
&+ (M_1-M_2-M_3+M_4)\sin\psi]\,,
\end{aligned}
\qquad (11)
$$

$$
\begin{aligned}
\ddot{y}_C = {} & \frac{-2m_1R^2}{A}(\rho_1-\rho_2)\sin\psi\,\dot{\psi}^2 \\
& -\frac{4J_1}{A}\left[\frac{m_0\alpha}{J}(\rho_1-\rho_2)^2\cos^2\psi-1\right]\dot{x}_C\dot{\psi} \\
& -\frac{2J_1m_0\alpha}{J\cdot A}(\rho_1-\rho_2)^2\sin 2\psi\,\dot{y}_C\dot{\psi} \\
& +\frac{2\alpha}{J\cdot R}(\rho_1-\rho_2)[l(M_2-M_1+M_4-M_3)+\rho_2(M_2-M_1) \\
& +\rho_1(M_4-M_3)-2\alpha(\rho_1-\rho_2)(M_1-M_2-M_3+M_4)]\cos\psi \\
& +\frac{R}{A}[(M_1+M_2+M_3+M_4)\sin\psi-(M_1-M_2-M_3+M_4)\cos\psi]\,,
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
\ddot{\psi} = {} & -\frac{2J_1m_0}{J\cdot A}(\rho_1-\rho_2)(\dot{x}_C\cos\psi+\dot{y}_C\sin\psi)\dot{\psi} \\
& +\frac{1}{J\cdot R}[l(M_2-M_1+M_4-M_3)+\rho_2(M_2-M_1) \\
& +\rho_1(M_4-M_3)-2\alpha(\rho_1-\rho_2)(M_1-M_2-M_3+M_4)]\,,
\end{aligned} \tag{13}
$$

$A = mR^2 + 4J_1$, $B = m_1R^2 + J_1$ and $\alpha = \frac{B}{A}$. Also, the mass moments of inertia is defined as following, where $J = J_0 + 4J_2 + 2m_1(\rho_1^2 + \rho_2^2 + 2l^2) + \frac{4J_1}{R^2}((l+\rho_1)^2 + (l+\rho_2)^2) - \frac{4B}{R^2}\alpha(\rho_1-\rho_2)^2$, $J_0 = \frac{m_0}{12}(4l^2 + (\rho_1+\rho_2)^2)$, $J_1 = \frac{m_1R^2}{2}$ and $J_2 = \frac{m_1R^2}{4}$.

The previous second order differential equations have been numerically integrated using MATLAB®. Three different scenarios have been assumed for the position of the center of mass for the 4WMV during the motion

1. Center of mass is identical with the geometrical center of the vehicle ($\rho_1 = \rho_2$)
2. Center of mass is shifted to near from the frontal axle of the vehicle ($\rho_1 > \rho_2$)
3. Center of mass is shifted to near from the rear axle of the vehicle ($\rho_1 < \rho_2$)

For more understanding for the vehicle behavior during the motion, a virtual reality model has been developed using the Simulink 3D animation toolbox. The structure of the SIMULINK® model for the virtual reality environment has been used to realize the motion of the vehicle. A virtual model for the 4WMV has been built and the trajectory for the motion of the mass center will be plotted during the simulation, see Fig. 4.

Fig. 4 The 3D virtual simulation environment using Simulink® and Simulink® 3D animation toolbox

In the virtual reality model, the change in the position coordinates of the center of mass has presented live during the simulation to track the change in the positon coordinates and compare them by the change progress for other simulated cases.

4. SIMULATION RESULTS

By applying the numerical integration methods in MATLAB®, the resulted motion trajectories have been plotted and the change of the rotation angle ψ along the simulation time has been demonstrated. The motion of the center of mass $(x_C\ , y_C)$ has been represented for different cases of the driving moments M_1 , M_2 , M_3 and M_4. The assumed simulation time is limited to 2 seconds. Different driving scenarios have been assumed to investigate the behavior of the four wheeled Mencanum vehicle in the life time of the handicapped persons.

4.1 Simple motion trajectories

The simple trajectories are covering all the principle essential motion directions such as forward, backward, left and right and even the translational diagonal motion such as diagonal forward left, diagonal forward right and vice versa. As this study aims to declare the effect of shifting the center of mass on the resulted motion trajectory, the effected trajectories is presented in this section. The effect of shifting the center of mass leads to deviation in the transitional and rotational motion of the vehicle such as during the translation to the left as shown in Fig. 5.

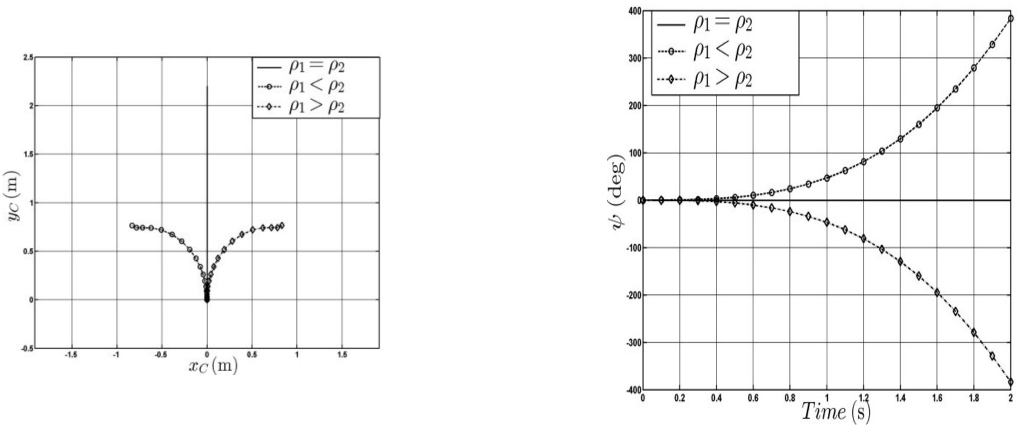

Fig. 5 The effect of shifting the center of mass on the simple trajectories - left

In case the center of mass is aligned to the geometrical center of the 4WMV, then the vehicle will translate to left without rotation. Otherwise, the deviation appears in the resulted trajectory and is combined by continuous rotation relative to the z-axis by angle ψ. The deviation and the continuous rotation are resulted from the unbalanced force components between the contact rollers of the wheels and the motion plane. The center of mass presents translation in the direction of the effect of the resultant force and the moment of inertia causes the rotation of the center of the mass. The same effect appears during the diagonal motion as presented in Fig. 6

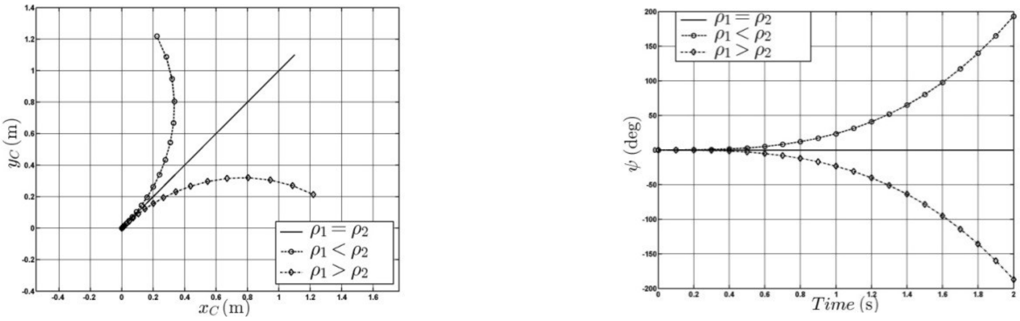

Fig. 6 The effect of shifting the center of mass on the simple trajectories – forward –left diagonal motion

The resulted effect appears in the most of the possible motion scenarios. The question now, is this effect extends to cover also the motion in complex trajectories.

4.2 Complex motion trajectories

It has been found, that the complex trajectories differ completely following the resultant force and the moment of the inertia as well. New effect has been appeared in the case of the complex trajectories, which is the continuous change of the center of the rotation for the whole vehicle. The variable position of the center of rotation is resulted from shifting the center of the mass and the whole position of the vehicle continuously with the time.

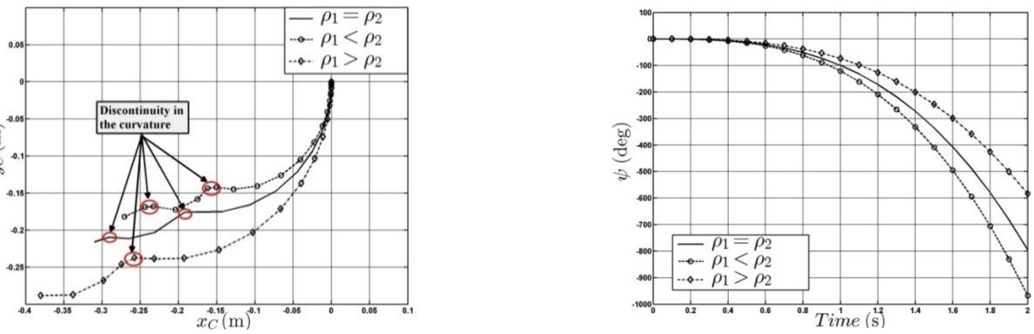

Fig. 7 The effect of shifting the center of mass on the complex trajectories – lateral arc motion

In Fig. 7-left, the center of mass has showed discontinuity of the curved motion. That is resulted from the continuous change in the center of the rotation. The deviation in the whole trajectory is in the direction of the resultant force, which is resulted from the unbalanced distribution for the vehicle weight. The combination between the translation and the rotation during the motion is still the dominate phenomena here.

5. NEW TREND LINES FOR THE HANDICAPPED LOCOMOTION SYSTEMS

In the future, new concepts for the handicapped vehicles will be developed. As a part from this study, new concepts based on the Mecanum wheels are presented. Deploying of the combination between the tracked chained vehicle and the steering concept has been proposed.

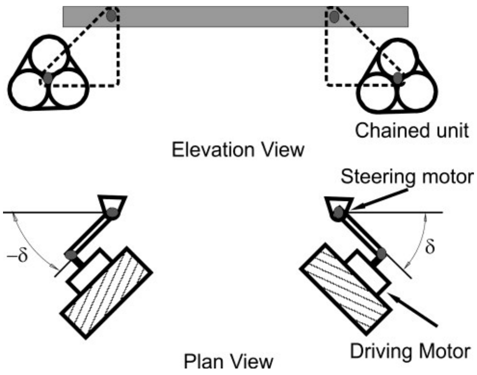

Fig. 8 The Chained –Mecanum wheeled vehicle

New concept combine between the benefits of the tracked chained wheel units and the separate severability of each unit has been introduced in Fig. 8. Also, a new mechanical design for adapting the inclination angle of the wheels rollers for the Mecanum wheel has been proposed like in Fig. 9. (See also [6])

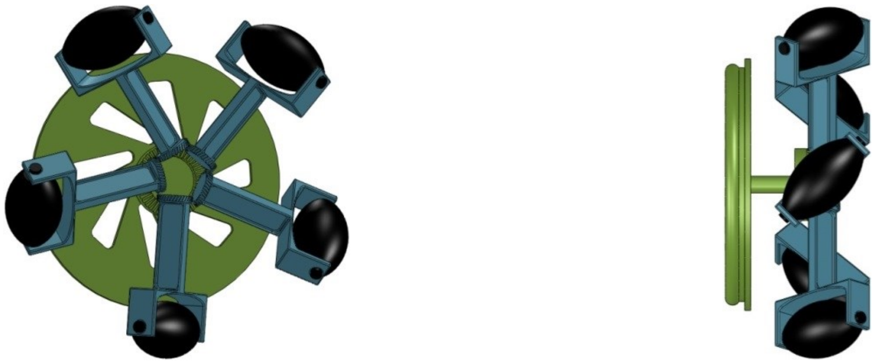

Fig. 9 Mecanum wheel with rotating rollers

The proposed mechanical design has the advantages of both the Omnidirectional wheel and the steering in one unit but suffers from the insufficient stability during the translation, where the change of the steering angle can lead to missing the contact between the wheel and the motion plan, which can in the worst case leads to the turn over.

6. CONCLUSION

In this study, the four wheeled Mecanum vehicle, which is subclass from the omnidirectional wheels, has been modelled from the kinematic and dynamic point of views. The dynamic model consists of three second order differential equations transferred into the explicit form and has been numerically integrated to estimate the translation and the rotation of the vehicle center of mass. The effect of shifting the center of mass, due to the unbalanced distributions of the vehicle weight on the driving axles, has been presented. New suggestions for new mechanical designs for the locomotion systems of the handicapped have been discussed, to initialize new motivation for more progress and development in this field.

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CONTACTS

Prof. Dr.-Ing. habil. K. Zimmermann	klaus.zimmermann@tu-ilmenau.de
M. Eng. M. Abdelrahman	mohamed.abdelrahman@tu-ilmenau.de
Dipl.-Ing. F. Becker	felix.becker@tu-ilmenau.de