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DYNAMICAL BEHAVIOR OF WINDOW REGULATOR SYSTEMS

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ABSTRACT

Modern electrical power window regulator for vehicle door is made of electrical drive, mechanism, electronic hardware and controlling software. Due to the increasing system complexity, method is being searched to analyze system behavior and influencing factors. In this article, a mechanical model is introduced to simplify window regulator systems. In the model, glass mass and moment of inertia of electrical drive armature are essential components, because they possess the most kinetic energy of system during moving. To build up model, moment of inertia is firstly converted into effective mass. With it, mechanical model, with two degrees of freedom, is formed in the way that tow masses are connected by spring and damper. The driving force in model has a constant component and a periodical component and the direction of friction force between door and glass depends only on the velocity of glass. Both driving force and friction force are assumed to be small in comparison with elastic force. The system is investigated by means of method of averaging. In the end, numerical calculation is presented and the outcomes from different combination of driving force are discussed.

Index Terms - Window regulator, method of averaging

1. INTRODUCTION

Window regulator is the mechanism in vehicle doors to lift up and pull down window glass. Its design evolves from metallic lever system in the early age to nowadays cable driving system, shown in Fig. 1 (1 - Electrical drive and electronics, 2 - Cable drum and housing (covered), 3 - Compensation spring, 4 - Bowden cable and cable, 5 - Pulley, 6 - Guiding rail, 7 - Slider (Glass carrier)). In comparison with manual window lifting system, electrical window regulators provide more comfort, with function, for example auto-up or so-called one-touch closing. However, it draws attention in aspect of safety. It leads to the invention of anti-trap function. The robust realization of anti-trap function in each single trapping event relies on the cooperation of mechanical components, electrical drive, electronic hardware, software and algorithms. Technically, modern development methods are under searching [3]. The external pressure is higher requirements, for instance, light weighted design (less CO_2 emission), higher durability, better acoustic performance and child finger protection for 4mm open glass. Internally, the development cycle is getting shorter and shorter. Under such circumstance, the earlier a design failure is found, the less cost and effort would be involved. For all these reasons, deeper understandings of window regulator systems are required.

In this paper, we studied the dynamic behaviour of window regulator system during transition from start-up to stationary state. A better understanding of effecting factors to such oscillation process benefits, from one side, the improvement of mechanical components and electrical drives and, from another side, the advancement of anti-trap function.

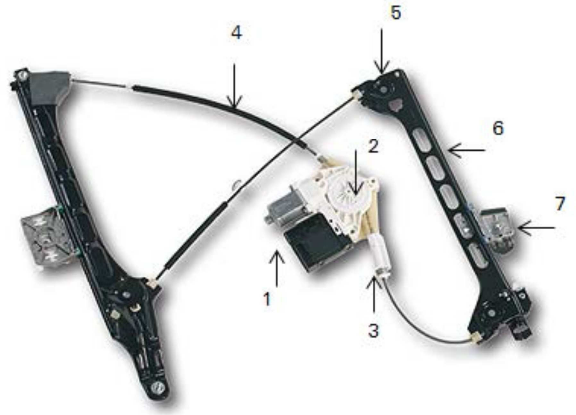

Figure 1: Window regulator

2. MECHANICAL MODEL

First of all, window regulator system is transformed into a lumped model. Window regulator transfers the lifting power from electrical drive to glass and at the same time transforms rotational movement into translational movement. If slack is ignored, a spring and a damper are introduced to represent its mechanical structure. The spring rate is denoted with c and the damping factor is denoted with μ. For the convenience, all movement is analyzed in form of translational movement. Therefore, The masses, M_1 and M_2, stand each for the equivalent mass of armature inertia in electrical drive and the mass of window glass. Only two masses are taken into consideration, because the most kinetic energy is distributed in these two components during the motion of glass, seen in Tab. 1.

	Speed	Mass/Inertia	Kinetic Energy
Glass	0.12 m/s	3 kg	0.022 J
Drum	5.4 rad/s	4.9e-6 kg·m^2	0.073e-03 J
Gear	5.4 rad/s	12e-6 kg·m^2	0.186e-03 J
Armature	398 rad/s	9.9e-6 kg·m^2	0.7848 J

Table 1: kinetic energy of components

According to energy conservation low, moment of inertia of electrical motor armature can be converted to effective mass with Eq. (1), where J_1 is moment of inertia of armature, i is the worm gear ratio, η is the worm gear efficiency and r is the gear wheel radius. Verification can be performed by comparing the kinetic energy of obtained mass at speed of glass and the production of kinetic energy of armature with consideration of gear efficiency.

$$M_1 = \frac{\eta i^2 J_1}{r^2}. \tag{1}$$

With simplified mechanism and converted inertia, the mechanical model of window regulator system is represented in Fig. 2. In the figure, x_1^* and x_2^* correspond each the position of masses M_1 and M_2. The external driving force F_a applies to the mass M_1, simulating driving torque of electrical drive, while the value F_r represents Coulomb's dry friction force, which is depending on the velocity of mass M_2.

The force equilibriums at masses M_1 and M_2 give the two following differential equations,

$$M_1\,\ddot{x}_1^* + c(x_1^* - x_2^*) + \mu(\dot{x}_1^* - \dot{x}_2^*) = F_a, \tag{2}$$

$$M_2\,\ddot{x}_2^* + c(x_2^* - x_1^*) + \mu(\dot{x}_2^* - \dot{x}_1^*) = F_r. \tag{3}$$

When F_a and F_r take difference forms, their combination can imitate various working situation of window regulator systems. We assume that the friction and the excitation forces are small in comparison to the spring force. In the following sections, the model is analyzed with method of averaging.

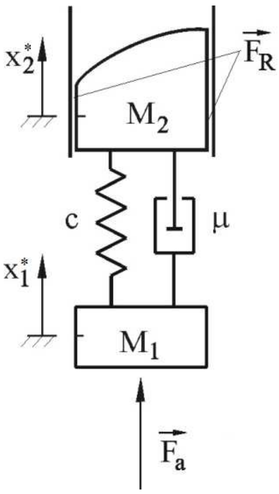

Figure 2: Model of window regulator

3. THEORETICAL INVESTIGATION – METHOD OF AVERAGING

The Coulomb's dry friction at M_2 takes form as $-F_r sgn(\dot{x}_2^*)$, where F_r is constant. The external driving force, applied to mass M_1, contains two parts. One part is constant force, F_{a0}. The other part is described by a harmonic function on time t^* with angular frequency ω and constant amplitude of F_a [1]. With this assumption, system equations are

$$M_1\,\ddot{x}_1^* + c(x_1^* - x_2^*) + \mu(\dot{x}_1^* - \dot{x}_2^*) = F_{a0} + F_a sin(\omega t^*), \tag{1}$$

$$M_2\,\ddot{x}_2^* + c(x_2^* - x_1^*) + \mu(\dot{x}_2^* - \dot{x}_1^*) = -F_r sgn(\dot{x}_2^*). \tag{2}$$

For further investigation, we introduce that system with dimension variables (denoted by asterisk) above is converted into system with dimensionless variables below:

$$t = t^*\omega_0, \quad x_i = \frac{x_i^*}{L}, i = 1,2, \quad \dot{x}_i = \frac{\dot{x}_i^*}{L\cdot\omega_0}, i = 1,2, \quad \ddot{x}_i = \frac{\ddot{x}_i^*}{L\cdot\omega_0^2}, i = 1,2,$$

$$\omega_0^2 = c\frac{M_2 + M_1}{2M_2M_1}, \quad \nu = \frac{\omega}{\omega_0}, \quad \varepsilon = \frac{F_r}{ML\omega_0^2}, \quad \alpha = \frac{\mu L\omega_0}{F_r\, m_1 m_2}, \quad \beta = \frac{F_a}{F_r}, \quad \gamma = \frac{F_{a0}}{F_r},$$

$$m_1 = \frac{M_1}{M}, \quad m_2 = \frac{M_2}{M}, \quad m_1 + m_2 = 1.$$

The value L is the scale of length and the value ω_0 is the scale of time, used for the conversion from dimensional system to dimensionless system. Substituting the dimensional variable in Eq. (4) - (5), we obtain the system in dimensionless variables

$$\ddot{x}_1 + 2m_2(x_1 - x_2) + m_2\varepsilon\alpha(\dot{x}_1 - \dot{x}_2) = \frac{\varepsilon}{m_1}\gamma + \frac{\varepsilon}{m_1}\beta sin(\nu t), \tag{3}$$

$$\ddot{x}_2 + 2m_1(x_2 - x_1) + m_1\varepsilon\alpha(\dot{x}_2 - \dot{x}_1) = -\frac{\varepsilon}{m_2} sgn(\dot{x}_2). \tag{4}$$

The method of averaging [2] is applied, so that the system is transformed into standard form. For this purpose, we rewrite the system in Eq. (6) - (7) as

$$m_2\ddot{x}_2 + m_1\ddot{x}_1 = \varepsilon\,[\gamma + \beta\, sin(\nu t) - sgn(\dot{x}_2)], \tag{5}$$

$$(\ddot{x}_2 - \ddot{x}_1) + 2(x_2 - x_1) = -\varepsilon\left[\frac{1}{m_2} sgn(\dot{x}_2) + \alpha(\dot{x}_2 - \dot{x}_1) + \frac{\varepsilon}{m_1}\gamma + \beta\frac{1}{m_1} sin(\nu t)\right]. \tag{6}$$

Introducing new variables, velocity of center of mass $V(t) = m_1\dot{x}_1 + m_2\dot{x}_2$ and relative displacement $z(t) = x_2 - x_1$. We obtain

$$\dot{V} = \varepsilon[\gamma + \beta\, sin(\nu t) - sgn(\dot{x}_2)], \tag{7}$$

$$\ddot{z} + 2z = -\varepsilon\left[\alpha\dot{z} + \frac{1}{m_2} sgn(\dot{x}_2) + \beta\frac{1}{m_1} sin(\nu t) + \gamma\frac{1}{m_1}\right]. \tag{8}$$

For unperturbed system, ν in (10) is equal to constant, $z(t) = x_2 - x_1$ changes harmonically with constant amplitude. According to it, we provide the general solution of Eq. (10) in the form: $z(t) = a(\mathrm{t})\cos(\varphi)$, where $\varphi = \sqrt{2}t + \theta$. Then,

$$\dot{\mathrm{z}} = -\mathrm{a}\sqrt{2}\sin(\varphi), \tag{9}$$

$$\dot{\mathrm{z}} = \dot{\mathrm{a}}\cos(\varphi) - \mathrm{a}\left(\sqrt{2} + \dot{\theta}\right)\sin(\varphi). \tag{10}$$

It yields

$$\dot{\theta} = \frac{\dot{a}\cos(\varphi)}{a\sin(\varphi)}, \qquad \dot{x}_2 = V - m_1 a\sqrt{2}\sin(\varphi). \tag{11}$$

Therefore, the system in Eq. (10) - (11) has form

$$\dot{V} = \varepsilon\left[\gamma + \beta sin(\nu t) - sgn\left(V - m_1 a\sqrt{2}\sin(\varphi)\right)\right], \tag{12}$$

$$\dot{a} = \varepsilon\frac{\sin(\varphi)}{\sqrt{2}}\left[-\alpha a\sqrt{2}\sin(\varphi) + \frac{1}{m_2} sgn\left(V - m_1 a\sqrt{2}\sin(\varphi)\right) + \beta\frac{1}{m_1} sin(\nu t) + \gamma\frac{1}{m_1}\right], \tag{13}$$

$$\dot{\varphi} = \dot{\theta} + \sqrt{2} = \varepsilon\frac{\cos(\varphi)}{\sqrt{2}a}\left[-\alpha a\sqrt{2}\sin(\varphi) + \frac{1}{m_2} sgn\left(V - m_1 a\sqrt{2}\sin(\varphi)\right) + \beta\frac{1}{m_1} sin(\nu t) + \gamma\frac{1}{m_1}\right] + \sqrt{2}. \tag{14}$$

In the case of $\beta = 0$, the periodic component of driving force is equal zero. The stable state of system can achieve when $\dot{V} = 0$, $\dot{a} = 0$ and $\dot{\varphi} = 0$. For $\dot{V} = 0$, we obtain

$$\gamma = sgn\left(V - m_1 a\sqrt{2}\sin(\varphi)\right). \tag{15}$$

It means that the constant component of driving force F_{a0} can be equal to either F_r or $-F_r$. The up-lifting process of window regulator is interested, therefore, $V > 0$ and $\gamma = 1$. $\dot{a} = 0$ and $\dot{\varphi} = 0$ can only be achieved, when $\varphi = n\pi, n \epsilon \boldsymbol{R}$. By simplifying Eq. (17) and applying $\dot{\varphi} = 0$, it gives

$$a = -\frac{\varepsilon}{2}(\frac{1}{m_1} + \frac{1}{m_2}). \tag{16}$$

Substitute ε, it yields

$$a = -\frac{F_r}{cL}. \tag{17}$$

By multiplying L, it gives the dimensional stationary amplitude of a.

In the case of $\beta \neq 0$, the system behavior is investigated in vicinity of the main resonance frequency: $\psi = (\sqrt{2} + \varepsilon\Delta)t, \nu = \sqrt{2} + \varepsilon\Delta$. After introducing new slow variable, $\xi = \psi - \varphi, \dot{\xi} = -\dot{\theta} + \varepsilon\Delta$, the system has standard form:

$$\dot{V} = \varepsilon\left[\gamma + \beta sin(\nu t) - sgn\left(V - m_1 a\sqrt{2}\sin(\varphi)\right)\right], \tag{18}$$

$$\dot{a} = \varepsilon\frac{\sin(\varphi)}{\sqrt{2}}\left[-\alpha a\sqrt{2}\sin(\varphi) + \frac{1}{m_2}sgn\left(V - m_1 a\sqrt{2}\sin(\varphi)\right) + \beta\frac{1}{m_1}sin(\nu t) + \gamma\frac{1}{m_1}\right], \tag{19}$$

$$\dot{\xi} = -\varepsilon\frac{\cos(\varphi)}{\sqrt{2}a}\left[-\alpha a\sqrt{2}\sin(\varphi) + \frac{1}{m_2}sgn\left(V - m_1 a\sqrt{2}\sin(\varphi)\right) + \beta\frac{1}{m_1}sin(\nu t) + \gamma\frac{1}{m_1}\right] + \varepsilon\Delta. \tag{20}$$

We assume that $\varepsilon \ll 1$. Then in the system above, the variables, V, a and ξ, are slow variables. According to the method of averaging, the procedure $\langle \cdots \rangle = \int_0^{2\pi}(\cdots)d\varphi$ is applied to the system of Eq. (21) - (23).

$$\dot{V} = \begin{cases} \varepsilon(\gamma + 1), & V < -m_1 a\sqrt{2}, \\ \varepsilon\left[\gamma - \frac{2}{\pi}\arcsin\left(\frac{V}{m_1 a\sqrt{2}}\right)\right], & |V| \leq m_1 a\sqrt{2}, \\ \varepsilon(\gamma - 1), & V > m_1 a\sqrt{2}, \end{cases} \tag{21}$$

$$\dot{a} = \begin{cases} -\varepsilon\frac{1}{\sqrt{2}}\left(\alpha a\sqrt{2}\frac{1}{2} - \beta\frac{1}{m_1}\frac{1}{2}cos\xi\right), & V < -m_1 a\sqrt{2}, \\ -\varepsilon\frac{1}{\sqrt{2}}\left[\alpha a\sqrt{2}\frac{1}{2} + \frac{1}{m_2}\frac{2}{\pi}\sqrt{1 - \frac{V^2}{2m_1^2 a^2}} - \beta\frac{1}{m_1}\frac{1}{2}cos\xi\right], & |V| \leq m_1 a\sqrt{2}, \\ -\varepsilon\frac{1}{\sqrt{2}}\left(\alpha a\sqrt{2}\frac{1}{2} - \beta\frac{1}{m_1}\frac{1}{2}cos\xi\right), & V > m_1 a\sqrt{2}, \end{cases} \tag{22}$$

$$\dot{\xi} = -\dot{\theta} + \varepsilon\Delta = -\varepsilon\frac{1}{\sqrt{2}a}\left(\beta\frac{1}{m_1}\frac{1}{2}sin\xi\right) + \varepsilon\Delta. \tag{23}$$

Where $cos\left[\arcsin\left(\frac{V}{m_1 a\sqrt{2}}\right)\right] = \sqrt{1 - \frac{V^2}{2m_1^2 a^2}}$.

The stationary motion of the system exists if $\dot{V} = 0$, $\dot{a} = 0$ and $\dot{\xi} = 0$. The value $\dot{V}$ can be equal to zero only if $|V| \leq m_1 a\sqrt{2}$. By eliminating $cos\xi$ and $sin\xi$ with $cos^2(\xi) + sin^2(\xi) = 1$, Eq. (25) - (26) give

$$2m_1^2 m_2^2 \pi^2(\alpha^2 + 4\Delta^2)a^2 + 8\sqrt{2}m_1^2 m_2 \pi\alpha a\cos(\frac{\pi\gamma}{2}) + \left(16m_1^2\cos^2(\frac{\pi\gamma}{2}) - m_2^2\pi^2\beta^2\right) = 0. \tag{24}$$

For the positivity of amplitude, the restriction, $\beta > \frac{4m_1}{\pi m_2}\cos(\frac{\pi\gamma}{2})$, should be implied. Under this condition, the stationary amplitude of a is

$$a = \frac{-4\sqrt{2}m_1\alpha\text{os}(\frac{\pi\gamma}{2}) + \sqrt{2\left[m_2^2\pi^2\beta^2(\alpha^2 + 4\Delta^2) - 64\Delta^2 m_1^2\cos^2(\frac{\pi\gamma}{2})\right]}}{2m_1 m_2 \pi(\alpha^2 + 4\Delta^2)}. \tag{25}$$

With known stationary amplitude of a, the stationary value of phase ξ can be calculated by either $\dot{a} = 0$ in Eq. (25) or $\dot{\xi} = 0$ in Eq. (26).

4. NUMERICAL CALCULATION

The typical values from real window regulator systems are $M_1 = 45\,kg$, $M_2 = 5\,kg$, $c = 2 \times 10^4\,\frac{N}{m}$, $\mu = 80\,\frac{N}{m/s}$, $F_r = 10\,N$. In the case of $\beta = 0$, F_{a0} must be equal to either $\mathrm{F_r}$, so as to a reach stable amplitude during up-lifting process of window regulator, according to the analysis previously. Thus, F_{a0} is equal to 10 N. Then, the dimensional stationary amplitude of a yields to be -0.5×10^3 m, by eliminating L in Eq. (20). It is verified by the numerical solutions of dimensional system in Eq. (4)-(5). Fig. 3 shows the solved V and a. The initial condition for the solution is $\mathrm{x}_2^*(0) = 0$ and $\mathrm{x}_1^*(0) = 0$.

In the case of $\beta \neq 0$, F_a and ω have each typical value of 200N and $75.8s^{-1}$. ω_0 is calculated to have value of $47.1s^{-1}$. With Coulomb's dry friction force $\mathrm{F_r}$ is 10N, we obtain $\beta = 20$. According to the restriction of Eq. 28, β must be greater than $\frac{4m_1}{\pi m_2}\cos(\frac{\pi\gamma}{2})$ and γ can only vary in range between -1 and 1, so that stationary amplitude of a exists. The maximal value of $\frac{4m_1}{\pi m_2}\cos(\frac{\pi\gamma}{2})$ is 11.5, when γ is 0. Thus, there exists stationary amplitude of a, with the system setup. According to [4], the scale of length L is equal to $\frac{\mathrm{F_a}}{\mathrm{M_1}\omega_0^2|2-\nu^2|}$. In this case, L has value of 3.3×10^{-3}m.

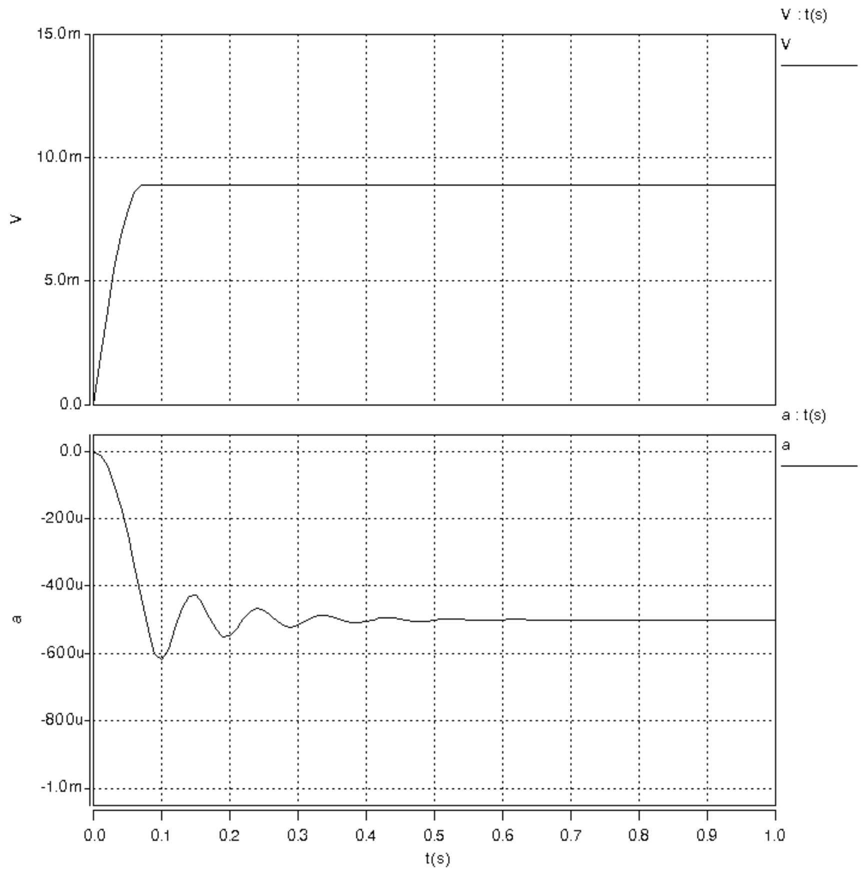

Fig. 3 Numerical solutions of the dimensional system in Eq. (4)-(5)

In one case, $F_{a0} = 0$. As a result, $\gamma = 0$. The left side of Fig. 4 shows the results, after solving the exact system in Eq. (15) – (17) and averaged system in Eq. (24) – (26) with initial condition, $V(0) = 0, a(0) = 0.01, \xi(0) = 0$. From the numerical solution of averaged system, stationary a is 0.45, which is identical to the calculation in Eq. (28). The dimensional value of a here is $1.5 \times 10^{-2} m$. In a more general case, $F_{a0} \neq 0$. As an example, F_{a0} is given to be 5. As a result, $\gamma = 0.5$. The right side of Fig. 4 shows the numerical solution, with the same initial condition. From the solution of averaged equation, stationary a is 0.62, which is identical to the calculation in Eq. (28). The dimensional value of a here is $2.1 \times 10^{-2} m$. Such system setup and combination of driving force and friction force simulates the system respond of window regulator under disturbance from electrical drive. The vibration in electrical drive, which is usually caused by the irregularity of its gear transmission, can lead into the unstable moving speed of window glass. The instability can be troublesome in realizing anti-trap function; in extreme case it can cause failure, such as false reversing of window regulator. Results of calculations on the basis of mathematical model will qualitatively be coordinated with experiment.

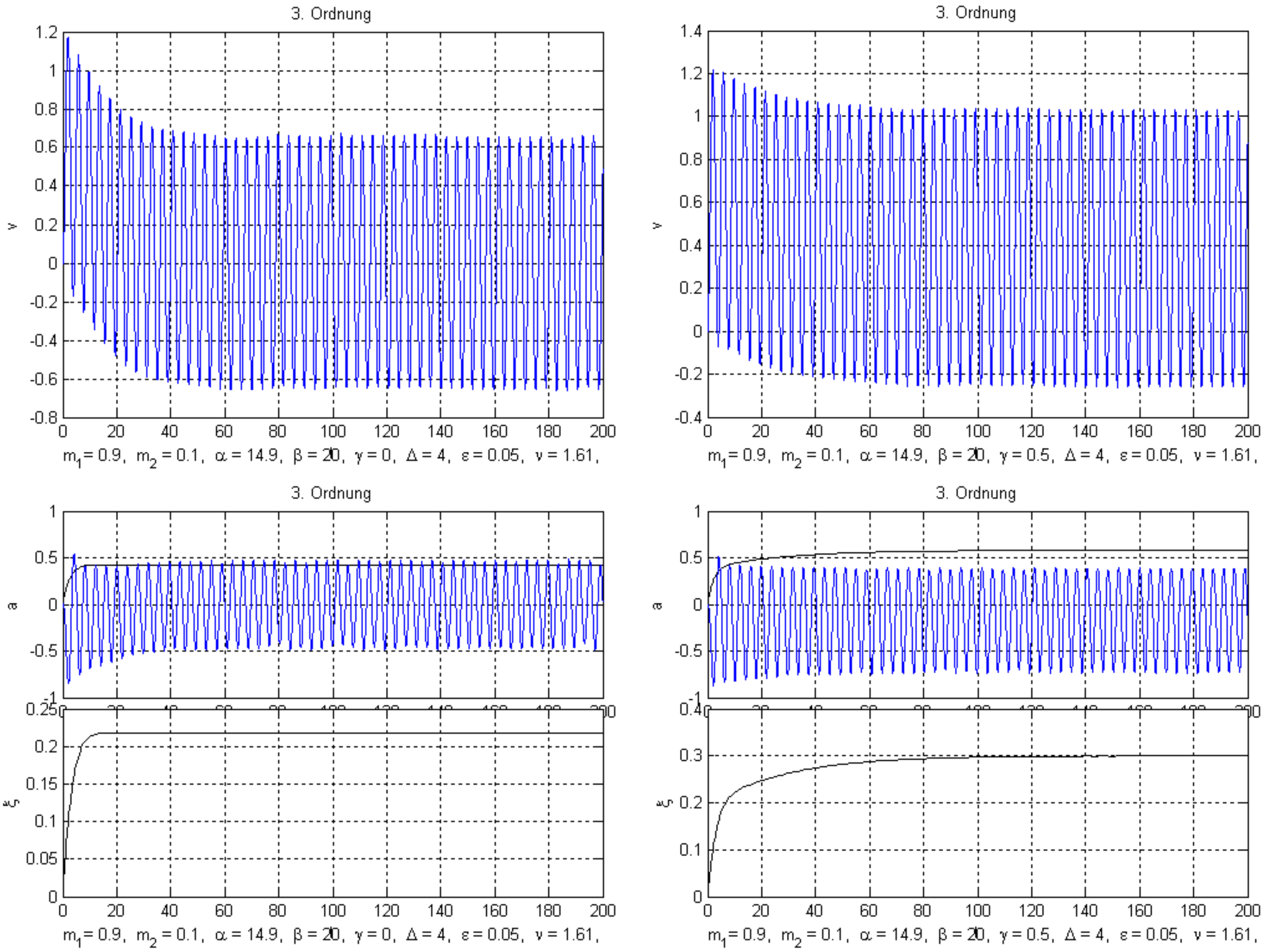

Fig. 4 Numerical solutions of the exact system (15)–(17) and the averaged system (24)-(26)

5. SUMMARY AND OUTLOOK

The paper presents a simple mathematical model of window regulator system. It is assumed that system is driven by an excitation, which has constant and periodic parts. The standard form of system is converted. Depending on driving force, system is analyzed respectively. The "averaged" system is deduced with applying the method of averaging to the exact system. Then the numerical solutions of system are computed and presented in graph. The stationary amplitude is compared between numerical solution of asymptotical approximation equation system and calculation from obtained analytical formula. The two values are equal to each other. In the end, the practical meanings are briefly discussed.

Although the model of driving force has its generality, the approach to analyze the dynamical behavior of window regulator systems can be advanced with more complicated model setup. For example, the mass 1 can be replaced by a model of electrical drive, so as to test the compatibility of electrical drive and window regulator. It is also interesting to find out, when a drive with smaller inertia is implemented, how window regulator system should be adjusted

REFERENCES

[1] Andronov, A.A., Vitt A.A., Khaikin S.E. Theory of oscillators. Dover Publications, 2011, p. 864

[2] Bogoljubow, N.N., Mitropolski, Yu. A. Asymptotic Methods in the Theory of Non-Linear Oscillations., Gordon and Breach, 1961, p. 537

[3] Petkun, S. Simulation added development on mechatronic products on the basis of car electrically power window regulator simulation. Proc. of 47. IWK, Ilmenau, Germany, 2002.

[4] Zimmermann, K., Zeidis, I., Bolotnik, M., Pivovarov, M. Dynamics of a two-module vibration-driven system moving along a rough horizontal plane. Multibody System Dynamics, 22, 2009, p.199-219.

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