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VECTORIAL TOLERANCES FOR THE UNCERTAINTY ANALYSIS OF PRECISION MEASUREMENT DEVICES

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ABSTRACT

Increasing demands on precision measurement devices require a detailed analysis of the existing measurement uncertainty, especially when the required measuring accuracy is in nanometre dimension. Such measuring devices consist of several sensors, drives, guidance, a corner mirror, a metrological frame and other precision elements. For the analysis of uncertainty budgets a vectorial metrological model can be used. Important influencing factors on the measurement uncertainty are the expected form, position and dimension tolerances. Therefore, an important part of the investigation in the uncertainty is a tolerance analysis. It seems obvious that it may be useful to feed an uncertainty analysis based on a vectorial metrological model with tolerances also based on a vectorial model. Thus, a considerable amount of information transfer between, for instance, conventional tolerance parameters and vectorial parameters as required by the uncertainty analysis could be avoided.

Index Terms – vectorial tolerances, CAD, metrological model

1. INTRODUCTION

Increasing demands on precision measurement devices require a detailed analysis of the existing measurement uncertainty, especially when the required measuring accuracy is in nanometre dimension. Such measuring devices consist of several sensors, drives, guidance, a corner mirror, a metrological frame and other precision elements. For the analysis of uncertainty budgets a vectorial metrological model can be used. The model describes the characteristics of the three measurement chains by closed vector chains for each measurement point on the surface of the object. By means of a modular model approach sub-models can be easily included in the metrological main model. Furthermore, cross-coupling effects arising between the measuring axes can be taken into account. These models provide a basis for the expression of uncertainty according to the Guide to the Expression of Uncertainty in Measurement (GUM) or by means of the Monte-Carlo-Method. Important influencing factors on the measurement uncertainty are the expected form, position and dimension tolerances. Therefore, an important part of the investigation in the uncertainty is a tolerance analysis. It seems obvious that it may be useful to feed an uncertainty analysis based on a vectorial metrological model with tolerances also based on a vectorial model. Thus, a considerable amount of information transfer between, for instance, conventional tolerance parameters and vectorial parameters as required by the uncertainty analysis could be avoided.

Currently, the permitted deviations of geometry design parameters as well as of positions and orientations of elements are described by tolerances in 2D-drawings and/or by adding semantic annotations to digital 3D-product-models [1]. Base are usually standards of tolerancing (e.g. by ISO, ASME). Since CAD-systems can only evaluate linear, one-dimensional tolerance chains, additional CAx-components (CAT – Computer-Aided Tolerancing systems, e.g. 3DCS, VSA, CETOL) are often necessary for advanced tolerance representation, analysis and synthesis (see Figure 1).

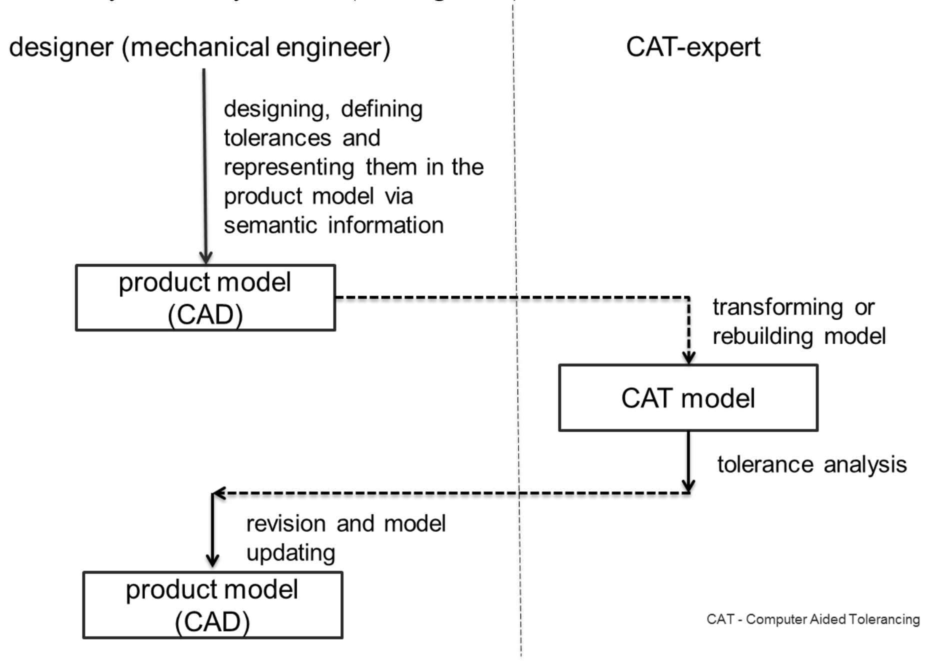


Figure 1: Alternation between CAD and CAT

By direct representation of mathematically evaluable tolerances in the CAD-model the analysis of the impact of deviations along the tolerance chain can be done directly in the CAD-system. The analysis of the tolerance chain can be done by support of standard methods in the CAD-system. In the paper the representation of the vectorial tolerances in the CAD-model is explained. A major motivation for the integration in the CAD-model is the similarity of the vectorial tolerance representation to the B-Rep (Boundary representation) description in current CAD-systems.

A main focus is the handling of partially closed tolerance loops. Most of the CAT-systems only handle open tolerance chains. In contrast a real system often consists of partially closed tolerance loops. Even if the total tolerance chain can be considered as an open chain many technical products have partially closed tolerance loops just in the joints.

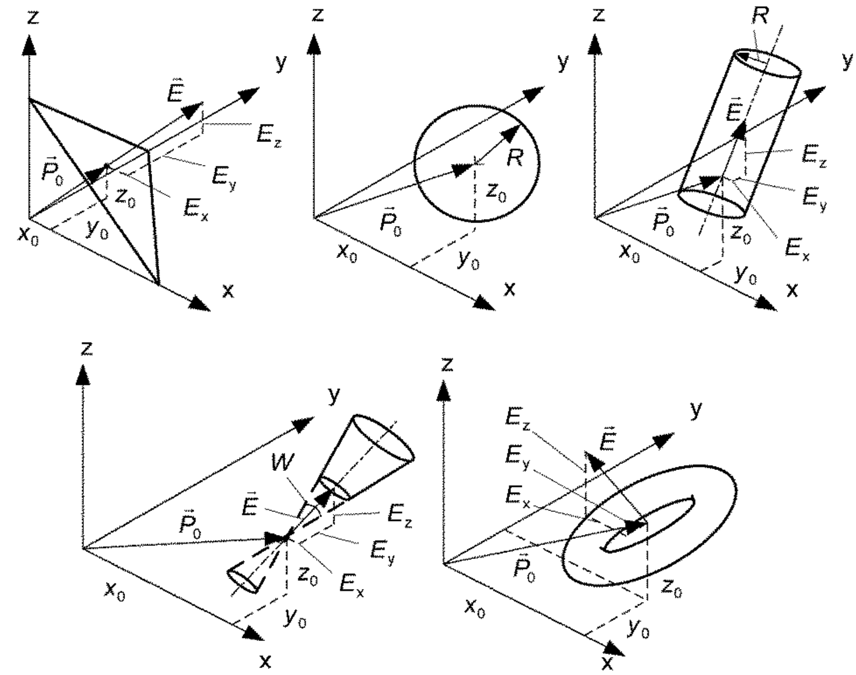
2. CONCEPT OF TOLERANCE REPRESENTATION

There exist several types of tolerance representations. The most known and standardized types are the ISO and ASME tolerances (e.g. ISO 1101:2012, ASME Y14.5M-2009), which divide tolerances into dimension, form and position tolerances. These are usually represented using semantic information in 2D-drawings or 3D-models. The tolerances as standardized by ISO or ASME have advantages for conventional manufacturing and conventional metrological inspection (e.g. two-point measurement and use of measuring gauges). However, they are not directly mathematically evaluable. Besides the standardized tolerances, science has investigated several different tolerance representations, often concentrated on proper mathematical evaluation:

- Tolerance Zones by Requicha (1983) [2]
- Vectorial tolerancing by Krimmel, Martinsen (1993) [3]

- Technologically and Topologically Related Surface (TTRS) model by Gaunet (1994) [4], [5], [6]
- Model of Geometric Face Tolerancing and Hybrid Model by Weber, Stark, Thome, Britten (1993) [7], [8]

Many concepts use vectorial tolerances (e.g. [9]). The main idea originally comes from coordinate measuring technology in the 1980s. First investigations using vectorial tolerances in 3D-CAD were done during the 90s. One difficulty of the investigation was that the CAD-systems were not ready for an implementation yet. In the meantime, the CAD technology is quite sophisticated, so a new and extended attempt of realization looks promising.
In contrast to standard tolerance specifications, vectorial tolerance representations address only the surfaces of components. Current investigations on tolerance representation and analysis focus on five standard surfaces (plane, cylinder, sphere, cone and torus – also see Figure 2); although due to their wide-spread application torus-type surfaces play a secondary role.



surface type	+/- tolerance position vector	+/- tolerance orientation vector	+/- tolerance size
plane	Δx_0 Δy_0 Δz_0	ΔE_x ΔE_y ΔE_z	
sphere	Δx_0 Δy_0 Δz_0		ΔR
cylinder	Δx_0 Δy_0 Δz_0	ΔE_x ΔE_y ΔE_z	ΔR
cone	Δx_0 Δy_0 Δz_0	ΔE_x ΔE_y ΔE_z	ΔW
torus	Δx_0 Δy_0 Δz_0	ΔE_x ΔE_y ΔE_z	ΔR_1 ΔR_2

Figure 2: Surface types and parameters for vectorial tolerances (according to [9] and [10])

A surface is described in the three-dimensional Euclidean space by a mathematical description for the geometry, a nominal position and orientation as well as the allowed deviations. So for each surface type up to two tolerance vectors exist: One for the position and

one for the orientation of the surface; only a sphere does not need an orientation tolerance vector. Some surface types need an additional size tolerance parameter (see Figure 2). The surfaces in the tolerance representation are boundless (except of sphere and torus, which are closed surfaces). The bounded faces of a solid body – its topology – are results of intersections of several boundless surfaces. Since the vectorial tolerance representation is almost similar to the B-Rep (boundary representation) description in current CAD-systems, it can be integrated directly into current parametric 3D-CAD-systems. The tolerance parameters can be represented by surface-type specific attribute containers, which are attached to the desired nominal surface at the part level.
The analysis of the tolerance chain can be done using standard geometry manipulation methods in the CAD-system itself, using the CAD-API (application programming interface). This means that for analysis and visualization the deviation-affected surfaces can be re-parameterized ("moved") by CAD-API methods within the limits defined by the tolerances. In the case of SolidWorks this necessary API-function is called "InsertMoveFace2". This function moves the desired face for specific translation and rotation parameters (see Figure 3).

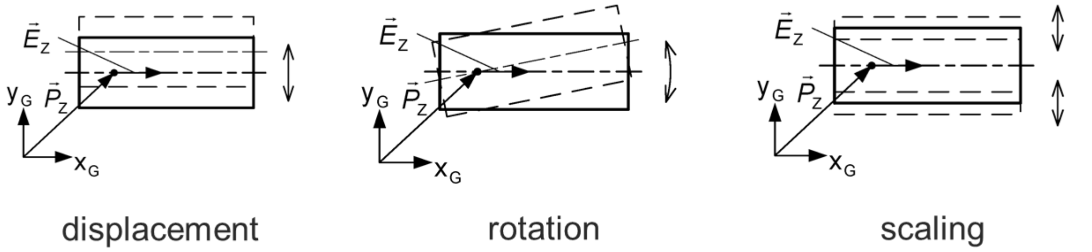


Figure 3: Necessary transformation of surfaces (pictures according to [9])

In order to ensure a consistent geometry, the CAD-system updates the B-Rep-models of the parts automatically as long as the topology remains unchanged. Usually, the deviations are very small. Therefore, for most parts the topology is maintained. Also the alignment of the parts can be updated automatically as long as the defined mates (e.g. "coincident") are still valid and the tolerance chain is open (details are explained in the following section). Consequently, it also becomes possible to analyse a tolerance chain across several components.

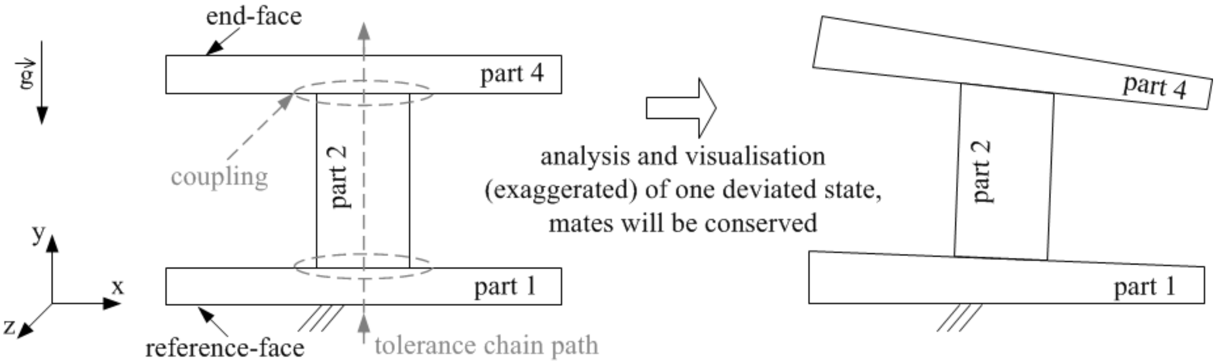


Figure 4: Realignment of deviation-affected parts for an open tolerance chain

Since all deviation-affected surfaces are described by displacements of ideal-geometric replacement elements, currently form tolerances cannot be represented using the concept of vectorial tolerancing. There exist several approaches in literature to handle form tolerances [11], which will be addressed in the further work.

3. PARTIALLY CLOSED TOLERANCE LOOPS

3.1 Motivation

For tolerance analysis of open tolerance chains commercially available CAT tools can already be used. In contrast, a real product often consists of partially closed tolerance loops. Even if the overall tolerance chain can be considered as an open chain, many technical products have partially closed tolerance loops. Closed tolerance loops exist either in cinematically closed chains (e.g. crank-rocker mechanisms) and in over determined systems (e.g. joints, base frames – see Figure 5) [12]. For the analysis of partially closed tolerance loops these CAT tools cannot be used. So in the following part a concept for handling partially closed tolerance loops is presented.

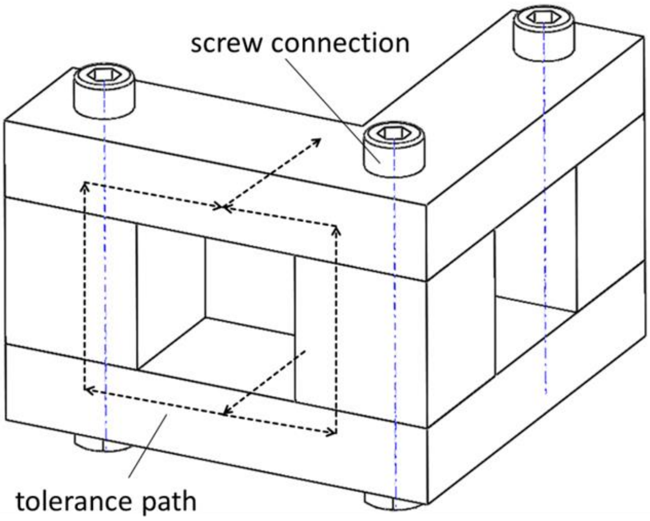


Figure 5: Simplified model of a metrological frame of a precision measuring machine with closed tolerance loops

3.2 Concept of analysis

Each tolerance chain is defined by a start- and an end-point. The mechanical engineer determines these points according to the functional chain he/she wants to analyse. The tolerance chain between these two points is – besides deviations of the individual parts – strongly determined by the couplings of the mated parts in the assembly. In the case, that all couplings are in series, the tolerance chain is considered as "open" (in analogy to the series connection of rigidities in engineering mechanics).

More formal, an open tolerance chain can be defined as:

- Open tolerance chains exist in technical products if each component is coupled with a neighbouring component via one mating surface pair only and, starting from an arbitrary component, this component cannot be reached via an alternative chain of couplings.

An open tolerance chain is exemplified in Figure 4. Start- and end-point of the tolerance chain are arbitrary points of the marked faces. It is clear that the orientations of the individual parts are mainly determined by the change in position of the coupling faces – their influence is much stronger than the influence of the position and orientation deviations of the faces of each individual part. The mates at the couplings can be retained.

In reality, most tolerance chains are not fully "open", but result from a concatenation of several open tolerance chains plus some partially closed tolerance loops. A partially closed tolerance loop exists when the tolerance chain path is branched at the couplings, i.e. two or more individual tolerance chains exist in parallel (analogy to the parallel connection of rigidities in engineering mechanics).

More formal, a partially closed tolerance loop can be defined as:

- Partially closed tolerance loops exist in technical products if components have two or more couplings to the same neighbouring component and, starting from an arbitrary component, this component may be reached by the chain of couplings again.

Such a partially closed tolerance loop is shown in Figure 5 and Figure 6.

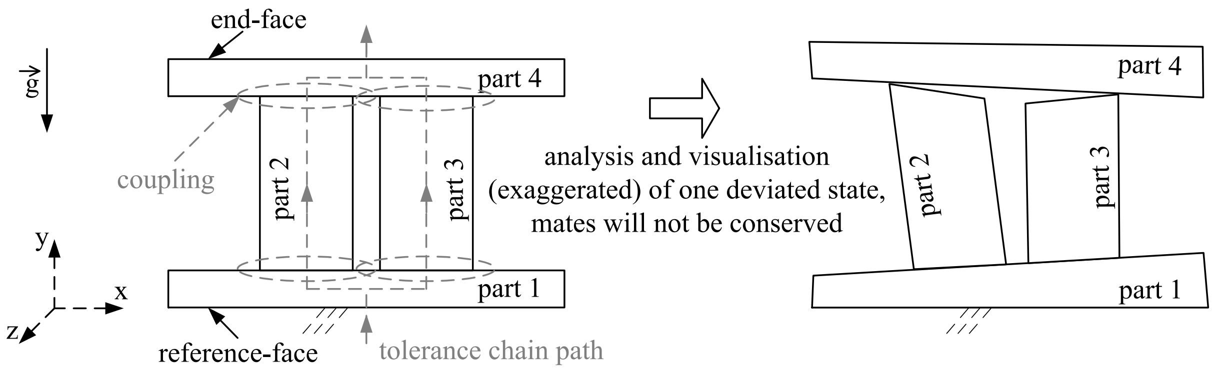


Figure 6: Orientation of the parts (updated geometry) in the case of a partially closed tolerance loop (here without consideration of forces and friction at the couplings)

For ease of illustration, in the example in Figure 6 only the coupling faces especially of part 2 and part 3 (coupling to part 4) are provided with position deviations (exaggerated presentation). The difference to the consideration of an open tolerance chain is that the relative orientations of part 4 to part 2 and to part 3 result from the interaction of both position changes of the coupling surfaces of part 2 and part 3.

The system according to Figure 6 is over determined. Therefore the mates at the couplings are no longer valid. Consequently, the orientation of part 4 must be re-determined based on the position-changes of the coupling faces from part 2 and part 3.

For further considerations some simplifications are introduced. For the investigation deviation-affected ideal rigid components, that have faces without form tolerances, are assumed. Moreover, no additional external forces (e.g. caused by screw connections) act on the individual components except gravity.

The basic idea of the concept presented here is based on the statically determinate placement of ideal rigid bodies on three points. These three points define a supporting triangle in such a way, that the toleranced components or assemblies (connection of components) re-align themselves.

In order to obtain a valid three-point pattern, in a first step the coupling faces are provided with deviations and then all possible patterns are deduced.

A large number of possible support triangles result from all these point-patterns. Now the best suitable support triangle for the re-alignment of the deviation-affected components has to be determined. An essential condition is to guarantee a safe stand of the components, which is fulfilled if the component's projected centre of gravity lies within the respective support triangle. In assemblies, the combined centre of gravity must be taken into account (see Figure 7, "intersection").

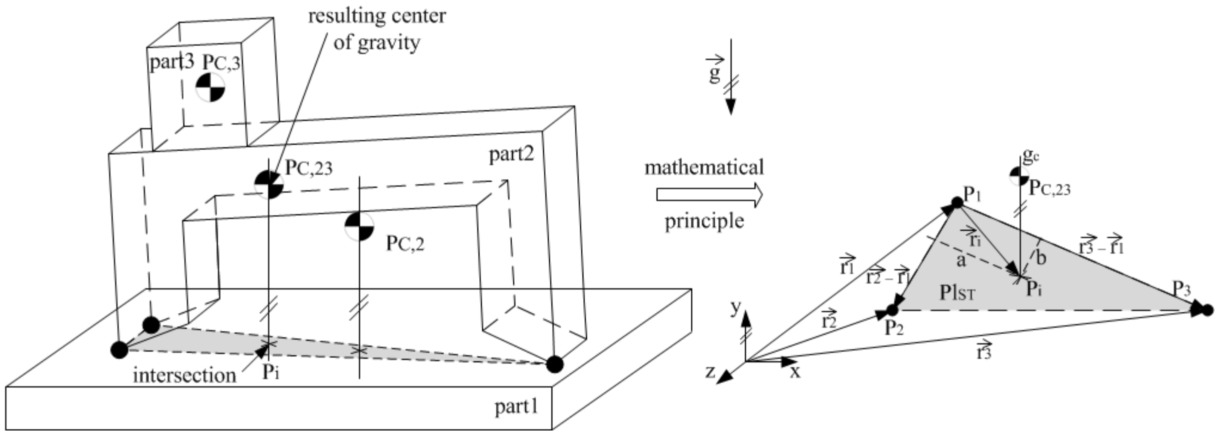


Figure 7: Orientation of a deviation-affected part on a support triangle in a partially closed tolerance loop [13]

For mathematical verification of this essential condition, a straight line, as a first step, is formed, which runs through the centre of gravity P_C (either only for one component or an assembly) in the direction of gravity. Furthermore, a plane Pl_{ST} is derived by the three points of the support triangle. Then, the intersection point P_i between the straight line and the plane is determined. The position of this intersection point can be described by the vector $\vec{r}_i$, which is characterized by the following equation (1):

$$\vec{r}_i = a \cdot (\vec{r}_3 - \vec{r}_1) + b \cdot (\vec{r}_2 - \vec{r}_1) \qquad (1)$$

The sum of the parameters a and b determines, whether the intersection is located inside the supporting triangle or outside. For this, the following mathematical correlation (2) is used:

$$a + b < 1 \qquad [a, b \in R] \qquad (2)$$

Only if the sum is less than one, the intersection point is located inside the supporting triangle. For this case, all possible support triangles are analysed. For further aptitude tests (e.g. volumetric intersections after re-alignment), only support triangles are considered where the intersection point lies within the support triangle and thus a safe stand for the deviation-affected components or assemblies is guaranteed.

At this point it is important to mention again, that this concept does not work on the original (geometrically ideal) component mates: Instead, new mates for the alignment of the deviation-affected parts in a partially closed tolerance loop are constituted by point-to-face, point-to-line or point-to-point contacts.

4. COMBINATION OF TECHNOLOGICAL TOLERANCES AND THERMALLY- AND LOAD-INDUCED DEFORMATIONS

For many applications, especially in the area of precision measurement, beside technological tolerances thermally- and load-induced deformations have to be considered. So the combination of technological tolerances with thermally- and load-induced deformations has to be analysed during the design process.

Therefore the technological tolerances have to be assigned by the engineer in the 3D-CAD-system. The deviations can be determined by the above explained method. Then the

resulting deviated faces between the limits given by the technological tolerances (e.g. for worst-case-scenarios) can be visualised in the 3D-CAD-system.
For the determination of the thermally- and load-induced deformations a FE-analysis is necessary, in which the deformed faces are simulated. Then these deformed faces are combined with the deviations from the technological tolerances and this is brought together in the 3D-CAD-system (see Figure 8 and Figure 9), so that the engineer can also evaluate the effect of the combined deviations of the CAD-model.

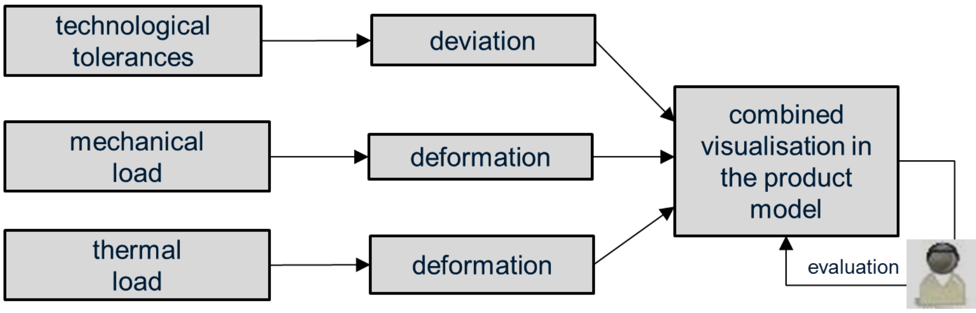


Figure 8: Process flow for a combined visualisation of deviations and deformations

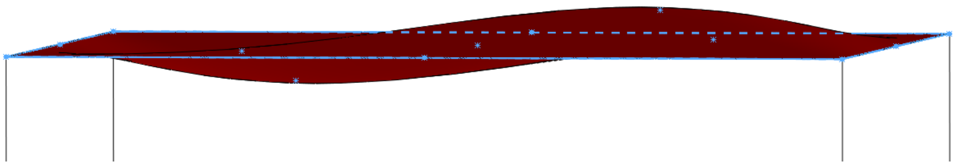


Figure 9: Superposition of deviations and thermally- and load-induced deformations

5. IMPLEMENTATION

The explained concept was implemented as a prototype in the CAD-system SolidWorks using the CAD-API. In a first step the CAD-model is scanned. During this process surface-type specific attribute containers are generated for all surfaces (unless this has been performed earlier). These containers can be visualised to the CAD user, if required. The engineer can define dimension tolerances as usual directly together with the respective dimension definitions. For the definition of position tolerances a special user interface exists as explained in section 2.3. The datum reference(s) also can be defined using this interface.
Through user interaction the tolerance analysis can be started. During this analysis the deviation-affected surfaces are moved (translation and/or rotation) according to the tolerance vectors, using CAD-API functions (see Figure 10). The result of the analysis can be seen directly for discrete deviations on the CAD-model.

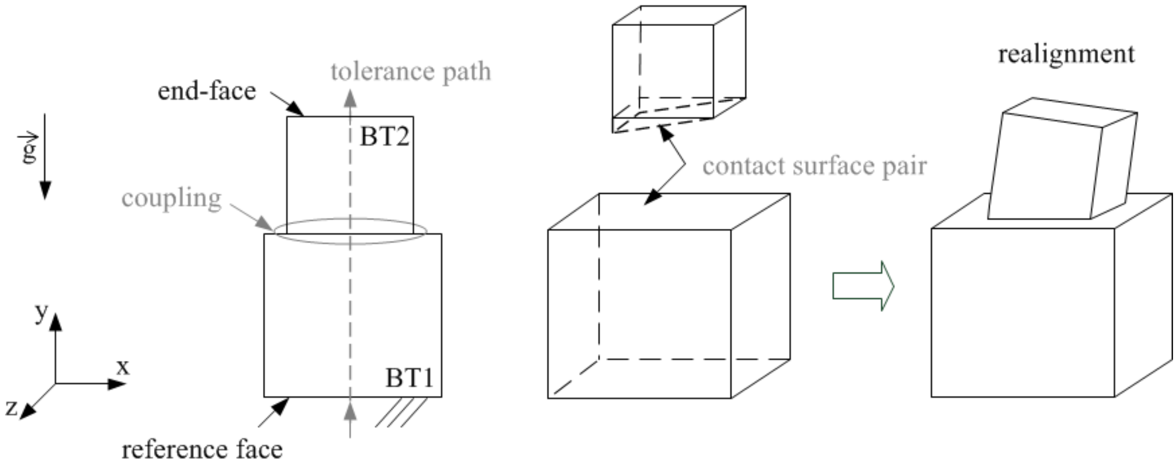


Figure 10: Vectorial tolerance analysis and visualisation [14]

6. EXAMPLE APPLICATION

Tolerances and tolerance chains exist in each geometrical model of a technical product. One focus of the current investigations lies on systems in the area of precision measuring and positioning. For such systems an error analysis has to be performed during the design process in order to minimise the measurement uncertainty. Important influencing factors are the expected form, position and dimension tolerances. Although precision machines as a whole are cinematically well constrained (i.e. no cinematic over-determination) there exist several closed tolerance loops inside the machines. One example is the metrological frame of the Nanopositioning and Nanomeasuring Machine (see Figure 11) [15]. This frame connects the measuring tip with the three-dimensional mechanism moving the probe below the (non-moving) tip. Therewith the tolerance chain of the frame influences the measuring accuracy. The tolerance chain of the metrological frame was modelled using the software tool described in this paper. Without the possibility to calculate partially closed tolerance loops the CAD-model has to be modified, so that all closed tolerance loops have to be opened. Considering partially closed tolerance loops and deleting the standard mates in the CAD-model a more realistic tolerance chain and its consequences for the measurement uncertainty of the machine can be calculated.

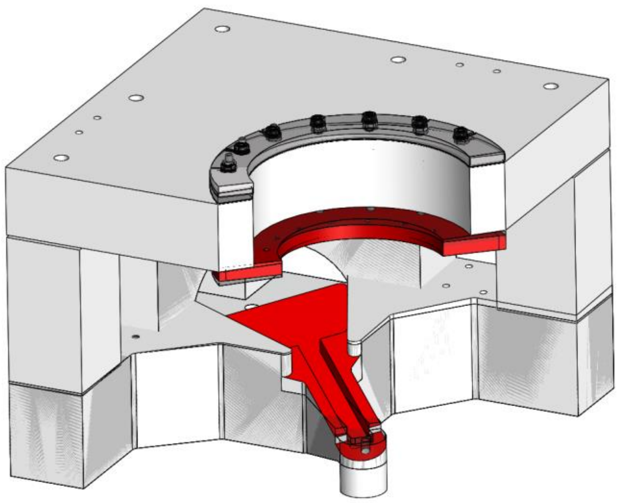


Figure 11: Metrological frame (model simplified) with start- and end-tolerance face (in red: deviated surfaces at the end of the relevant tolerance chain)

7. SUMMARY AND OUTLOOK

In this paper a method and a tool for precision measurement applications are presented, which enable tolerance representation and analysis directly in the CAD-system using vectorial tolerances. A major motivation for the integration in the CAD-model is the similarity of the vectorial tolerance representation to the B-Rep (Boundary representation) description in current CAD-systems. The vectorial tolerance model can be integrated in an overall vectorial metrological model for an analysis of the uncertainty budget. In the ongoing research this integration should be done.
For the investigations on partially closed tolerance loops as presented in this paper a number of simplifications were made. In the ongoing research these points will also be addressed, i.e. the simplifications will be dropped. The research will focus on the impact of additional external forces and moments as well as consideration of form tolerances.

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