

## SIGNAL INTERPOLATION METHOD FOR QUADRATURE PHASE-SHIFTED FABRY-PEROT INTERFEROMETER

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### ABSTRACT

Quadrature phase-shifted Fabry-Perot interferometer (QFPI) is a kind of Fabry-Perot interferometer (FPI) which is applicable to displacement measurement. QFPI differs from the conventional FPI. It has an orthogonal interference signal. So the quantity and direction of the measured displacement can be obtained simultaneously.

In this research, the analysis and testing of QFPI have been demonstrated. And the signal processing and optimal parameters of different situation are investigated in this paper. By this signal processing and analysis, the interpolation error of QFPI can be reducing by 85%, comparing with conventional signal processing.

**Keywords** - Quadrature phase-shift, Fabry-Perot interferometer, interpolation method, subfrequency

### 1. INTRODUCTION

Extrinsic FPI for length measurements had been investigated for several years, especially for displacement and vibration measurements. Because of its high resolution and stable optical structure, FPI had been widely utilized in those measurement tasks.

In the earlier researches of FPI, its applications are only in quasi-static measurements, e.g. strain and thermal expansion measurements. For the conventional FPI, the direction cannot be determined instantly. Therefore, an additional aid for directional discrimination is necessary. For this reason, the conventional FPI which is using the fringe counting method (FCM) would not be able to determine the movement direction of the object [1, 2]. Figure 1 and 2 are the structure and interferometric signal of FCM FPI. This disadvantage is also one of the reasons that conventional FPI is hard to be utilized in dynamic length measurement.

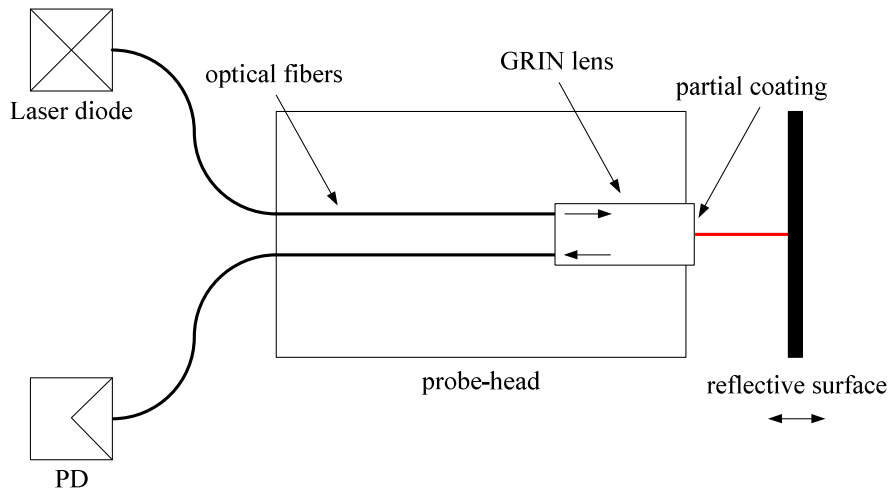


Fig. 1. FCM fiber FPI [2]

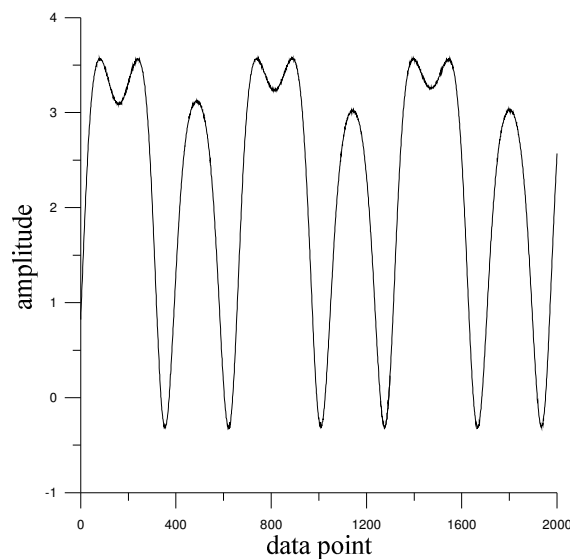


Fig. 2. Signal of FCM FPI

However some the novel investigations of FPI yield that quantity and direction of the measured displacement can be simultaneously acquired by QFPI [3, 4]. Therefore, FPI could be utilized for dynamic measurements.

In this research, some of the phase shift methods and the signal processing for QFPI have been investigated. Also some experimental results are shown as following. And the suggestions for QFPI design are proposed.

## 2. THEORY AND PRINCIPLE

### 2.1 Conventional Fabry-Perot interferometer

Fabry-Perot interferometer shown in Fig. 3 has common path structure where the measured displacement is precisely defined by the optical cavity, independent of an external reference arm of interferometer and not involved with a beam splitter in the optical path [5]. For this reason, the environmental effect will be obviously reduced. This is the key point that FPI is more insensitive to the environmental disturbances.

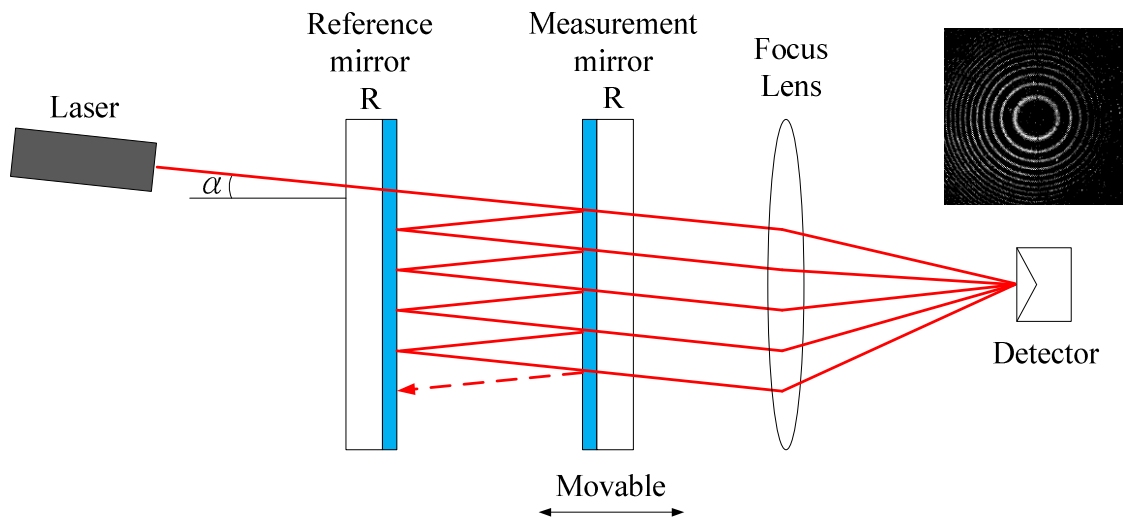


Fig. 3. Fabry-Perot interferometer

## 2.2 Quadrature phase-shifted Fabry-Perot interferometer

Quadrature phase-shifted Fabry-Perot interferometer is a kind of FPI which has the orthogonal signal pattern. Orthogonal signal is the most common way for the signal processing in industry application. Almost every signal processing of high speed commercial displacement measurement devices are based on this signal pattern. For this reason, QFPI have highly potential of high speed positioning in nanometer scale.

### 2.2.1 Spacial QFPI

Spacial QFPI is a kind of QFPI whose phase-shift is based on spacial arrangement of the sensor heads or detectors [3, 4, 6]. Fig. 4 and 5 are the basic arrangement of spacial QFPI. According to this arrangement, the orthogonal signal can be easily obtained. It is a benefit for the signal processing. Therefore, FPI has a chance to use to the displacement measurement in the industrial fields.

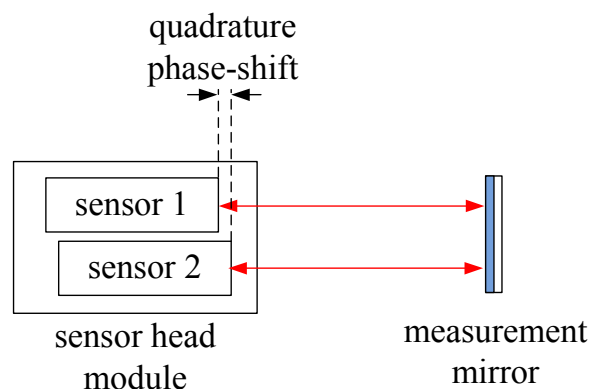


Fig. 4. Quadrature phase-shifted Fabry-Perot interferometer designed by K. A. Murphy [3, 4]

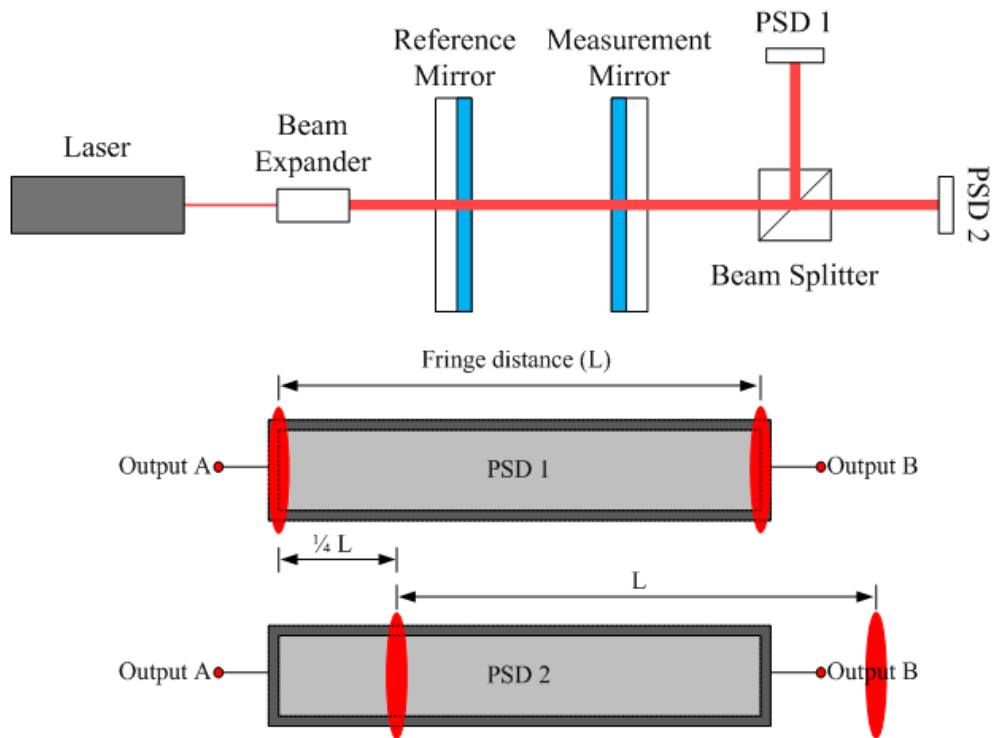


Fig. 5. Previous research of spacial QFPI [6]

$$I = \frac{A_0^2 \cdot T^2}{1 + R^2 - 2 \cdot R \cdot \cos(\delta)} \quad (\text{Eq. 1})$$

But there have some drawbacks of spacial QFPI which is about the phase-shift maintaining. In the design of QFPI, any tilt angle will change the arrangement for quartered phase-shift. This kind of phase-shift method is hard to against to the tilt angle of the measurement mirror.

### 2.2.2 Polarized QFPI

For the polarized QFPI, the optical structure is schemed as Fig. 6. The intensity equation is shown in Eq. 2 and 3. The optical design and theory are completely expressed in the reference [7]. This investigation demonstrates the method to obtain the orthogonal signal of QFPI by the one-eighth waveplate which will be favorable to simplify the signal processing.

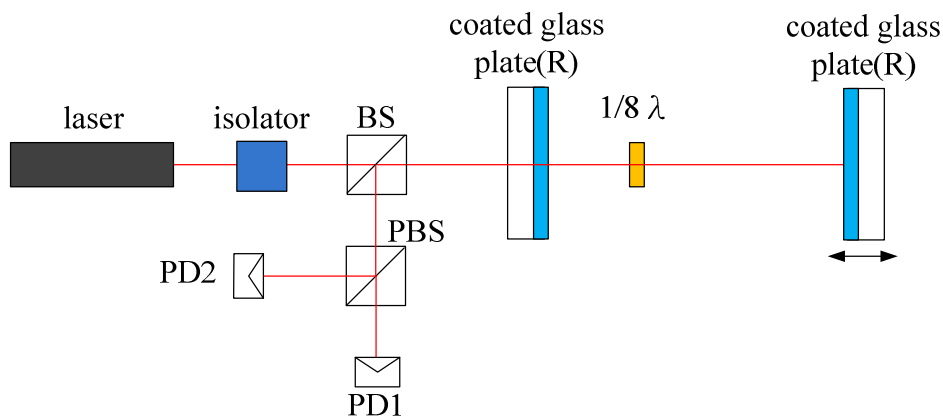


Fig. 6. Previous research of polarized QFPI [7]

$$I_s = \frac{1}{8} A_0^2 \times \left[ \frac{R + R \cdot T^2 - 2 \cdot R \cdot \cos(\delta + \frac{\pi}{4})}{1 + (R \cdot T)^2 - 2(R \cdot T) \cdot \cos(\delta + \frac{\pi}{4})} \right] \quad (\text{Eq. 2})$$

$$I_p = \frac{1}{8} A_0^2 \times \left[ \frac{R + R \cdot T^2 - 2 \cdot R \cdot \cos(\delta - \frac{\pi}{4})}{1 + (R \cdot T)^2 - 2(R \cdot T) \cdot \cos(\delta - \frac{\pi}{4})} \right] \quad (\text{Eq. 3})$$

The polarized QFPI is different from the spacial QFPI. The phase-shift mechanism is based on optical element and the interference beam propagates in the same spacial path. Therefore, the system has the ability to against the tilt angle of the measurement mirror.

### 2.3 Optical Design and Signal Processing

In the past studies, some quadrature phase-shifted FPI had been proposed. For most of them, the resonance cavity with lower reflectance has been arranged due to the higher finesse signal with discontinuous signal shown as Fig. 7. Nevertheless it is a disadvantage for signal processing. When the reflectance is reduced to a rather lower quantity, the amplitude of the interference signal will become indistinguishable. That will lead to the problem of signal processing. Hence in this investigation, the optimal reflectance has been recommended to set from 20% to 30% that depends on the measuring ranges and application conditions.

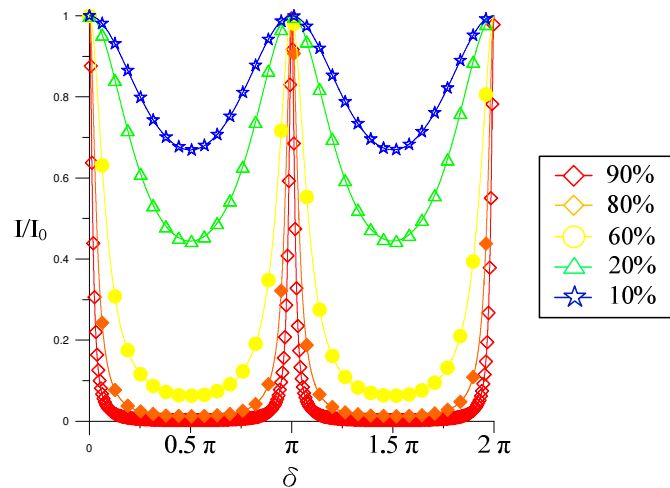


Fig. 7. Intensity distribution of a Fabry-Perot interferometer with the different reflectances

Generally, the commercial signal processing modules of the homodyne interferometer are designed for orthogonal sinusoidal signals. Due to the signal of the Fig. 8, the signal of Fabry-Perot interferometer is not a sinusoidal signal. Hence the Lissajou figure is not perfectly circular. It means that each equal interval of phase angle in the Lissajou figure signifies different displacement quantity. Hence, the interpolation error cannot be neglected, if the commercial signal processing modules is used. This error will also increase in accompanying with the higher finesse of the interference signals. For reducing the error, an appropriate model of the signal processing must be developed.

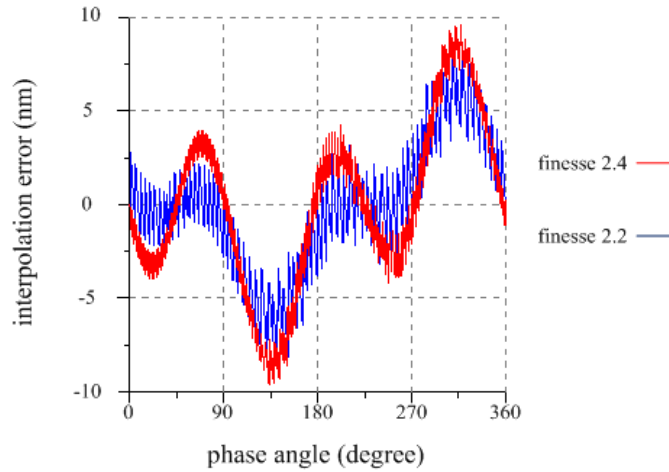


Fig. 8. Interpolation errors of interference signals with different finesses [7]

Here according to the Eq. 2, Eq. 3 and the simulation results shown in Fig. 8, the method of lookup table (LUT) can be established and determined for interpolation model of QFPI with low finesse. By the Fig. 9, the interpolation table can be accomplished as table 1. Here R is reflectance of two coated mirror and T' is the coefficient of intensity loss in the cavity [8]. In this investigation, the resolution of the LUT table is about 0.1 nm. This model will be able to minimize the interpolation errors for both types of measurement mirrors.

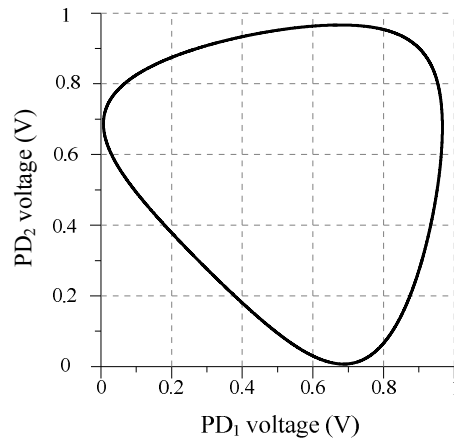


Fig. 9. Lissajou figure of the interference signal (R=25%, T'=50%)

Table 1 Table of LUT method

phase angle $\theta$ (rad)	displacement (nm)
$\theta < 0.000309$	0
$0.000309 \leq \theta < 0.002409$	0.1
$0.002409 \leq \theta < 0.004509$	0.2
$0.004509 \leq \theta < 0.006607$	0.3
⋮	
$6.275058 \leq \theta < 6.277172$	316.2
$6.277172 \leq \theta < 6.279280$	316.3
$6.279280 \leq \theta < 6.281388$	316.4
$6.281388 \leq \theta$	0

Following are some example for the optimization of the reflectance design. Fig. 10 and 11 show the Lissajou figures of different parameters. Fig. 10 shows the figures in ideal situation. As the investigation in the reference [4], the figure with reflectance of 3.5% is the most circles-like pattern of three Lissajou figures. The maximum error to a real circle is only 6.5%. When considering about the resolution, reflectance of 3.5% is no longer as an option of the signal processing. Because the amplitude of the signal is too small, the resolution will become extremely rough. At this situation, the reflectance of 22% will be suggested as the optimal reflectance. This reflectance has the reasonable pattern and maximum amplitude of the first quadrant. When considering about the cavity loss, the optimal suggestion reflectance will change to 21% as Fig. 11.

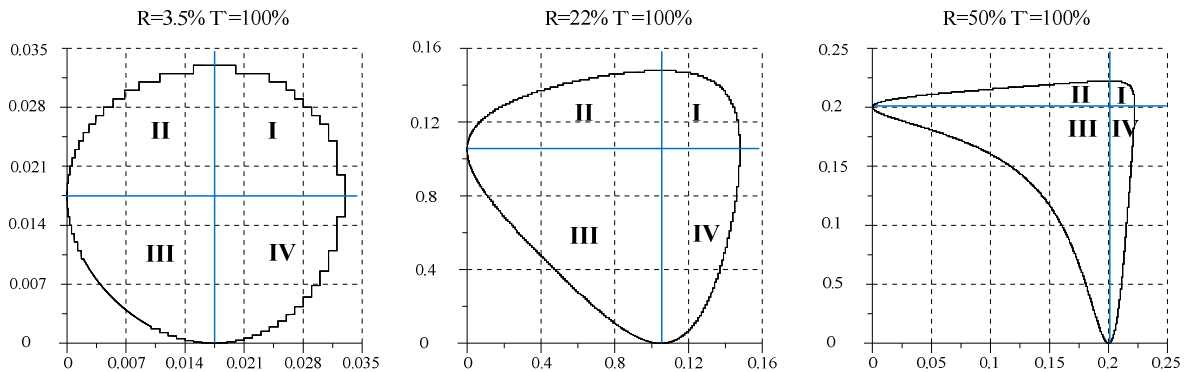


Fig. 10. Lissajou figures in ideal situation

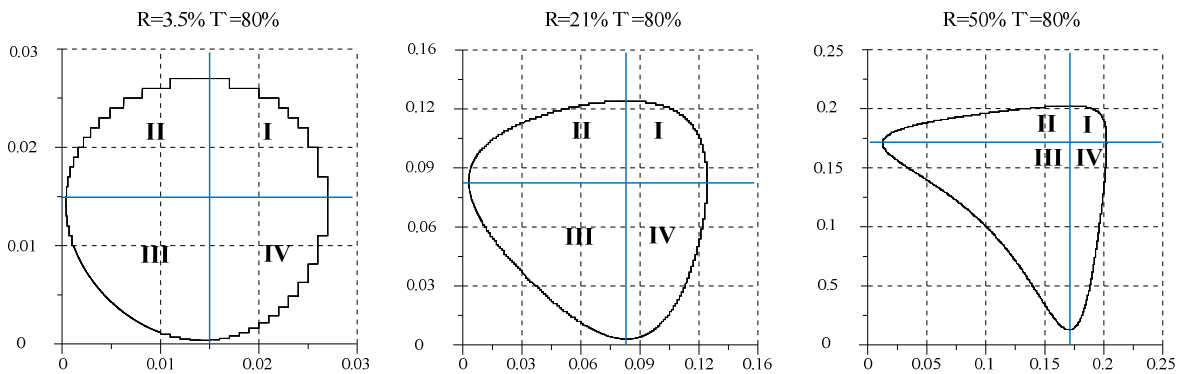


Fig. 10. Lissajou figures under cavity loss coefficient at 80%

### 3. EXPERIMENT DESIGN

During the displacement experiments under the ordinary environment, it is difficult to acquire the interpolation error of few nanometers. For verifying the interpolation models, vibration experiments are carried out for the comparison measurements with a commercial interferometer. The tiny vibration is generated by a piezo transducer with a sinusoidal motion. Figure 11 demonstrates the displacement measurement of the self-developed interferometric system whose signals are processed by the conventional processing model and LUT model during two periods. For the conventional model, the waveform is not exact sinusoidal type and there are some evident deviations due to the interpolation error. For the LUT model, the interpolation error is significantly reduced, although the signal is not a perfect sinusoidal.

In order to investigate the influence of the LUT model on the interpolation error, the measured data are processed by Fast Fourier Transform (FFT). Fig. 12 is vibration experiment setup. The measurement mirrors of commercial interferometer and the QFPI are fixed on the

same metal plate. The metal plate is driven by a piezo transducer with the specific frequency of the sinusoidal wave of 100 Hz. The measurands of two interferometers are processed by FFT. By result of experiment, the ability of LUT method can be confirmed.

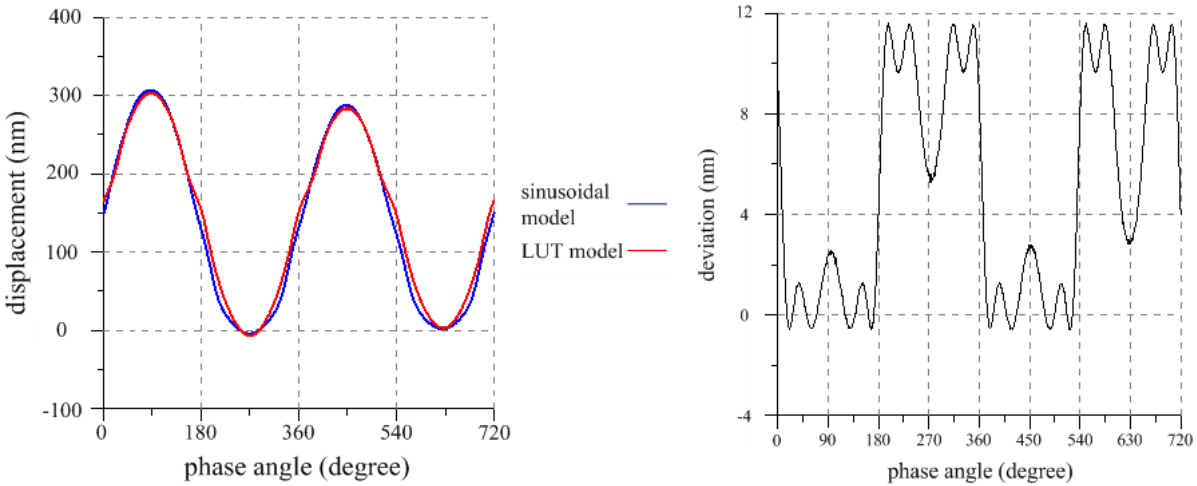


Fig. 11. Interferometric signals with different interpolation models (sinusoidal and LUT model)

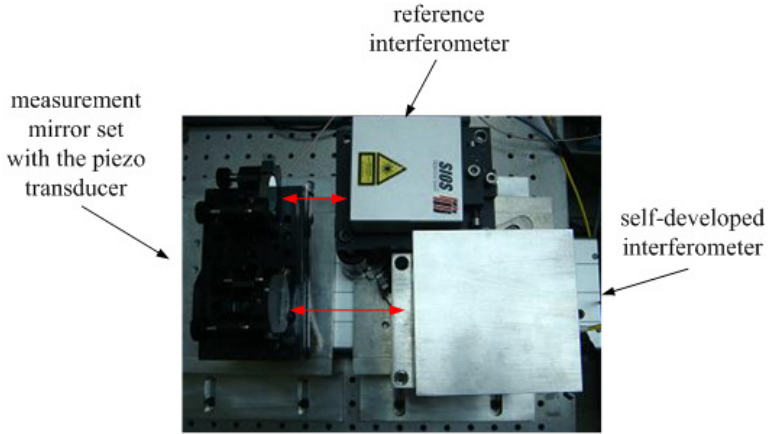


Fig. 12. Interferometric signals with different interpolation models (sinusoidal and LUT model)

**4. RESULTS AND ANALYSIS**

Fig. 13 and 14 are the corresponding frequency spectrums of two interferometers. In the frequency spectrum from the sinusoidal processing model, some subfrequencies can be observed (Fig. 13-a) due to the deviations from perfectly sinusoidal signal. From the spectrums of the commercial interferometer (Fig. 14), the subfrequencies do not exist. In the frequency spectrums obtained by the LUT model (Fig. 13-b), the subfrequencies have been obviously reduced and are not observable. The result shows that about 85% amplitude of maximal subfrequency can be reduced by the LUT method in this case.



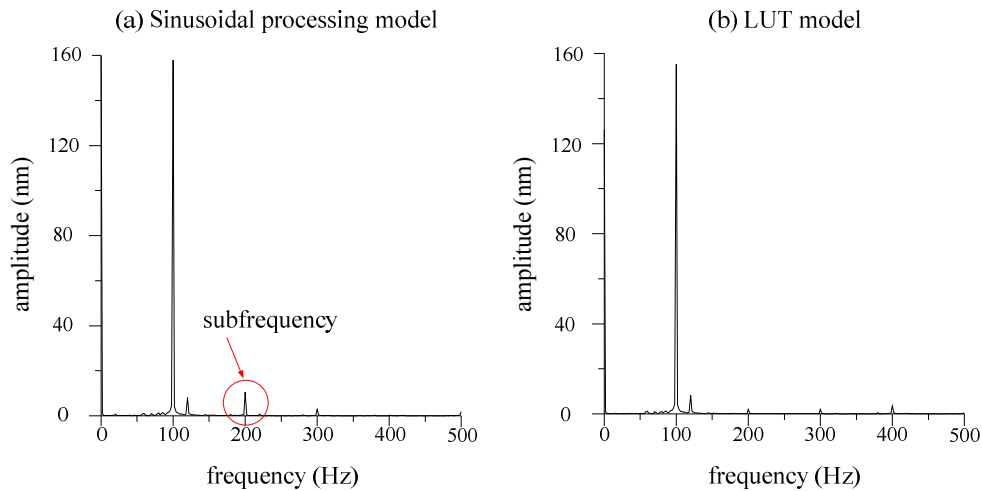


Fig. 13. Frequency spectrum of QFPI with different signal processing

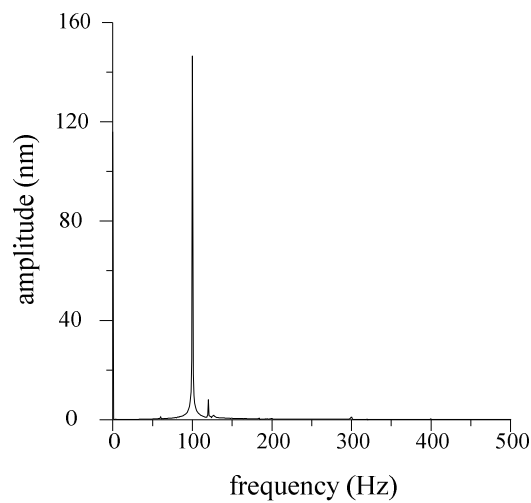


Fig. 14. Frequency spectrum of commercial interferometer

## 5. CONCLUSIONS

In this study, the method of ideal parameters design for QFPI has been presented. Also the signal interpolation method has been investigated. The proposed signal processing method for QFPI is feasible in displacement measurements.

According to the experimental results, the LUT model should be more suitable for the QFPI. With this model, the subfrequency amplitude of 8.2 nm can be declined to 1.1 nm. The relative reductions will be about 85%. That would be quite indispensable for the measurements of the nanometer order.

In future work, the measurement and positioning in nanometer scale with QFPI and LUT method should be performed.

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