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Zuerst erschienen in:
DGaO-Proceedings. - Erlangen-Nürnberg: Dt. Gesellschaft für angewandte Optik, ISSN 1614-8436. - Bd. 114.2013, P54, insg. 2 S.

URN: urn:nbn:de 0278-2013 -P054-3

An optimization method for radial NURBS surfaces

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We study the use of non-uniform rational B-splines (NURBS) for describing axially symmetric surfaces in the design of imaging optical systems. We suggest a representation using a base sphere with an additional NURBS curve. We present an optimization method for such surfaces based on the local structure of NURBS.

1 Introduction

Freeform optical surfaces provide additional degrees of freedom for the design of imaging systems. This enables systems with fewer optical elements and leads to more compact and lightweight systems, while at the same time the image quality is improved. Initially, NURBS have been developed for the (computer aided) design of freeform surfaces such as car bodies. There are few applications in the design of imaging systems. Chase explored the influence of degree and internal knots on rms spot size and optimization time for a Cassegrain-type telescope [1]. Ott designed a head-up display [2].

Instead of directly representing an axially symmetric surface by a NURBS curve, we add such a curve to a base sphere. We extend the work by Zhao [3] on an optimization method based on the local structure of NURBS. We implement radial NURBS surfaces as a user defined DLL for Zemax for use with Zemax's built-in optimization and in our own optical simulation framework for use with the new optimization method.

2 NURBS curves

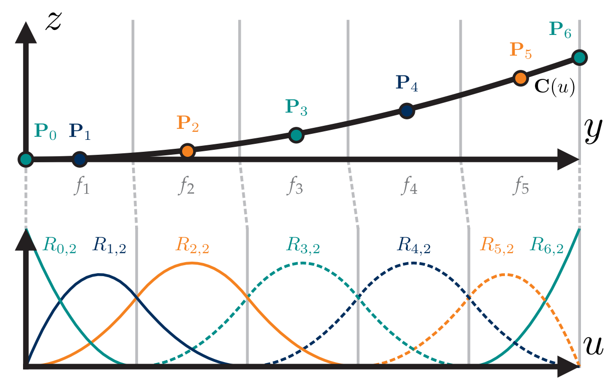

Fig. 1 *A 2nd-degree NURBS curve with $n = 6$ and corresponding $\mathbf{P}_i$ and $R_{i,2}$.*

A NURBS curve has a parametric form and can be described in a compact notation by [4]

$$\mathbf{C}(u) = \sum_{i=0}^{n} R_{i,p}(u)\mathbf{P}_i =: \begin{pmatrix} y(u) \\ z(u) \end{pmatrix},$$

where the $\{\mathbf{P}_i\}$ are the control points and the $\{R_{i,p}\}$ are piecewise rational functions of degree p on $[0, 1]$. Each control point $\mathbf{P}_i$ has also an associated weight w_i hidden within $R_{i,p}$. Such a curve in shown in Fig. 1.

The NURBS curve can also be combined with a base sphere, which for radius $R \neq \infty$ can be written in parametric form as

$$\mathbf{C}_s(u) = \begin{pmatrix} y(u) \\ z(u) + z_s(u) \end{pmatrix}$$
$$\text{with } z_s(u) = R - \operatorname{sgn}(r) \cdot \sqrt{R^2 - y(u)^2}.$$

Using a base sphere has some advantages over the plain NURBS curve: The z-coordinates of the control points are a humanly readable upper limit on the sag departure from the sphere, an important criterion for manufacturing and testing. The ability to start with an initial sphere shape simplifies usage.

3 Piecewise optimization

The idea of piecewise optimization is to optimize only a subset of variables using only a part of the merit function at a time. This is feasible because of the limited local support of the control points. For convenience the principal algorithm is shown for case $p = 2$ and $n = 6$:

```
REPEAT
    FOR Step FROM 1 TO 5
        OPTIMIZE [only respective variables
                  and merit function parts
                  as in Table 1]
UNTIL CONVERGENCE
```

Step:	Merit function for methods 1 to 3		
Var.	1, 2 & 3 term	2 & 3 add. term	3 add. term
$\boldsymbol{x_1}, \boldsymbol{x_2}$	$f_1(x_0, \boldsymbol{x_1}, \boldsymbol{x_2})$	$+f_2(\boldsymbol{x_1}, \boldsymbol{x_2}, x_3)$	$+f_3(\boldsymbol{x_2}, x_3, x_4)$
2: $\boldsymbol{x_3}$	$f_2(x_1, x_2, \boldsymbol{x_3})$	$+f_3(x_2, \boldsymbol{x_3}, x_4)$	$+f_4(\boldsymbol{x_3}, x_4, x_5)$
3: $\boldsymbol{x_4}$	$f_3(x_2, x_3, \boldsymbol{x_4})$	$+f_4(x_3, \boldsymbol{x_4}, x_5)$	$+f_5(\boldsymbol{x_4}, x_5, x_6)$
4: $\boldsymbol{x_5}$	$f_4(x_3, x_4, \boldsymbol{x_5})$	$+f_5(x_4, \boldsymbol{x_5}, x_6)$	
5: $\boldsymbol{x_6}$	$f_5(x_4, x_5, \boldsymbol{x_6})$		

Tab. 1 *Variables and merit function parts for steps of piecewise optimization in case of $p = 2$ and $n = 6$.*

The x_i in Table 1 contain the variables of control point $\mathbf{P}_i$, i.e. a subset of $\{y_i, z_i, w_i\}$. The functions f_i are the parts of the merit function (rms spot size) for which the rays intersect the corresponding segment of the NURBS curve (see Fig. 1). Three different methods of piecewise optimization, named 1 to 3, are considered, which use the first column; the first two columns; and all three columns of the merit function parts of Tab. 1, respectively.

For a control point, y, z and w are not independent, therefore only two of the three should be used as variables for optimization. Method 1 (used by Zhao [3]) does not attain a local minimum. It reaches its final result after only one iteration of the REPEAT-loop. Methods 2 and 3 attain a local minimum. A convergence proof for such *block coordinate descent methods*, applicable to method 3, is given by Bertsekas [5].

Methods 2 and 3 can be "approximated" when for each OPTIMIZE, instead of performing a complete optimization, only the first iteration is performed.

4 Results

Systems used for evaluation are U.S. patents 5,636,065 (1st embodiment, see Fig. 2), 5,754,347 (2nd and 4th) and 6,028,713 (1st) with the aspheric surface replaced by our radial NURBS surface. Only parameters of the NURBS surface are set as variables. We vary p from 2 to 4 and n from 3 to 8.

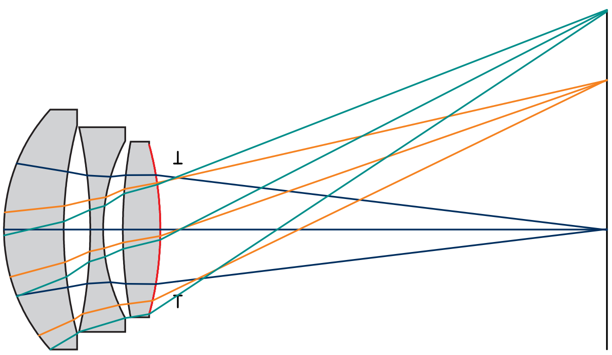

Fig. 2 *One of the systems, U.S. patent 5,636,065 (1st).*

Table 2 shows parameters of the first system with $p = 2$ and $n = 4$ after optimization. Because of the strong convex hull property of NURBS curves, it is immediately observable that the sag maximum departure from the base sphere is less than $0.0178\ \mathrm{mm}$.

	y	z	w
$\mathbf{P}_0$	0.0000	0.0000	1.0000
$\mathbf{P}_1$	0.0895	0.0000	1.0985
$\mathbf{P}_2$	8.0000	0.0082	1.2210
$\mathbf{P}_3$	12.0000	0.0178	0.9227
$\mathbf{P}_4$	16.0000	-0.0064	1.0773

Tab. 2 *Parameters after optimization.*

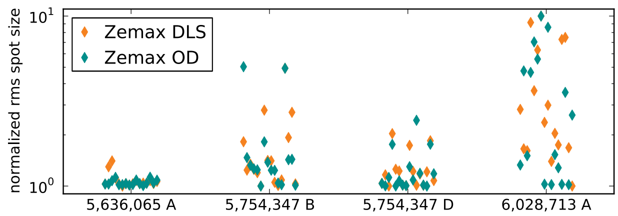

Fig. 3 *Optimizing with Zemax, using different p and n.*

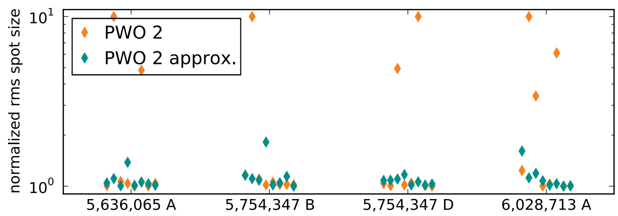

Fig. 4 *Optimizing with piecewise optimization method 2, using different p and n.*

Optimizing with Zemax leads to bad local minima for many configurations (Fig. 3), whereas piecewise optimization method 2 using approximation leads to good local minima for all configurations (Fig. 4).

5 Concluding remarks

When using NURBS to represent axially symmetric surfaces, the proposed representation with base sphere makes the sag maximum departure from the sphere directly visible. Piecewise optimization with approximation is suitable for optimizing such surfaces. A drawback is that it cannot be used to simultaneously optimize other surfaces.

The next step is to find possible applications of such NURBS surfaces, which might be wide angle objectives where on the outer surfaces a control point would only influence a limited region of the field; or off-axis systems where the control points could be concentrated in the area of interest.

6 Acknowledgments

The authors would like to thank the DFG for the financial support through the project "Verallgemeinerte optische Abbildungssysteme" (FKZ: HO 2667/1-1).

7 *

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