Working Papers on Global Financial Markets

No. 12

The Effect of the Interbank Network Structure on Contagion and Financial Stability

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GLOBAL FINANCIAL MARKETS

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September 2010

The Effect of the Interbank Network Structure on Contagion and Financial Stability[1]

In the wake of the financial crisis it has become clear that there is a need for macroprudential oversight in addition to the existing microprudential banking supervision. One of the lessons from the crisis is that the network structure of the banking system has to be taken into account to assess systemic risk. There exists, however, no analysis on the influence of the network topology on contagion in financial networks. This paper therefore compares contagion in Barabási-Albert (scale-free) with Watts-Strogatz (small-world) and random networks. A network model of banks, a firm- and household-sector as well as a central bank is used. Banks optimize a portfolio of risky investments and risk-free excess reserves according to their risk and liquidity preferences. They form a network via interbank loans and face a stochastic supply of household deposits. Contagion effects from the default of a large bank are studied in different network topologies. The results indicate that contagion is more severe in random and scale-free networks than in small-world networks. This situation changes when the central bank is not active in which case small-world networks are less stable than scale-free and random networks. It is also shown that interbank liquidity above a certain threshold leads to endogenous instability, regardless of the network topology. The results further indicate that network heterogeneity does not contribute to financial instability.

Keywords: systemic risk, contagion, interbank markets, network models
JEL classification: C63, E52, E58, G01, G21

I. Introduction

In normal times, banks with excess liquidity provide short-term loans without collateral as interbank loans to banks with a liquidity deficit. This interconnection between banks can lead to an enhanced liquidity allocation and increased risk sharing amongst them as Allen and Gale (2000) show. However, interbank networks display a "robust-yet-fragile" behaviour as for example Haldane (2009) and Gai and Kapadia (2008) argue: the interconnections that serve as a mutual insurance in nor-

[1]The author wishes to thank Markus Pasche, Monika Bucher, Jenny Poschmann, Esti VanWykde-Vries, the members of the working group seminar of the department for market analysis and portfolios of Deutsche Bundesbank and the members of the department for Financial Stability at the South African Reserve Bank for helpful discussions and comments.

mal times can amplify shocks from the insolvency of a bank in a crisis[2]. The insolvency of the US investment bank, Lehman Brothers, resulted in liquidity-hoarding of many banks and ultimately led to the breakdown of interbank markets. As a result the risk premia for unsecured interbank loans increased drastically and resulted in a massive impairment of bank's liquidity provision (see e.g. Heider et al. (2009) and Brunnermeier (2008)). Central banks were forced to undertake unprecedented non-standard measures to reduce money market spreads and ensure liquidity provision to the banking system[3]. Despite this fact the majority of models of interbank markets do not include the central bank as a key actor.

Even though the immediate threat from the crisis seems to have ceased, systemic risk is still a major concern. Bandt et al. (2009) distinguish between a broad and a narrow sense of systemic risk. In their nomenclature, contagion effects on interbank markets pose a systemic risk in the narrow sense, whereas the broad sense of systemic risk is characterized as a common shock that affects many financial institutions or markets. There exists a vast literature focussing on systemic risk in the narrow sense. A number of authors, however, argue that systemic risk in the broad sense is not subordinate to contagion (see e.g. Georg and Poschmann (2010) for an overview) but instead poses the greater threat to systemic stability. Also the Financial Stability Board (FSB), the International Monetary Fund (IMF) and the Bank for International Settlements (BIS) define systemic risk in a broad sense. According to the definition of the International Monetary Fund et al. (2009) systemic risk is *"[...]a risk of disruption to financial services that is (i) caused by an impairment of all or parts of the financial system and (ii) has the potential to have serious negative consequences for the real economy."* emphasizing the possible impact of systemic risk onto the real economy[4]. The key task now is to identify systemic importance of financial institutions and markets in practice. To do so, the International Monetary Fund et al. (2009) propose that: *"Three key criteria that are helpful in identifying the systemic importance of markets and institutions are: (i) size (the volume of financial services provided by the individual component of the financial system); (ii) substitutability (the extent to which other components of the system can provide the same services in the event of a failure); and (iii) interconnectedness (linkages with other components of the system)."* The European Central Bank (2009) suggests that systemic risk can be described as the risk of experiencing a strong systemic event that adversely affects a number of systemically important intermediaries or markets. The trigger of the event could either be a shock from outside or from inside the

[2] A number of authors argue in the same direction. See e.g. Fernando (2003), and Cifuentes et al. (2005).

[3] To motivate central bank interventions, already Goodfriend and King (1988) could show that open market operations enhance the liquidity provision in the financial system. More recently, Allen et al. (2009) and Freixas et al. (2010) could show that central bank intervention can increase the efficiency of interbank markets.

[4] See also the background paper of the Financial Stability Board et al. (2009) and the update of the Financial Stability Board (2010).

financial system. The systemic event is strong when the intermediaries concerned fail or when the markets concerned become dysfunctional. Since all these different dimensions of a systemic event interact with each other, it is clear that systemic risk is a highly complex phenomenon. In its analysis, the European Central Bank (ECB) focuses on three main forms of systemic risk namely contagion risk, the risk of macroeconomic shocks causing simultaneous problems at many financial institutions or markets and the risk of the abrupt unraveling of imbalances that have built up over time. It was recently emphasized by Borio (2010) that the distinction between the time- and cross-sectional dimensions of aggregate risk is critical. In the time-dimension leading indicators of financial distress are needed, while in the cross-sectional dimension a robust quantification of the contribution of each institution to systemic risk is necessary.

According to Acharya and Yorulmazer (2003) as well as Nier et al. (2007), informational contagion is, in addition to contagion and common shocks, a third form of systemic risk that has to be taken into account. Especially in times of crises financial markets exhibit a herding behaviour and the insolvency of a bank can increase the cost of borrowing for the remaining banks. The insolvency of the US investment bank Lehman Brothers in September 2008 led to a breakdown of interbank markets not only because of the direct losses that were associated with its default, but mainly because it was a signal to financial markets that there was a problem with their risk-perception. This signal led to a surge in risk-awareness and risk-aversion and ultimately to the breakdown of interbank money markets. While informational contagion clearly deserves more attention, currently there exists no model to properly assess, measure and forecast it.

A number of suggestions on how to assess systemic risk originating from contagion and common shocks exist in the literature. Brunnermeier et al. (2009) propose to apply leverage, maturity mismatch or the rate of expansion to measure systemic risk. Lehar (2005) estimates the risk of a common shock by the correlation between institution's asset portfolios. Acharya et al. (2009) recommends to measure an institution's contribution to aggregate risk based on its marginal value-at-risk and its marginal expected shortfall. Haldane (2009) suggests to measure contagion based on the interconnectedness of each institution within the financial system, whereas Tarashev et al. (2009) propose to apply the Shapley value methodology to asses the systemic importance of a financial institution. Thomson (2009) provides a scoring model to categorize each institution according to its contribution to systemic risk. Eligible criteria are size, contagion, correlation, concentration and economic conditions. This paper wants to focus on global properties of financial systems to assess their inherent stability properties and analyze how monetary policy impulses are transmitted in different network types.

The rest of this paper is organized as follows. Section II will review some developments from network theory to assess systemic risk. Section III describes our model, while section IV describes the results of our numerical simulations. Section V concludes.

II. Network theory and systemic risk

A new approach to assess systemic risk in financial markets originates from network theory and has been widely applied to ecology, neuroscience, biochemistry, epidemology, social sciences and computer science. The neural network of the worm C-Elegans, the structure of the world-wide-web, the power grid of the United States and the spreading of the HI virus have all been analysed using network theory. The increase in computing power in recent years has led to a vast increase in the research of large and complex systems and some of the results, especially from Epidemology, can be applied to the analysis of financial networks. A financial network consists of a set of banks (nodes) and a set of relationships (edges) between the banks. Even though many relationships exist between banks, this paper focuses on relationships that stem from interbank lending. For the originating (lending) bank the loan will be on the asset side of its balance sheet, while the receiving (borrowing) bank will hold the loan as a liability.

As for example Allen and Babus (2008) argue, linkages between financial institutions can stem from both the asset side (through holding similiar portfolios) and the liabilities side (by sharing the same mass of depositors). These linkages can be direct (as in the case of interbank loans) and indirect (as in the case of similiar portfolios). The authors investigate the resilience of financial networks to shocks and the formation of financial networks. Network theory has also been successfully applied in the analysis of payment systems (see e.g. Soramäki and Galbiati (2008) or Markose et al. (2010)). Castrén and Kavonius (2009) apply network theory to study accounting-based balance sheet interlinkages at a sectoral level. Canedo and Jaramillo (2009) propose a network model to analyse systemic risk in the banking system and seek to obtain the probability distribution of losses for the financial system resulting both from the shock/contagion process. Nier et al. (2007) construct a network model of banking systems and find that (i) the banking system is more resilient to contagious defaults if its banks are better capitalized and this effect is non-linear; (ii) the effect of the degree of connectivity is non-monotonic; (iii) the size of interbank liabilities tend to increase the risk of a knock-on default; and (iv) more concentrated banking systems are shown to be prone to larger systemic risk. In Gai and Kapadia (2009) the authors investigate systemic crises with a network model and show that on the one hand the risk of systemic crises is reduced with increasing connectivity on the interbank market. On the other hand, however, the magnitude of systemic crises increases at the same time.

To describe the toplogy of a network, some notions from graph theory are helpful. The starting point is the definition of a graph.

Definition 1 *A (un)directed graph $G(V, E)$ consists of a nonempty set V of vertices and a set of (un)ordered pairs of vertices E called edges. If i and j are vertices of G, then the pair ij is said to join i and j.*

One sometimes speaks of graphs as networks and the two terms are often used interchangably. Since the focus of this paper is on interbank markets, the nodes of a network are (commercial) banks and the edges are interbank loans between two banks. For every graph a matrix of bilateral exposures which describes the exposure of bank i to bank j can be constructed.

Definition 2 *The matrix of bilateral exposures $W(G) = [w_{ij}]$ of an interbank market G with n banks is the $n \times n$ matrix whose entries w_{ij} denote bank i's exposure to bank j. The assets a_i and liabilities l_i of bank i are given by $a_i = \sum_{j=1}^{n} w_{ij}$ and $l_j = \sum_{j=1}^{n} w_{ji}$.*

Closely related to the matrix of bilateral exposures is the adjacency matrix that describes the structure of the network without referring to the details of the exposures.

Definition 3 *The entries a_{ij} of the adjacency matrix $A(G)$ are one if there is an exposure between i and j and zero otherwise.*

One can define the interconnectedness of a node as the in- and out-degree of the node.

Definition 4 *The in-degree $d_{in}(i)$ and out-degree $d_{out}(i)$ of a node i are defined as:*

$$d_{in}(i) = \sum_{j=1}^{n} a_{ji} \quad , \quad d_{out}(i) = \sum_{j=1}^{n} a_{ij} \tag{1}$$

and give a measure for the interconnectedness of the node i in a directed graph $G(V, E)$. The two degrees are equal for directed graphs.

One can define the size of a node i analogously to its interconnectedness in terms of the value in- and out-degree.

Definition 5 *The value in- and out-degree of a node are defined as:*

$$vdc_{in}(i) = \frac{\sum_{j=1}^{n} w_{ji}}{\sum_{k=1}^{n} \sum_{j=1}^{n} w_{kj}} \in [0, 1] \tag{2}$$

$$vdc_{out}(i) = \frac{\sum_{j=1}^{n} w_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} w_{jk}} \in [0, 1] \tag{3}$$

and give a measure for the size of the node. The value in-degree is a measure for the liabilities of a node while the value out-degree is a measure for its assets.

A quantity that can be used to characterise a network is its average path length. The average path length of a network is defined as the average length of shortest paths for all pairs of nodes $i, j \in V$. Another commonly used quantity to describe the topology of a network is the clustering coefficient, introduced by Watts and Strogatz (1998) in their seminal work on small-world networks. Given three nodes i, j and k, with i lending to j and j lending to k, then the clustering coefficient can be interpreted as the probability that i lends to k as well. For $i \in V$, one define the number of opposite edges of i as:

$$m(i) := |\{j, k\} \in E : \{i, j\} \in E \text{ and } \{i, k\} \in E| \tag{4}$$

and the number of potential opposite edges of i as:

$$t(i) := d(i)(d(i) - 1) \tag{5}$$

where $d(i) = d_{in}(i) + d_{out}(i)$ is the degree of the vertex i. The clustering coefficient of a node i is then defined as:

$$c(i) := \frac{m(i)}{t(i)} \tag{6}$$

and the clustering coefficient of the whole network $G = (V, E)$ is defined as:

$$C(G) := \frac{1}{|V'|} \sum_{i \in V'} c(i) \tag{7}$$

where V' is the set of nodes i with $d(i) \geq 2$. The average path length of the whole network can be defined for individual nodes. The single source shortest path length of a given node i is defined as the average distance of this node to every other node in the network.

It is possible to distinguish between a number of networks by looking at their average path length and clustering coefficient. One extreme type of networks are regular networks which exhibit a large clustering coefficient and a large average path length. The other extreme are random networks which exhibit a small clustering coefficient and a small average path length. Watts and Strogatz (1998) define an algorithm that generates a network which is between these two extremes. They could show that the so-called *"small-world networks"* exhibit both, a large clustering coefficient and small average path length. A large number of real networks like the neural network of the worm Caenorhabditis elegans, the power grid of the western United States, and the collaboration graph of film actors are small-world networks. From a systemic risk perspective, small-world networks are interesting, as it is reasonable to assume that the short average path length and high cluster-

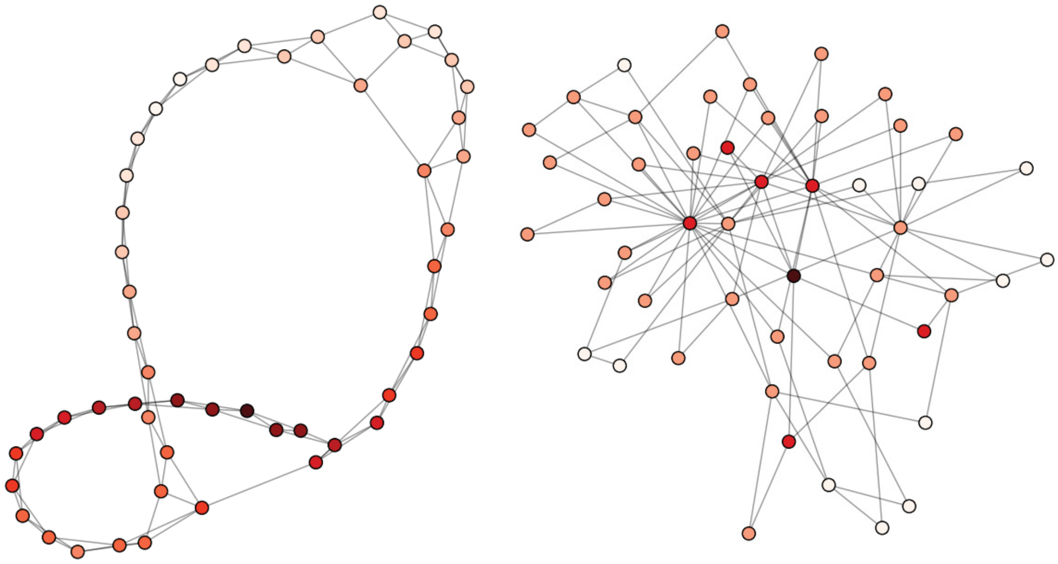

Figure 1: On the left: a small-world network that was created using the algorithm of Watts and Strogatz (1998) with $N = 50$, $k = 4$ and $\beta = 0.05$. On the right: a scale-free network that was created using the methodology introduced in Barabási and Albert (1999) with $N = 50$ and $m = 2$. The colour is an indication for the single source shortest path length of the node and ranges from white (large) to red (short).

ing of small-world networks make them more vulnerable to contagion effects than random or regular networks. Small-world networks can be created by using the algorithm defined in Watts and Strogatz (1998). Starting point is a regular networks of N nodes where each node is connected to its m neighbours. The algorithm now loops over all links in the network and rewires each link with a probability β. For small values of β (about 0.01 to 0.2) the average path length drops much faster than the clustering coefficient so one can have a situation of short average path length and high clustering. On the left side of Figure 1 is a small-world network with $N = 100$, $m = 4$ and $p = 0.05$ shown.

Another interesting class of networks are scale-free networks. They are characterized by a logarithmically growing average path length and approximately algebraically decaying distribution of node-degree (in the case of an undirected network). They were originally introduced by Barabási and Albert (1999) to describe a large number of real-life networks as e.g. social networks, computer networks and the world wide web. To generate a scale-free network one starts with an initial node and continues to add further nodes to the network until the total number of nodes is reached. Each new node is connected to k other nodes in the network with a probability that is proportional to the degree of the existing node. When thinking about financial networks, this preferential attachment resembles the fact that larger and more interconnected banks are generally more trusted by other market participants and therefore form central hubs in the network. On the right side of Figure 1 a scale-free network with $N = 50$ and $k = 2$ is shown.

A typical feature of scale-free networks is their degree-distribution, as it typically follows a power-law. The exponent of the power-law can be measured and characterises the network topology for different networks. Boss et al. (2004) show that the degree distribution of the Austrian interbank market follows a power law with an exponent of $\gamma = -1.87$. Cajueiro and Tabak (2007) analyze the topology of the Brazilian interbank market. They show that the Brazilian interbank market employs a scale-free toplogy and is characterized by money-center banks. Iori et al. (2008) and Manna and Iazzetta (2009) report that the Italian interbank market shows a similiar scale-free behaviour. Cont and Moussa (2009) show that a scale-free interbank network will behave like a small-world network when Credit Default Swaps (CDS) are introduced. In this sense a CDS acts as a "short-cut" from one part of the network to another. This paper therefore focuses on these three classes of networks (random, scale-free and small-world) to analyze their effect on systemic risk through contagion effects.

III. The Model

III.1. Balance Sheets

This paper uses the model developed in Georg and Poschmann (2010), which follows the earlier works of Iori et al. (2006) and Nier et al. (2007) as well as Georg and Pasche (2008) and briefly review its main features here. Starting point is the balance sheet of a bank k that holds risky investments I^k (which banks expect to give a positive return of ρ^+ with probability p and a negative return ρ^- with probability $(1-p)$) and riskless excess reserves E^k as assets at every point in (simulation-) time $t = 1 \dots \tau$. The investments of bank k have a random maturity $\tau_I^k > 0$ and we assume that each bank finds enough investment opportunities according to its preferences. The bank refinances this portfolio by deposits D^k (which are stochastic and have a maturity of zero), from which it has to hold a certain fraction rD^k of required reserves at the central bank, fixed banking capital BC^k, interbank loans L^k and central bank loans LC^k. Interbank loans and central bank loans are assumed to have a maturity of $\tau_L^k = \tau_{LC}^k = 0$. The maturity mismatch between investments and deposits is the standard maturity transformation of commercial banks. A bank can have excess liquidity ($L < 0$) or demand for liquidity ($L > 0$), depending on its balance sheet at time t. The same holds true for central bank loans, where the bank can use either the main refinancing operations to obtain loans, or the deposit facility to loan liquidity to the central bank. The balance sheet of the commercial bank therefore reads as:

$$I_t^k + E_t^k = (1-r)D_t^k + BC_t^k + L_t^k + LC_t^k \tag{8}$$

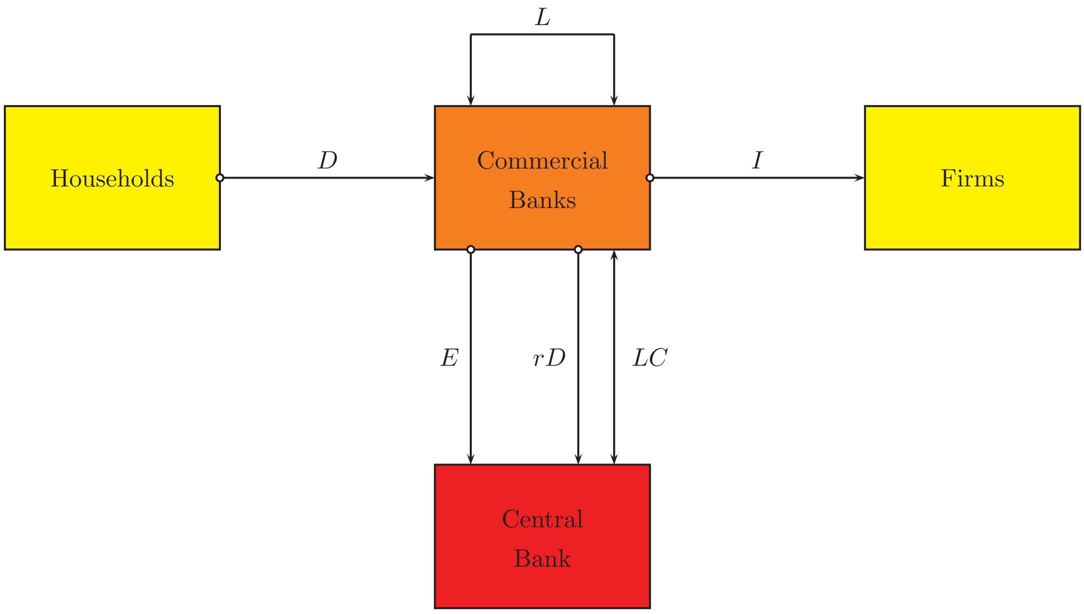

Figure 2: Interaction dynamics of the model. The private sector (household/firms), the banking sector (commercial banks) and the central bank interact via the exchange of deposits, investments, loans, excess- and required reserves and central bank loans. Arrows indicate the direction of fund flows.

The interest rate for deposits at a bank is r^b and the interest rate for central bank loans is r^b. Note that we have not distinguished between an interest rate for the lending and deposit facility and therefore the interest rate on the interbank market will be equal to the interest rate for central bank loans. This in turn also means that the central bank will pay an interest of r^b on excess reserves, as they are deposited at the deposit facility. In figure 2 we have shown the flow of funds for the model.

Banks are characterised by a constant relative risk aversion utility function:

$$u^k = \frac{1}{1-\theta^k}\left(\xi\left(V^k(1+\lambda^k\mu^k - \frac{1}{2}\theta^k(\lambda^k)^2(\sigma^2)^k)\right)\right)^{(1-\theta^k)} \tag{9}$$

where λ^k is the fraction of the risky part of the portfolio, μ^k is the expected return of the portfolio and θ^k is the banks risk aversion parameter. ξ is a scaling parameter that can be introduced into the utility function and is used to scale an ordinal utility function to a cardinal scale. Unless otherwise stated this parameter will be set to $\xi = 1$. $V_t^k = I_t^k + E_t^k$ denotes the bank's portfolio volume and is refinanced at the rate r^b. The risky part of the portfolio and the bank's portfolio volume are obtained by maximising the utility function with respect to λ_k and V^k. Since banks obtain financing on the interbank market and at the central bank at the same interest rate, this refinancing cost is equal to the main refinancing rate. It is possible to introduce a spread between the lending and deposit facility and therefore allowing the interest rate on the interbank market to stochastically vary around the main refinancing rate. For the sake of simplicity we want to exclude this possibility. Note that no explicit market for central bank money is modelled. The central bank can determine the amount and interest rate of central bank liquidity. In times of crisis the central bank might be willing to accomodate all liquidity demands by commercial banks as the full allotment policy of the ECB shows.

III.2. Time Evolution

The update algorithm described in Georg and Poschmann (2010) is used to determine how the model evolves over time. The simulation starts by generating N banks that are endowed with initial values for all balance sheet variables. The banks form an interbank network by issuing (and demanding) loans to each other and therefore redistributing liquidity. In a more realistic model banks will determine the level of the interbank loans that they grant to other banks from portfolio calculus in order to diversify their risk. In the model described in this paper the demand and supply of interbank loans, however, arises only due to deposit fluctuations and losses on assets. At the initialization stage every bank is connected to a number of other banks, according to the underlying network topology. This interconnection means that those two banks can, if both agree, exchange liquidity. The update step is described in detail in Georg and Poschmann (2010) and is repeated for all N banks τ times. The update step starts with banks receiving the required and excess reserves, plus interest payments from the central bank. Banks receive a stochastic return for all investments which might be either positive or negative (in the negative case they suffer losses on their assets). All investments that have a maturity of $\tau_I = 0$ are repaid to the banks. The banks then pay interest for all deposits that were deposited by the households in the previous period. After that the banks can either receive further deposits from the households or suffer deposit withdrawings ΔD_t^k. At the end of the first period, all interbank and central bank loans plus interests are paid either to, or by bank k.

Then a liquidity check is performed and all banks that have insufficient liquid funds to pay the interest on deposits and interbank and central bank loans are marked as illiquid and are removed from the system. It is assumed that a defaulted bank can resolve its investments in order to pay off its depositors (and that every difference is paid by some form of deposit insurance). However, as a defaulted bank will not be able to repay all its depositors it is assumed that all its interbank liabilities have to be written off and that the banks who issued interbank loans to a defaulted bank suffer losses on their assets. All banks that pass the liquidity check now transfer required reserves for their actual deposit level to the central bank. Then the bank will plan its optimal investment and excess reserve level according to its preferences. From this it determines its liquidity demand (or surplus). Banks will try to satisfy their liquidity needs (and likewise their liquidity provision) by going to the interbank market first and then to the central bank. This is due to the fact that banks have to provide adequate collateral if they want to obtain liquidity from the central bank and therefore have a preference to obtain interbank funds that are normally given without collateral. Profitable banks will accumulate a liquidity surplus over time and it is assumed that those bansk will pay dividends to their shareholders before they provide liquidity on the interbank market or deposit it at the standing facility of the reserve bank. Finally all investments are transferred to the firm sector and all excess reserves are transferred to the central bank.

However, as it is not possible for banks to enter the interbank market in this model, the number of banks will continuously decrease until very few, or no banks are left. In real financial systems there is not only the possibility of market exits but also of market entries, which will lead to a steady state with a larger number of active banks. The value of the model presented here lays not in its potential to describe the full financial system, but rather in its ability to analyse how the global topology of the interbank network influences the impact of the insolvency of a large bank. This is especially relevant in times of crises when the insolvencies of banks happen at a much faster rate than the number of banks entering the financial system.

III.3. Model Parameters

There are eighteen model parameters that control the numerical simulation. If not stated otherwise, numerical simulations were performed with the parameters given in this section. Simulation were conducted with $N = 100$ banks and $\tau = 1000$ update steps. Each simulation was repeated 100 times to average out stochastic effects. The deposit interest rate was chosen to be $r^d = 0.02$ and the main refinancing rate as $r^b = 0.04$. The required reserve rate is $r = 0.02$. $\beta = 0.01$ and $k = 10$ is used to generate small-world networks. Barabási-Albert (BA) networks are created with $m_0 = 4$ and $m = 1$ while random networks are a special case of Watts-Strogatz (WS) networks with $\beta = 1$ and $k = 10$. Perfect interconnection in random networks is assumed, if not stated otherwise.

The probability that an investment is successful is given by $p_f = 0.97$. The return for a successful returned credit is $\rho_f^+ = 0.09$ and in case a credit defaults, the negative return on the investment is $\rho_f^- = -0.05$. To plan their optimal portfolio, the banks have an expected investment success probability p_b and expected credit return ρ_b^+. It is assumed, that these expected values correspond to the "true" values determined by the real economy. The optimal portfolio structure and volume of a bank depends also on its risk aversion parameter θ. θ was chosen as $\theta \in [1.67, 2.0]$ randomly for each bank to account for a simple form of heterogeneity in the banking sector.

Deposit fluctuations ΔD_t^k were modelled as in Georg and Poschmann (2010) as:

$$\Delta D_t^k = (1 - \gamma^k + 2\gamma^k x)\underline{D}_{t-1}^k \qquad (10)$$

with $\gamma^k = 0.1$ can be interpreted as a scaling parameter for the level of deposit fluctuations and x being a random variable with $x \in [0, 1]$. The fraction of a banks investments that the central bank accepts as securities is set to $\alpha^k = 0.8$, assuming that banks invest only in assets which have a good rating. The level of dividends β^k that a bank pays to its shareholders was chosen as $\beta^k = 0.99$.

IV. Results

IV.1. Contagion in Barabási-Albert, Watts-Strogatz and random networks

To analyse the effect of different network topologies on contagion, the model described above was simulated on WS-, BA- and random networks. At a pre-defined shock-time of $\tau = 400$ the largest bank in terms of interbank exposures (that is, the bank with the largest value in-degree) was selected and suffered an exogenous loss on its assets that reduced its banking capital BC below the capital adequacy ratio and led to the insolvency of this bank. The results of the simulations are shown in Figure 3, where it can be seen that up to the shock time all three network topologies performed comparably well. The impact of the shock, however, is different in the three topologies. In the Barabási-Albert network 14.21 ± 2.08 banks[5] went into insolvency when the biggest bank in the system defaulted, while in the Watts-Strogatz network it is 6.85 ± 1.18 banks and in the random case 17.98 ± 2.43 banks. The WS network performs comparatively to the BA network and both perform better than the random network. This situation changes when the central bank is inactive which is depicted in the lower part of Figure 3. In the case of no central bank activity, the random network and the BA network perform similarly, while the WS case performs worse (in terms of the number of insolvencies which are used as a measure for financial instability). The volume of interbank loans, normalized to the number of active banks in the system, is significantly higher in the case of no central bank activity. Note that this is in line with Georg and Poschmann (2010) but more dramatic in our case, as we have used a different parameter set.

[5] Note that every simulation was repeated a number of times to average out stochastic effects. This accounts for the non-integer number of bank.

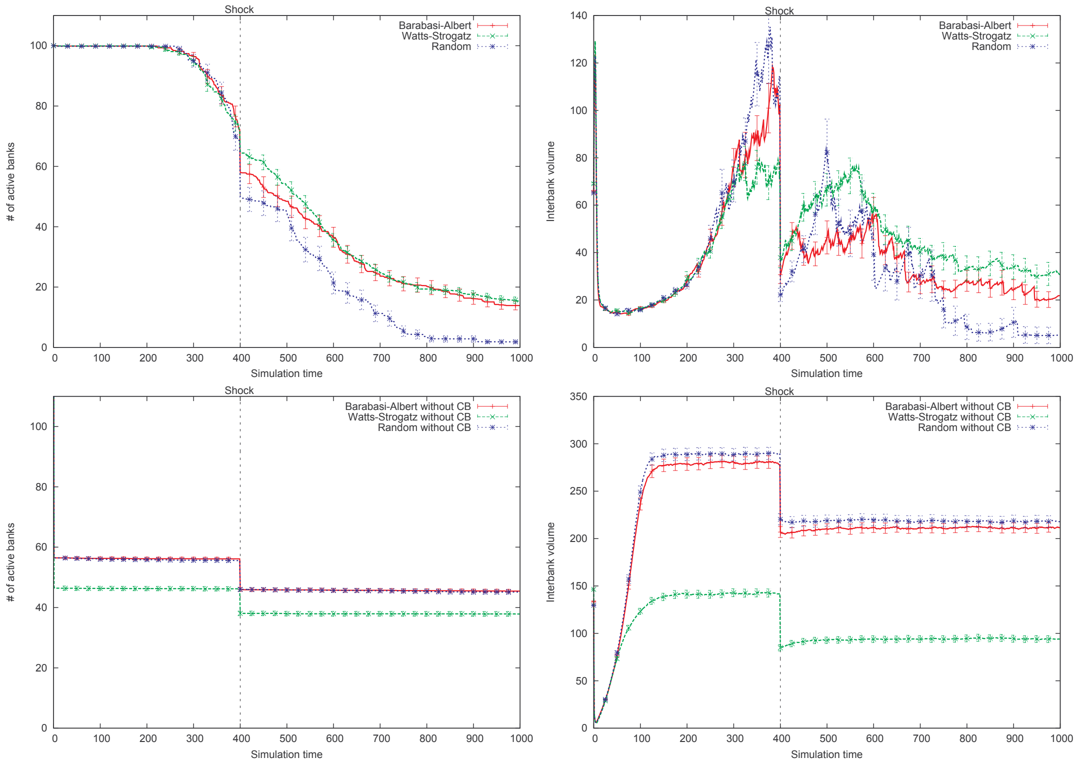

Figure 3: Left: The effect of different network topologies on financial stability. Right: the effect of different network topologies on the interbank trading volume. Top: the case of central bank activity with $\alpha = 0.8$. Bottom: no central bank activity. At time $\tau = 400$ we simulated the insolvency of the largest bank in the system and used the parameters from section III.3 otherwise.

IV.2. Liquidity and financial stability

The effect of bank default depends on the total interbank market volume since at times of high market volume the insolvent bank with the largest exposure will have the largest impact on the banks that have borrowed to the insolvent bank. In the above section the default time was chosen to be in a situation of high interbank market volume, reflecting the fact that the interbank market volume in many countries increased prior the the 2007/2008 financial crisis. The above simulations were repeated with a shock at time $\tau = 200$ when the interbank market volume is only about 25% than at time $\tau = 400$. In the Barabási-Albert network there were 3.95 ± 0.29 insolvencies, while in the Watts-Strogatz network there were 4.03 ± 0.32 insolvencies and in the random case there were 4.10 ± 0.20 insolvencies. In order to analyse the differences between the three network types one has to select a time for the exogenous shock where interbank lending is at a substantial level since at small interbank lending volumes there is no difference visible.

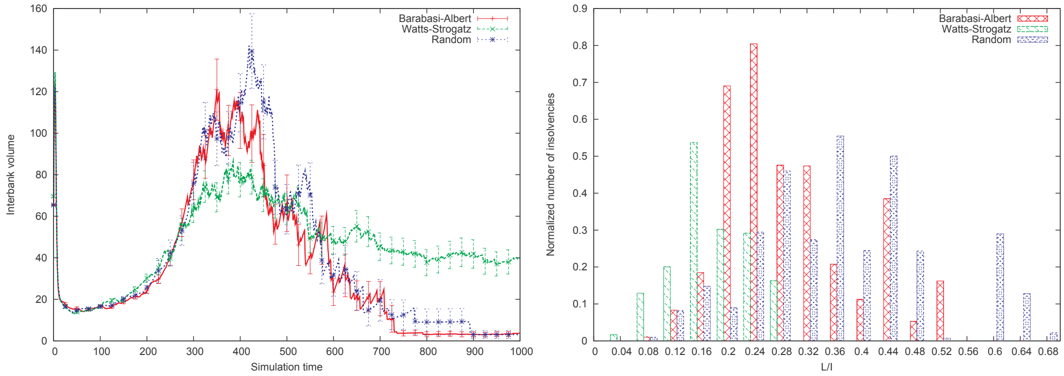

Figure 4: Left: Interbank lending over simulation time for different network topologies. Right: normalized number of insolvencies over the fraction L/I of interbank loans over investment level at which the insolvencies occured. We used the parameters defined in section III.3.

This result gives rise to the conclusion that there is a negative relationship between the amount of interbank loans and financial stability[6] (measured as the number of active banks over time). In the literature it has been argued that the interbank market exhibits a knife-edge property[7]: in normal times the mutual exposures lead to an enhanced risk sharing amongst the financial institutions, while in times of crisis the very same exposures lead to contagion effects.

As can be seen from Figure 3 every shock will lead to a drastic reduction in interbank lending. To further investigate the knife-edge property of interbank markets it is instructive to repeat the above simulations without an exogenous shock. On the left hand side of Figure 4 the interbank volume for the three topologies is shown. Irrespective of the actual network topology, all three simulations exhibit a similar behaviour. Until time $\tau \approx 400$ one can see an increase in interbank lending up to a point, where the volume of interbank lending exceeds a certain fraction of the investment volume. After this point the level of interbank lending decreases, which is due to a drastic decrease in the number of active banks.

To quantify this effect, the number of insolvencies (normalized by the number of active banks in the system) as well as the fraction L/I at each point in time were measured. The results of this measurement are shown in form of a histogram for the three cases of a BA, WS and random network on the right hand side of figure

[6]In a recent paper Boissay (2010) comes to the same conclusion using a general equilibrium model.The model financial market becomes fragile when the liquidity available exceeds the liquidity absorption capacity of the economy, which is determined by productivity in the real sector.
[7]See e.g. Haldane (2009), as well as Fernando (2003); Cifuentes et al. (2005) and Gai and Kapadia (2008).

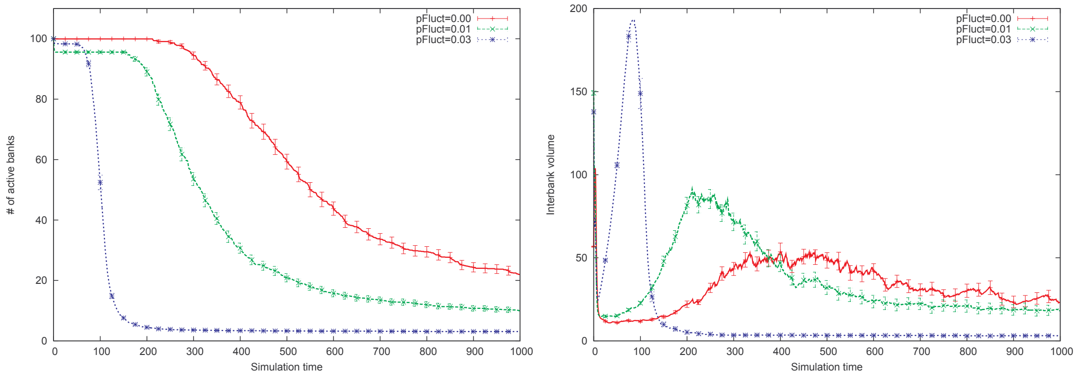

Figure 5: The effect of network heterogeneity (i). The effect of banks investment screening. Left: number of active banks over simulation time. Right: The amount of interbank loan volume over simulation time. We have used $ra_{fluct} = 0.01, 0.02, 0.03$ *and the parameters from section III.3 otherwise.*

4. One can see that the distributions of insolvencies peak around a certain amount of L/I. It is possible to fit a normal distribution to the histogram data in order to obtain the value of L/I where the most insolvencies occur. In the Watts-Strogatz case this mean of the distribution is at about $L/I = 0.179$, while in the Barabási-Albert case it is at $L/I = 0.249$ and in the random case at $L/I = 0.355$. The results indicate that there is an "upper limit" to interbank lending in the sense that larger values of interbank lending endogenously lead to financial instability. As long as interbank lending is low, insolvencies cause no problem for systemic stability since their impact is limited. As the amount of interbank lending increases, possible contagion effects increase as well, until finally there are only the most resilient banks (e.g. those with the most banking capital) left. The results in this paper indicate that different networks are differently resilient to large amounts of interbank loans. While the WS case is the least resilient to large values of L/I, its short average path length and high clustering makes it easier for banks to obtain funds or lend excess liquidity. In this sense WS networks will on the one hand lead to a more enhanced liquidity allocation than BA and random networks. On the other hand, however, WS networks are more prone to contagion at large interbank loan volumes.

IV.3. The effect of network heterogeneity

In Georg and Poschmann (2010) we analysed the effect of network heterogeneity on financial stability. We assumed that banks could differ only in the risk aversion parameter and that all banks faced the same investment opportunities. In this case systemic stability is driven mainly by the fraction of banks with a large risk aversion.

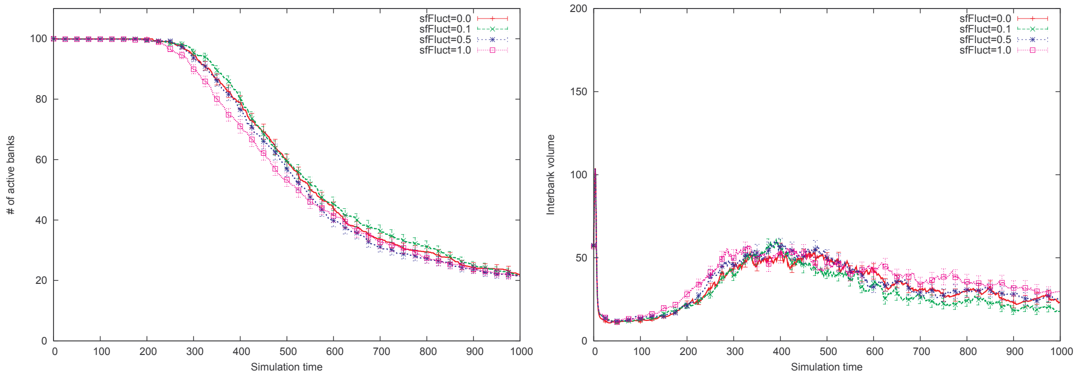

Figure 6: The effect of network heterogeneity (ii). The effect of a differing mass of depositors. Left: number of active banks over simulation time. Right: The amount of interbank loan volume over simulation time. We have used $sf_{fluct} = 0.0, 0.1, 0.5, 1.0$ *and the parameters from section III.3 otherwise.*

This paper now wants to allow for the possibility that some banks have a better screening mechanism for investments than others. The rationale behind this is, that some banks have better ways to ex-ante assess the default probability of an investment (that ultimately is defined only after it defaulted or not) than others. Therefore, the parameter ra_{fluct} that determines the fluctuations in bank's risk assessment is introduced. The larger this parameter is, the larger is the number of banks that are too optimistic about their investments. The results[8] are shown in Figure 5 for a Watts-Strogatz network and are in line with our previous results.

In Figure 6 the effect of banks having a different mass of depositors are analysed for a Watts-Strogatz topology. This is done by allowing the possibility of different banks face different scale factors $\gamma'^k = sf_{fluct}\gamma^k$ of household deposits. As it can be seen from Figure 6 the effect of this type of heterogeneity is negligible.

The third way heterogeneity can arrive in the presented model is through a larger variation in the size of the banks. This is done by allowing the scaling parameter ξ in the utility function to vary over a wider range $\xi := \xi + \xi_{fluct}$. The results of this analysis are shown in Figure 7 and indicate that the financial stability in very heterogenous systems (in terms of bank size) does not differ considerably from more homogenous systems. To interpret this result in the context of the discussion about institutions that are too-big-to-fail (TBTF), one has to note that this result does not mean there is no problem with TBTF. It is merely shown that banking systems

[8] See Figure 4 in Georg and Poschmann (2010) and the discussion about the role of expectations in the model.

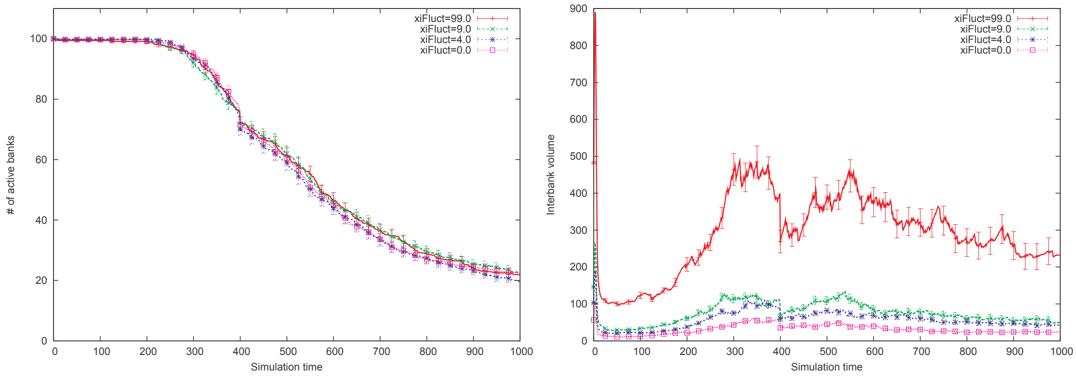

Figure 7: The effect of network heterogeneity (iii). The effect of heterogenous size. Left: number of active banks over simulation time. Right: The amount of interbank loan volume over simulation time. We have used $\xi_{fluct} = 0.0, 4.0, 9.0, 99.0$ *and the parameters from section III.3 otherwise.*

with heterogenous size of banks are not necessarily more prone to contagion. At the core of the TBTF discussion, however, is the observation that banks that are deemed "too-big-to-fail" have an incentive for taking excess risk by implicitely assuming that they will be bailed out should they default. This paper analyses only systemic risk that arises through contagion and neglects the possibility of informational contagion[9] which will effectively lead to a larger systemic importance for larger banks.

IV.4. The impact of the clustering coefficient and the average path length on contagion

Finally,this paper analyses the effect of a different clustering coefficient and a different average path length on financial stability. Therefore, various simulations of WS networks with varying β parameter are performed and the clustering coefficient, average path length and the impact of a shock (where the largest bank goes into insolvency) is measured. The results of these simulations are depicted in Figure 8. Note that there is a correlation between the clustering coefficient and the average path length which makes it impossible to isolate the influence of a variation in clustering or average path length on financial stability. As shown by Watts and Strogatz (1998) in the region where $\beta_{WS} = 0.005, \ldots, 0.1$ the clustering coefficient stays approximately constant, while average path length drops drastically. In the region where $\beta_{WS} = 0.2, \ldots, 0.7$ the average path length does not change much, while the clustering coefficient drops drastically.

As can be seen in Figure 8 there is a tendency for shocks to be more severe in

[9]For a discussion on this see e.g. Acharya and Yorulmazer (2003), Bandt et al. (2009).

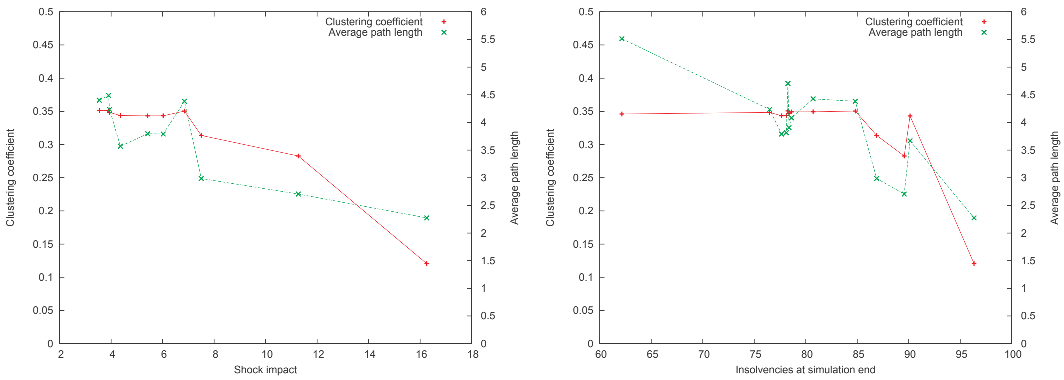

Figure 8: The relationship between clustering coefficient and average path length and financial stability. Left: clustering coefficient (left scale) and average path length (right scale) versus the impact of the insolvency of the largest bank in the system. Right: clustering coefficient (left scale) and average path length (right scale) versus the total number of insolvencies at the end of the simulation. We used $\beta_{WS} = 0.005, 0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7$ *and the parameters from section III.3 otherwise.*

situations where clustering and average path length are low. The same tendency is observable when analyzing the total number of insolvencies instead of the impact of a shock to the system. It is intuitively clear that the average path length is negatively correlated with financial instability, since shocks can spread easier in networks with short average path lengths. The role of the clustering coefficient is less clear, however. By definition the clustering coefficient gives the probability that two banks A and B are connected with each other if they are both connected to a third bank, bank C. In the presented model each bank is at every point in time either a provider or a receipient of liquidity. This is a simplification of reality where banks can and will be provider and receipient of interbank liquidity at the same time. One consequence of this behaviour is that in the case of high clustering, a bank with a liquidity deficit will have more sources of funding than in the case of low clustering. This in turn will lead to an effective reduction of the average size of interbank exposures as the total liquidity demand is driven by either deposit outflows or losses on investment. This behaviour is depicted in Figure 9 where two simple networks consisting of four banks with different clustering coefficient are shown. During the initialization of the network, contractual agreements between the banks about their relationships and the possible direction of interbank flows are generated. These agreements define the structure of the interbank network.

At a given point in the simulation, each bank in the network has either a liquidity surplus or a deficit (or none, but that situation is very rare). The banks will then check their contracts with other banks to find possible partners for interbank transactions. This situation is depicted in the second column in Figure 9 where banks 1 and 4 have a liquidity surplus, while banks 2 and 3 are in need of liquidity. The solid lines denote actual interbank loans amongst the banks. Now assume that bank 2 goes into insolvency. In the case of high clustering it had two contractual partners, bank 1 and bank 3 (in the simulation these contractual partners are chosen upon initialization), each suffering losses on their loan books. In the case of low clustering, it is bank 1 that suffers the entire loss, which most likely is larger than in the case of high clustering. This will force bank 1 into a position where it is in need of liquidity itself. In the case of high clustering it can ask bank 4 for additional liquidity and maybe aquire the necessary funds. In the case of low clustering this possibility does not exist and bank 1 has to go into insolvency as well. The results hold true even in the presence of a central bank, as the central bank does not provide infinite liquidity, but only the amount that a bank can provide collateral for.

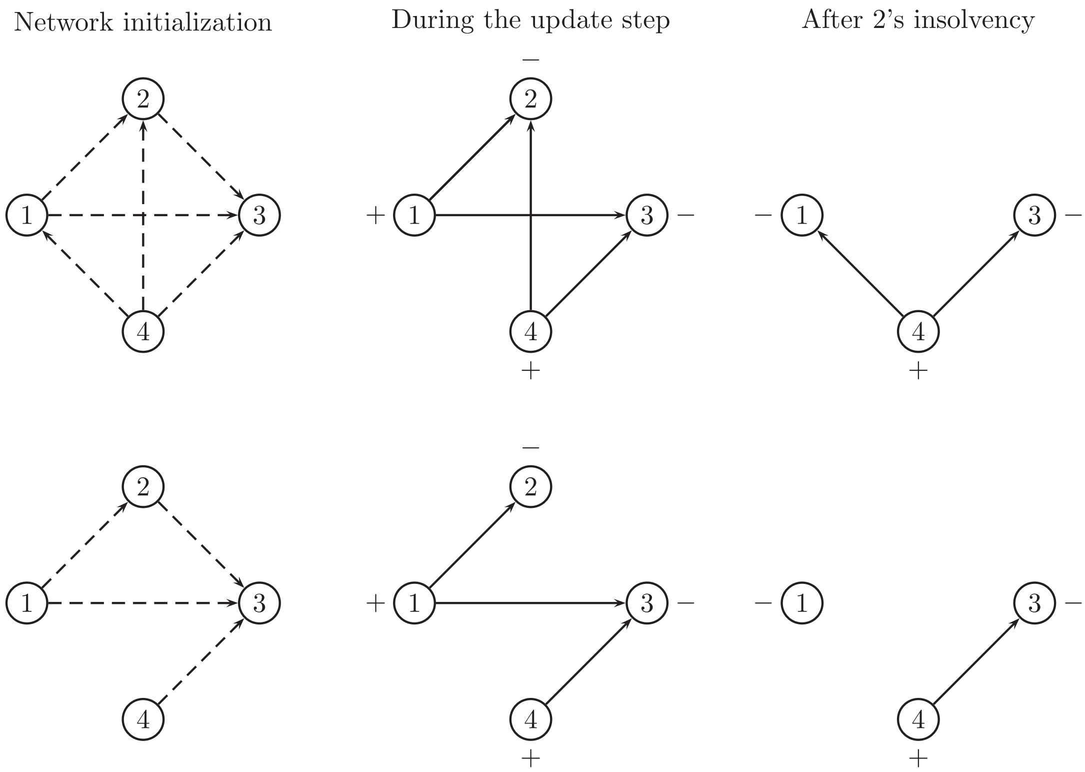

Figure 9: Comparison of high versus low clustering in interbank networks. Top: network with high clustering. Bottom: network with lower clustering. Left: the network at initialization stage where a dashed line indicates that the one bank is able to lend to the other. Middle: a realized network configuration where ± denotes liquidity surplus/shortage and solid lines denote interbank loans. Right: realized network configuration after bank 2 has gone into insolvency.

Note that this logic will change if banks have interbank assets and liabilities at the same time. The structure of the contractual relationships between banks can still remain the same, but for example bank 1 can be provider and receipient of liquidity at the same time.

V. Conclusion and Outlook

This paper analysed how contagion is transmitted in networks with different network topologies using a dynamic model of the banking sector, a firm- and household sector and the central bank. Intuition would suggest that contagion is generally more pressing in networks with short average path length and small clustering. This intuition holds true for interbank networks without central bank activity. If the central bank is active, this intuition does no longer hold true. Small-world networks are shown to be less stable than random and Barabási-Albert networks. This strengthens the argument of Georg and Poschmann (2010) that the central bank has to be taken into account when assessing systemic risk in interbank networks. One of the reasons is that central bank activity will ultimately change the network structure and flow of funds in the network. In most of the existing literature on network models of interbank markets, the central bank is not yet included. Therefore, further investigation of the role of the interbank network structure is required.

Another result of this paper is that large amounts of interbank liquidity endogenously lead to financial instability. The amount of liquidity that will endogenously trigger instability is, however, different for different network topologies. This is another example of the knife-edge property of interbank markets. In normal times the short average path length and large clustering of small-world networks will lead to an enhanced liquidity allocation in the system and effectively stabilizing it. While Barabási-Albert networks generally tend to be more prone to contagion it has to be emphasized that this is due to the money-center structure of BA networks and the fact that in our simulations always the largest bank is sent into insolvency. In real world networks this picture will change if one assumes a random bank becomes insolvent.

It has been argued in the literature that more heterogenous financial systems are more fragile than homogenous ones. In contrast to that, Haldane (2009) argues that the foundations for the financial crisis of 2007/2008 rather were complexity and homogeneity. Within the framework of this model, there is no evidence that heterogeneity somehow affects the stability of the system. It is rather the case that the number of banks who misprice the actual risk of an investment pose a threat to systemic stability. Systemic risk in complex financial networks arises whenever the riskiness of an investment is misjudged. This calls for improved credit rating and more transparent financial products.

Finally the impact of varying clustering coefficient and average path length on financial stability was analysed. Due to the fact that banks are either the provider or the receipient of liquidity at any point in time, shocks tend to be larger at low average path lengths and low clustering. This situation might change if the model framework is extended to allow banks to lend and borrow on the interbank market simultaneously. Even though there is a growing literature on network properties of large value payment systems and country specific interbank markets, there is no paper that compares contagion in different network topologies. One common factor in most of the country-specific analyses is that they aggregate interbank transactions for a given period of time (typically a month, or a year). On any given day, at any given point in time, the actual interbank network might look considerably different. There can be large volatility in the topology of the interbank network during a week, or even within a day. A related problem is that the real interbank network structure will look different than the structure that can be observed in large value payment systems, due to the existence of over-the-counter interbank trades. These data shortcomings make it increasingly difficult to use tools from network theory to analyse interbank markets. Therefore, one of the key tasks of policymakers should be to enhance transparency and data availability on interbank connections.

It is not only interbank loans that form connections or correlations amongst bank's portfolios. The same holds true for derivatives and most form of structured finance products such as credit default swaps. An interbank loan is a correlation between two banks where one bank's assets are another bank's liabilities. Derivatives and credit default swaps are similar in one respect: whenever one bank is short on a certain financial product, another bank (or a hedge fund, a pension fund, etc. in an extended framework) will be short on the same product. The correlation amongst multiple banks arises if they invest in similar asset classes. Even a small initial knock-on can then trigger large losses at multiple institutes at once, as for example Whelan (2009) argues. Network theory might be of limited use when analysing these kinds of multiple correlations, as it deals primarily with edges that join two nodes by definition. In this paper it has been argued that the structure of the interbank market indeed matters when assessing systemic risk. More research, however, is needed to fully understand the various correlations amongst banks and therefore to fully assess the network structure of the financial system.

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