ILMENAU UNIVERSITY OF TECHNOLOGY

Faculty of Electrical Engineering and Information Technology
Department of Electronic Measurement
Master of Science in Communications and Signal Processing

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Master Thesis

UWB Localization of People -Accuracy Aspects

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Course: MSCSP

Matriculation Number: 46066

Matriculation Year: 2009

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urn:nbn:de:gbv:ilm1-2012200296

TECHNICAL UNIVERSITY OF ILMENAU

Abstract

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by Salah Zaied

The Time Of Arrival (TOA) localization technique in Ultra-Wideband (UWB) wireless sensor networks (WSN) is one of the most promising position location techniques that can be used to estimate the position of passive target objects like people. TOA technique determine the time that the signal takes from the transmitting antenna, the passive target object and the receiving antenna. TOA is then transformed into range distance . TOA algorithm involves solving a non linear equations resulting from estimated TOA ranges measured from multiple receiving antennas.

This thesis analyzes the performance of four different passive TOA algorithms in wireless sensor networks. The assessment and comparison of these algorithms has been made for two different simulation scenarios in Additive White Gaussian Noise (AWGN), where a passive target object need to be localized. The simulation also considers a measure of accuracy and precision for TOA algorithms by applying the principal component analysis (PCA) to the covariance matrix of position estimates.

"Faith is taking the first step even when you don't see the whole staircase."

Martin Luther King, Jr.

Acknowledgements

I would like to sincerely thank my supervisor, Dr.Rudolf Zetik, for his guidance, advising, and support. He guided me through my master thesis by sharing his knowledge and experience with me. I would also like to thank the Master of Science in Communications and Signal Processing program team , especially Dipl. Ing. Florian Römer for his valuable input and discussions throughout his teaching courses that broaden my knowledge, and M.Sc. Nuan Song, with her help during my research projects, I learned how to define research problems and formalize them and how to write papers and present them.

Finally, I would especially like to thank my family for their love, encouragement and endless support.

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Abbreviations

AOA	Angle Of Arrival
AWGN	Additive White Gaussian Noise
CEP	Circular Error Probability
FCC	Federal Communication Commission
GDOP	Geometric Dilution Of Precision
PCA	Principal Component Analysis
RMSE	Root Mean Square Error
RN	Reference Node
RSS	Received Signal Strength
Rx	Receiving antenna
TDOA	Time Ddifference Of Arrival
TOA	Time Of Arrival
Tx	Transmitting antenna
UWB	Ultra-Wide Band
WSN	Wireless Sensor Network

1

Introduction

1.1 Motivation

Localization of people or finding the location of a passive target object plays an important role in many of wireless sensors networks (WSN) positioning systems applications. Such as, locating victims in avalanches or earthquakes, injured skier on ski slope, military personnel , fire fighters or lost children. Generally, these applications need very high accuracy requirements, low power consumption and low complexity which make ultra- wideband technology (UWB) the best candidate in these scenarios. Ultra-wideband (UWB) is a promising technology which became popular after the Federal Communications Commission (FCC) in the USA allowed the unlicensed use of UWB devices in February 2002 subject to emission constraints [5]. UWB technology is a response to the limitation in available spectrum, which in turn limits the data transmission rates and the accuracy of positioning systems. UWB uses very wide bandwidths (at least 500 MHz), but is restricted to very low transmitter power density to minimize the interference to other existing radio systems which use part of the same frequency band (3-10.7 GHz). UWB technology is mainly aimed at short range, high data rates links. However, the large bandwidths are ideal for indoor positioning systems, as the large bandwidths mitigate the effects of multipath propagation by allowing very fine time resolution [23].

In order to accurately determine the location of a target object, some measurements and signal parameters must be extracted at first stage [26]. Such parameters are: received signal strength (RSS), angle of arrival (AOA), and signal propagation delay. Although RSS measurements are easily available, but the major drawback of the method is that multi-path reflections, non-line-of-sight conditions, and other shadowing effects might lead to erroneous distance estimates. AOA measurements is an attractive method due to the simplicity of subsequent calculations. But the main drawback of this technique is the possibility of error in estimating the directions caused by multi-path reflections. Signal propagation delay based techniques estimate the object location based on the time it takes the signal to travel from the transmitter to receivers.

Positioning techniques based on signal propagation time can be further classified into Time of Arrival (TOA) and Time Difference of Arrival (TDOA). TOA employ the information of the absolute signal travel time from the transmitter to the receivers; this approach requires the knowledge of signal departure time and thus the synchronization between the transmitter and receivers. Such synchronization can be done by cable connections between the devices, or sophisticated wireless synchronization algorithms. TDOA is employed if there is no synchronization between the transmitter and the receivers. In that case, the receivers do not know the signal travel time and therefore employ the difference of signal travel times between the receivers. It is intuitive that TOA has better performance than the TDOA, since the TDOA loses information about the signal departure time. the second stage involves utilizing efficient algorithms to produce an unambiguous solution to the resulting nonlinear equations. Passive target object means that the object is just reflects signals stemming from separate transmitters, while active object means that the object carries transmitter and receiver.

In this thesis, a number of TOA algorithms for a passive target object are investigated and their performance is compared by analyzing the precision of algorithms for specific geometry scenarios in Additive white Gaussian noise (AWGN) channel.

1.2 Thesis Outline

UWB sensors feature very large bandwidth. This bandwidth allows very accurate localization of passive targets like people .These targets are localized by one transmitting node and several receiving nodes belonging to the infrastructure. There exist different localization approaches. The thesis concerns with a TOA based approach. The Main tasks of the thesis are summarized as follows:

- Mathematical analysis of different solutions for a system of 2-D dimensional nonlinear equations of the second order like: Taylor series linearization, intersection of ellipses and spherical interpolation.
- Programming of the solutions in Matlab.
- Performance analysis of the localization approaches in different simulation scenarios.

- Evaluation the precision of algorithms by Principal Component Analysis (PCA).

2

Localization of Objects

Localization or positioning estimation can be defined as a method of determining the geographic position of an object using the properties of propagated signals. The positioning system in UWB wireless sensor networks is based on the concept of fixed nodes called reference nodes (RN's) carrying transmitting or receiving antennas with known location and the object or the target node (T) whose position is required. Localization of people in static environment can be classified as the passive objects localization. The positioning system uses different approaches to estimate the position of a target node depending on the paramaters extracted from the signals traveling between the transmitting and receiving nodes. Such parameters are: received signal strength (RSS), angle of arrival (AOA), time of arrival and time difference of arrival of propagated signals.
In the following sections, an overview of positioning estimation approaches is presented in 2-D space.

2.1 Received signal strength (RSS)

RSS measurements provide information about the distance (range) between the reference node and the target node based on certain channel characteristics. The main idea behind the RSS-based approach is that if the relation between distance and power loss is known, the RSS measurement at the receiving node can be used to estimate the distance between it and the target node, assuming that the transmit power is known.
The distance between the reference node and the target node provides a circle of uncertainty for the position of the target node.Fig. 2.2a. However, due to inaccuracies in both RSS measurements and quantification of the distance versus path loss relation, distance estimates are subject to errors. Therefore, in reality, each RSS measurement defines an uncertainty area instead of a circle.

Commonly, the RSS technique cannot provide very accurate range estimates due to its heavy dependence on the channel parameters, which is also true for UWB systems.

2.2 Angle Of Arrival (AOA)

Unlike RSS measurement that provides range information between the reference node and the target node , an AOA measurement provides information about the direction of an incoming signal, hence the angle between them, as shown in Fig. 2.1.
Commonly, multiple antennas in the form of an antenna array are employed at the reference node (RN) in order to estimate the AOA of the signal arriving at that node. The angle information is obtained at the antenna array by measuring the differences in arrival times of an incoming signal at different antenna elements. When the distance between the transmitting and receiving nodes are sufficiently large, the incoming signal can be modeled as a planar wave-front. This results in ($l sin\frac{\alpha}{c}$) seconds difference between the arrival times at consecutive array elements, where (l) is the inter-element spacing, (α) is the AOA and (c) represents the speed of light. Therefore, estimation of the time differences of arrivals provides angle information.
For a narrowband signal, time difference can be represented as a phase shift. Therefore, the combinations of the phase-shifted versions of received signals at array elements can be tested for various angles in order to estimate the direction of signal arrival. However, for UWB systems, time-delayed versions of received signals should be considered since a time delay cannot be represented by a unique phase value for a UWB signal.

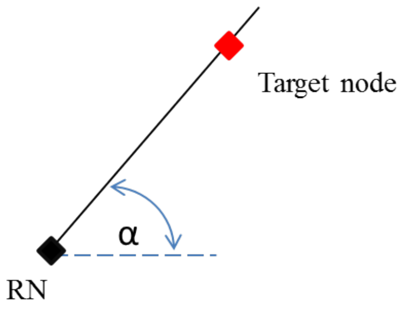
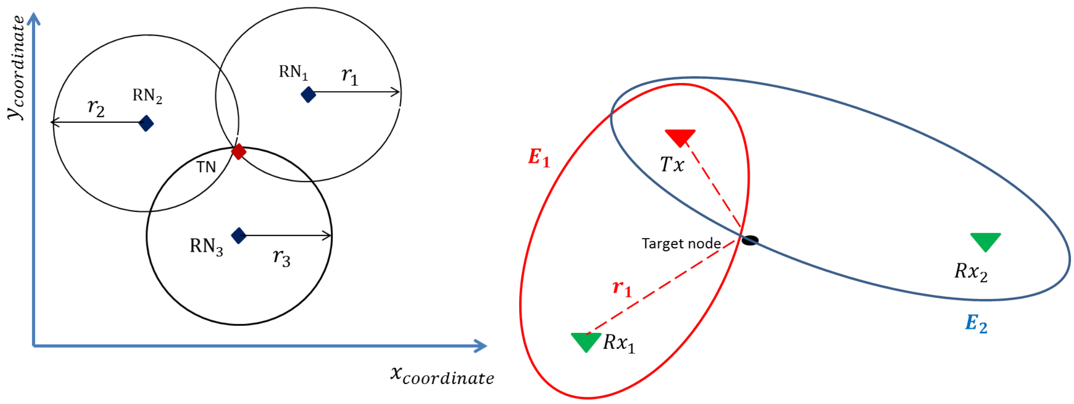


Figure 2.1: RN measures the AOA and determines the angle (α) between itself and the target node

2.3 Time Of Arrival (TOA)

TOA measurement provides information about the distance between the reference node and the target node by estimating the time of flight of a signal that travels from one node to the other. In case of active object, the estimated distance define a circle around the reference node. When measurements are made from multiple reference nodes with known locations, the circles described by the range measurements intersect at a unique point indicating the position location estimate of the target node as shown in Fig.2.2a .[1]. If the circles described by the range measurements intersect at more than one point, an ambiguous solution to the position location estimate results.
In the passive object case, where the target node is unable to carry a wireless transceiver and is just reflects the signals transmitted from reference nodes. In this case, TOA expresses the signal propagation time from the transmitting node (T_x) towards a target (T) and reflected from the target towards the receiving nodes (R_x). This defines en ellipse with foci in transmitting node and receiving node and the semi-major axis equal to the half of TOA estimated distance as shown in Fig 2.2b.



(a) Position estimation based on RSS and TOA (b) Position estimation based on TOA

Figure 2.2: Position estimation based on TOA

Although TOA seems to be a robust technique, it has a few drawbacks:

1. It requires all nodes (The reference nodes and target node in the active case. The transmitting node and receiving nodes in the passive case) to precisely synchronized. A small timing error may lead to a large error in the calculation of the TOA distance.

2. The transmitted signal must be labeled with a time stamp in order to allow the reference node to determine the time at which the signal was initiated at the target node. This additional time stamp increases the complexity of the transmitted signal and may lead to an additional source of error.

3. The positions of the reference nodes should be known; thus, either static nodes or GPS - equipped dynamic nodes should be used.

2.4 Time Difference Of Arrival (TDOA)

As the name suggests, TDOA estimation requires the measurement of the difference in time between the signals arriving at two reference nodes. Similar to TOA estimation, this method assumes that the positions of reference nodes are known. The TOA difference at the reference nodes can be represented by a hyperbola. A hyperbola is the locus of a point in a plane such that the difference of distances from two fixed points (the foci) is a constant.

Assuming 2-D space scenario, three reference nodes and two TDOA measurements are required to localize a target node as shown in the Fig. 2.3. The TDOA measurements are made with respect to the first reference node. For 3-D space case, the position of four anchor nodes and three TDOA measurements are required.

TDOA addresses the first drawback of TOA by removing the requirement of synchronizing the transmitting node clock with reviving node clock. In TDOA, all reference nodes receive the same signal transmitted or reflected by the target node. Therefore, as long as receiving node clocks are synchronized, the error in the arrival time at each node due to unsynchronized clocks is the same. Thus, in the TDOA technique, only receiving nodes clocks need to be synchronized to ensure minimum measurement error.

With respect to the second drawback of TOA, the transmitted signal from the target node in TDOA need not contain a time stamp, since a single TDOA measurement is the difference in the arrival time at the respective anchor nodes. This simplifies the structure of transmitted signals and removes potential sources of error.

The TDOA estimate in the absence of noise and interference restricts the possible target locations to a hyperboloid of revolution with the target node as the foci. If the number of unknowns, or coordinates of the target node to be determined, is equal to the number of range difference measurements, then the system is consistent and a unique solution exist. However, if redundant range difference measurements are made, then the system may be inconsistent and a unique solution may or may not exist. If the hyperbola determined from multiple receivers intersects at more than one point, then ambiguity in the estimated position exists. This location ambiguity may be resolved by using a priori information about the target node location or bearing measurements at one or more of the reference nodes, or redundant range difference measurements at additional reference node to generate additional hyperbolas.

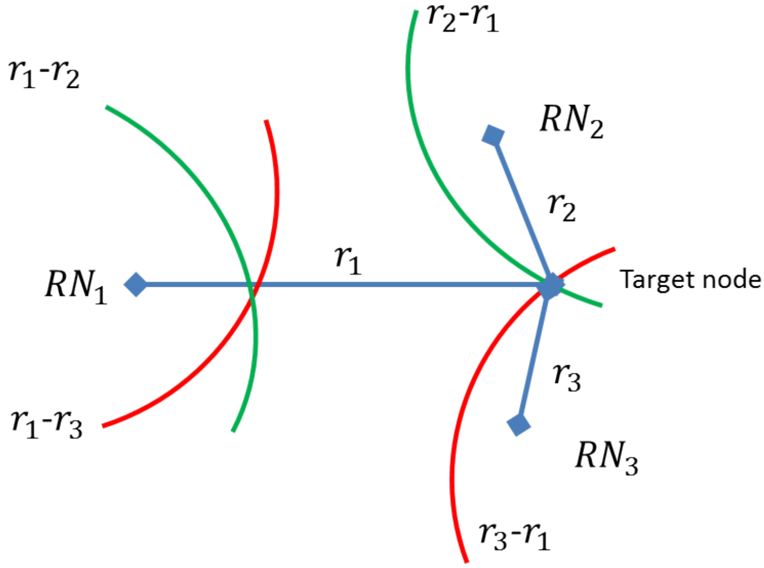


Figure 2.3: Position estimation based on TDOA

3

TOA Passive Location Estimation Algorithms

3.1 General Module

Assuming 2-D space as shown in Fig. 3.1. The passive target node (T) is localized by one transmitting antenna (T_x) and (N) receiving antennas $R_{x_i}(i = 1, 2..., N)$. (x, y) are the coordinates of the target node, (x_t, y_t) are the known coordinates of the transmitting antenna, and (x_i, y_i) are the known coordinates of the receiving antennas.
The range measurements related to the time of arrival (TOA) of the signal propagated from the transmitting node (T_X) towards a target (T) and reflected from the target towards each of the receiving nodes can be written as:

$$r_i = r_{Tx} + r_{Rx_i} \qquad i = 1, 2, 3..N\ . \tag{3.1}$$

$$r_i = \sqrt{(x_t - x)^2 + (y_t - y)^2} + \sqrt{(x_i - x)^2 + (y_i - y)^2} \qquad for\ \ i = 1, 2, 3, ...N. \tag{3.2}$$

This defines the set of nonlinear equations. The task of TOA algorithms is to transform these set of nonlinear equations into linear equations whose solution gives the 2-D coordinates of the target node.

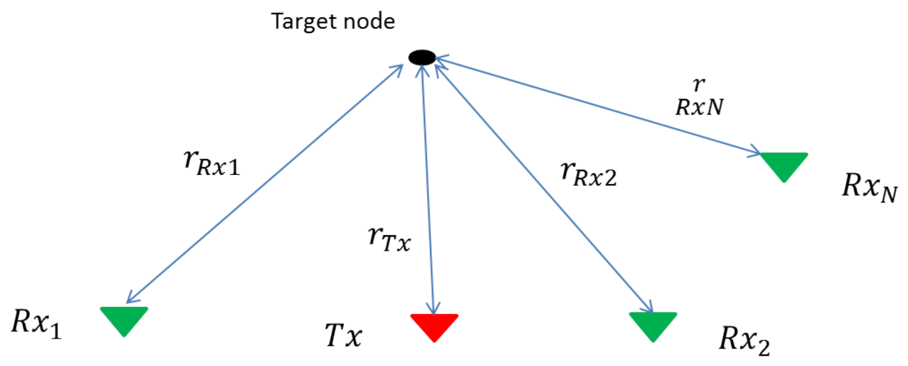


Figure 3.1: TOA Passive target node position estimation by N receivers

3.2 Algorithm.1:Taylor series estimation

TOA range measurements can be defined as a function :

$$\begin{aligned} f_i(x,y) &= \sqrt{(x_t-x)^2+(y_t-y)^2}+\sqrt{(x_i-x)^2+(y_i-y)^2} \\ &= r_i+\varepsilon_i \qquad\quad for\ \ i=1,2,3,...N. \end{aligned} \tag{3.3}$$

(r_i) is the TOA estimate for (ith) receiving antenna and (ε_i) is the corresponding range estimation error, and it is assumed independent and zero-mean Gaussian random variable.

The vector for the range estimation errors is:

$$\varepsilon = \Big[\varepsilon_1\varepsilon_2.......\varepsilon_N\Big]^{\mathbf{T}}, \tag{3.4}$$

the covariance matrix of range estimation errors is:

$$\mathbf{Q} = \mathbf{E}\{\varepsilon\varepsilon^{\mathbf{T}}\}. \tag{3.5}$$

Expanding equation (3.3) into Taylor series using initial estimation (x_v, y_v) and retaining the first two terms :

$$f_{i,v}+a_{i,1}\delta_x+a_{i,2}\delta_y \approx r_i+\varepsilon_i, \tag{3.6}$$

δ_x, δ_y are the location estimation errors to be determined and

$$f_{i,v} = f_i(x_v, y_v), \tag{3.7}$$

$$a_{i,1} = \frac{\partial f_i}{\partial x}\,|_{x_v,y_v} = \frac{x_t - x_v}{\sqrt{(x_t - x_v)^2 + (y_t - y_v)^2}} + \frac{x_i - x_v}{\sqrt{(x_i - x_v)^2 + (y_i - y_v)^2}}, \tag{3.8}$$

$$a_{i,2} = \frac{\partial f_i}{\partial y}\,|_{x_v,y_v} = \frac{y_1 - y_v}{\sqrt{(x_t - x_v)^2 + (y_t - y_v)^2}} + \frac{y_i - y_v}{\sqrt{(x_i - x_v)^2 + (y_i - y_v)^2}}. \tag{3.9}$$

Equation (3.6) can be written in matrix equation form as:

$$\boldsymbol{A\delta} = \boldsymbol{D} + \boldsymbol{e}, \tag{3.10}$$

where

$$\boldsymbol{A} = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ . & . \\ . & . \\ a_{N,1} & a_{N,2} \end{bmatrix}, \tag{3.11}$$

$$\boldsymbol{\delta} = \begin{bmatrix} \delta_x & \delta_y \end{bmatrix}^T, \boldsymbol{D} = \begin{bmatrix} r_1 - f_{1,v} \\ r_2 - f_{2,v} \\ . \\ . \\ r_N - f_{N,v} \end{bmatrix}, \boldsymbol{e} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ . \\ . \\ \varepsilon_N \end{bmatrix}. \tag{3.12}$$

The weight least square estimation of equation (3.10) is:

$$\boldsymbol{\delta} = (\boldsymbol{A}^T\boldsymbol{Q}^{-1}\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{Q}^{-1}\boldsymbol{D}. \tag{3.13}$$

From the initial position guess $(x_{v,}y_v)$ and $\boldsymbol{\delta}$ computed from (3.13), the location estimation can be updated according to :

$$x = x_v + \delta_x, \quad y = y_v + \delta_y. \tag{3.14}$$

The location estimation can be continually refined by iterating the above procedure until δ_x and δ_y are sufficiently small.

3.3 Algorithm.2: Method of intersection of ellipses

Another way to estimate the position of a passive target node is based on calculating the intersection of ellipses.
Consider 2-D space a localization system with one transmitting antenna with coordinates $T_x = (x_t, y_t)$ and two receiving antennas with coordinates $Rx_i = (x_i, y_i), i = 1,2$ as shown in Fig. 3.2. From the TOA range measurements (r_i) obtained its possible to construct two ellipses E_1 and E_2 corresponding to the two receiving antennas with foci $F_{1,1} = (x_t, y_t)$, $F_{1,2} = (x_1, y_1)$ for E_1 and $F_{2,1} = (x_t, y_t)$, $F_{2,2} = (x_2, y_2)$ for E_2 and semi-major axis ($a_i = \frac{r_i}{2}$). Now the problem is to find the intersection points of the two ellipses E_1 and E_2 using the Bézout determinant [7] .

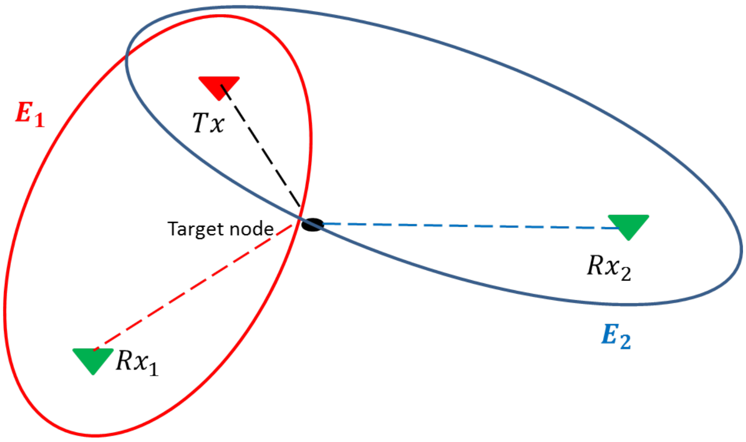


Figure 3.2: Intersection of two ellipses

3.3.1 Intersection of two ellipses

Consider positioning system with a single transmitting antenna T_x and two receiving antennas $Rx_i(i = 0,1)$ in 2-D space. Two ellipses are constructed E_0, E_1 corresponding to the two receiving antennas.
Let the ellipses E_i be defined by the quadratic equation

$$\begin{aligned} \boldsymbol{Q}_i(\boldsymbol{X}) &= \boldsymbol{X}^T \boldsymbol{A}_i \boldsymbol{X} + \boldsymbol{B}_i^T \boldsymbol{X} + \boldsymbol{C}_i \\ &= \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} a_{00}^{(i)} & a_{01}^{(i)} \\ a_{01}^{(i)} & a_{11}^{(i)} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_0^{(i)} & b_1^{(i)} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + c^{(i)} \\ &= 0. \end{aligned} \tag{3.15}$$

The two polynomials $f(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2$ and $g(x) = \beta_0 + \beta_1 x + \beta_2 x^2$ have a common root if and only if the B$\acute{e}$zout determinant is zero,

$$(\alpha_2\beta_1 - \alpha_1\beta_2)(\alpha_1\beta_0 - \alpha_0\beta_1) - (\alpha_2\beta_0 - \alpha_0\beta_2)^2 = 0. \tag{3.16}$$

This is constructed by the combinations

$$\alpha_2 g(x) - \beta_2 f(x) = (\alpha_2\beta_1 - \alpha_1\beta_2)x - (\alpha_2\beta_0 - \alpha_0\beta_2) = 0, \tag{3.17}$$

and

$$\beta_1 f(x) - \alpha_1 g(x) = (\alpha_2\beta_1 - \alpha_1\beta_2)x^2 - (\alpha_2\beta_0 - \alpha_0\beta_2) = 0, \tag{3.18}$$

solving the equation (3.17) for x and substituting it into the equation (3.18), when the B$\acute{e}$zout determinant is zero, the common root of $f(x)$ and $g(x)$ is

$$\bar{x} = \frac{\alpha_2\beta_0 - \alpha_0\beta_2}{\alpha_1\beta_2 - \alpha_2\beta_1}. \tag{3.19}$$

The ellipses equations can be written as quadratics in x whose coefficients are polynomials in y as:

$$\boldsymbol{Q}_i(x,y) = \left(a_{11}^{(i)}y^2 + b_1^{(i)}y + c^{(i)}\right) + \left(2a_{01}^{(i)}y + b_0^{(i)}\right)x + \left(a_{00}^{(i)}\right)x^2. \tag{3.20}$$

With f corresponding to Q_0 and g corresponding to Q_1

$$\begin{aligned} \alpha_0 &= a_{00}^{(0)}y^2 + b_1^{(0)}y + c^{(0)}, & \alpha_1 &= 2a_{01}^{(0)}y + b_0^{(0)}, & \alpha_2 &= a_{00}^{(0)} \\ \beta_0 &= a_{00}^{(1)}y^2 + b_1^{(1)}y + c^{(1)}, & \beta_1 &= 2a_{01}^{(1)}y + b_0^{(1)}, & \beta_2 &= a_{00}^{(1)}. \end{aligned} \tag{3.21}$$

The B$\acute{e}$zout determinant is fourth degree polynomial

$$R(y) = u_0 + u_1 y + u_2 y^2 + u_3 y^3 + u_4 y^4, \tag{3.22}$$

where

$$
\begin{aligned}
u_0 &= v_2 v_{10} - v_4^2, \\
u_1 &= v_0 v_{10} + v_2(v_7 + v_9) - 2v_3 v_4, \\
u_2 &= v_0(v_7 + v_9) + v_2(v_6 - v_8) - v_3^2 - 2v_1 v_4, \\
u_3 &= v_0(v_6 - v_8) + v_2 v_5 - 2v_1 v_3, \\
u_4 &= v_0 v_5 - v_1^2.
\end{aligned} \tag{3.23}
$$

With

$$
\begin{aligned}
v_0 &= 2\left(a_{00}^{(0)} a_{01}^{(1)} - a_{00}^{(1)} a_{01}^{(0)}\right), \\
v_1 &= a_{00}^{(0)} a_{11}^{(1)} - a_{00}^{(1)} a_{11}^{(0)}, \\
v_2 &= a_{00}^{(0)} b_0^{(1)} - a_{00}^{(1)} b_0^{(0)}, \\
v_3 &= a_{00}^{(0)} b_1^{(1)} - a_{00}^{(1)} b_1^{(0)}, \\
v_4 &= a_{00}^{(0)} c^{(1)} - a_{00}^{(1)} c^{(0)}, \\
v_5 &= 2\left(a_{01}^{(0)} a_{11}^{(1)} - a_{01}^{(1)} a_{11}^{(0)}\right), \\
v_6 &= 2\left(a_{01}^{(0)} b_1^{(1)} - a_{01}^{(1)} b_1^{(0)}\right), \\
v_7 &= 2\left(a_{01}^{(0)} c^{(1)} - a_{01}^{(1)} c^{(0)}\right), \\
v_8 &= a_{11}^{(0)} b_0^{(1)} - a_{11}^{(1)} b_0^{(0)}, \\
v_9 &= b_0^{(0)} b_1^{(1)} - b_0^{(1)} b_1^{(0)}, \\
v_{10} &= b_0^{(0)} c^{(1)} - b_0^{(1)} c^{(0)}.
\end{aligned} \tag{3.24}
$$

For each $\bar{y}$ solving $R(\bar{y}) = 0$, solve $Q_0(x, \bar{y}) = 0$ for up to two values $\bar{x}$. We keep only those $(\bar{x}, \bar{y})$ for which both $Q_0(\bar{x}, \bar{y}) = 0$ and $Q_1(\bar{x}, \bar{y}) = 0$. Some of the estimated positions may be excluded if they are a complex number or not in the observed area. This can be done by prior information about the target location [7].

3.3.2 Intersection of ellipses algorithm

For a positioning system consists of one transmitting antenna T_x and N $(N > 2)$ receiving antennas $Rx_i(i = 1, 2, 3..N)$, the position of the target node is estimated by computing the intersection of two ellipses for all possible pair of receiving antennas resulting $N(N-1)/2$ points , the final estimated position is determined by the arithmetic mean of theses points.

3.4 Algorithm.3: Method of least squares

In the method of least squares, the nonlinear system of equations (3.2) are transformed into a system of linear equations [20].

$$r_i = \sqrt{(x_t - x)^2 + (y_t - y)^2} + \sqrt{(x_i - x)^2 + (y_i - y)^2} \qquad for \;\; i = 1, 2, 3, ...N, \tag{3.25}$$

Let the range distance between the transmitting node and the target node is :

$$d_t = \sqrt{(x_t - x)^2 + (y_t - y)^2}, \tag{3.26}$$

and the range distance between the receiving nodes and the target node is :

$$d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2}. \tag{3.27}$$

Squaring the both sides of the equation (3.25) and substituting into the second part equations (3.26) and (3.27)

$$r_i^2 = d_t^2 + 2d_t d_i + (x_i - x)^2 + (y_i - y)^2 \qquad for \;\; i = 1, 2, 3, ...N, \tag{3.28}$$

$$r_i^2 = x^2 + y^2 - 2xx_i - 2yy_i + x_i^2 + y_i^2 + d_t^2 + 2d_t d_i, \tag{3.29}$$

rearranging the equation where

$$d_i^2 = (x_i - x)^2 + (y_i - y)^2 = x^2 + y^2 - 2xx_t - 2yy_t + x_t^2 + y_t^2 \tag{3.30}$$

$$r_i^2 = 2x(x_t - x_i) + 2y(y_t - y_i) + x_i^2 + y_i^2 - x_t^2 - y_t^2 + 2d_t^2 + 2d_t d_i, \tag{3.31}$$

$$r_i^2 - 2x(x_t - x_i) - 2y(y_t - y_i) - x_i^2 - y_i^2 + x_t^2 + y_t^2 = 2d_t^2 + 2d_t d_i, \tag{3.32}$$

$$r_i^2 - 2x(x_t - x_i) - 2y(y_t - y_i) - x_i^2 - y_i^2 + x_t^2 + y_t^2 = 2d_t(d_t + d_i), \tag{3.33}$$

$$r_i^2 - 2x(x_t - x_i) - 2y(y_t - y_i) - x_i^2 - y_i^2 + x_t^2 + y_t^2 = 2d_t r_i, \tag{3.34}$$

$$x(x_i - x_t) + y(y_i - y_t) - \frac{1}{2}(x_i^2 + y_i^2 - x_t^2 - y_t^2 - r_i^2) = d_t r_i. \tag{3.35}$$

Let:

$$p_i = \frac{1}{2}(x_i^2 + y_i^2 - x_t^2 - y_t^2 - r_i^2), \tag{3.36}$$

then the equation (3.35) can be written as :

$$x(x_i - x_t) + y(y_i - y_t) - p_i = d_t r_i. \tag{3.37}$$

To transform the set of non-linear equations in (3.37) into linear equations, the intersection between the pair of ellipses E_i and E_j where $(i = 1,2,3...N, j = 1,2,3...N)$ $(i \neq j)$ is calculated.
For ith ellipses , multiplying both sides of the equation (3.37) by r_j

$$x(x_i - x_t)r_j + y(y_i - y_t)r_j - p_i r_j = d_t r_i r_j, \tag{3.38}$$

and for jth ellipses , the equation (3.37) becomes :

$$x(x_j - x_t) + y(y_j - y_t) - p_j = d_t r_j, \tag{3.39}$$

and multiplying it by r_i

$$x(x_j - x_t)r_i + y(y_j - y_t)r_i - p_j r_i = d_t r_j r_i, \tag{3.40}$$

subtracting equation (3.40) from equation (3.38)

$$x\left[(x_i - x_t)r_j - (x_j - x_t)r_i\right] + y\left[(y_i - y_t)r_j - (y_j - y_t)r_i\right] = p_i r_j - p_j r_i, \quad for \;\; i = 1,2,3,...N, i \neq j \tag{3.41}$$

$$x\left[x_i r_j - x_j r_i + x_t r_i - x_t r_j\right] + y\left[y_i r_j - y_j r_i + y_t r_i - y_t r_j\right] = p_i r_j - p_j r_i. \quad for \;\; i = 1,2,3,...N, i \neq j. \tag{3.42}$$

Equation (3.42) can be written in matrix equation form

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}, \tag{3.43}$$

where

$$\boldsymbol{A} = \begin{bmatrix} (x_2 r_j - x_j r_2 + x_t r_2 - x_t r_j) & (y_2 r_j - y_j r_2 + y_t r_2 - y_t r_j) \\ (x_3 r_j - x_j r_3 + x_t r_3 - x_t r_j) & (y_3 r_j - y_j r_3 + y_t r_3 - y_t r_j) \\ \vdots & \vdots \\ (x_N r_j - x_j r_N + x_t r_N - x_t r_j) & (y_N r_j - y_j r_N + y_t r_N - y_t r_j) \end{bmatrix}, \quad \boldsymbol{x} = \begin{bmatrix} x \\ y \end{bmatrix},$$

$$\boldsymbol{b} = \begin{bmatrix} p_2 r_j - p_j r_2 \\ p_3 r_j - p_j r_3 \\ \vdots \\ p_N r_j - p_j r_N \end{bmatrix}.$$

The least squared solution of the matrix equation (3.43) results the estimated coordinates of the passive target node:

$$\hat{\boldsymbol{x}}_{LS} = (\boldsymbol{A}^T \boldsymbol{A})^{-1} \boldsymbol{A}^T \boldsymbol{b}, \tag{3.44}$$

3.5 Algorithm.4:Spherical interpolation method

Spherical interpolation method transforms a set of nonlinear equations into a system of linear equations with the auxiliary variable, which depends on the target position. The target position is then determined using the method of least squares [27] [28].
The Pythagoras's theorem is a special case of the more general theorem relating the lengths of sides in any triangle, the law of cosines:

$$a^2 + b^2 - 2ab\cos\theta = c^2. \tag{3.45}$$

The distances between the transmitting antenna $T_x = (x_t, y_t)$, the target node $T = (x, y)$ and the receiving antennas $R_{x_i} = (x_i, y_i), for\ \ i = 1, 2, 3, ...N$, can be written as :

$$s = |T_xT| = \sqrt{(x_t - x)^2 + (y_t - y)^2}, \tag{3.46}$$

$$s_i = |T_xR_{x_i}| = \sqrt{(x_t - x_i)^2 + (y_t - y_i)^2}, \tag{3.47}$$

$$q_i = |R_{x_i}T| = \sqrt{(x_i - x)^2 + (y_i - y)^2}. \tag{3.48}$$

In vector notation and assuming the transmitting antenna T_x at the origin $(x_t = 0, y_t = 0)$ as shown in Fig. 3.3. The vector $\boldsymbol{p}_i = \begin{bmatrix} x_i & y_i \end{bmatrix}^T$, start at T_x and end at R_{x_i}, and the vector $\boldsymbol{p} = \begin{bmatrix} x & y \end{bmatrix}^T$, start at T_x and end at T.

According to the Pythagoras' theorem this can be written as:

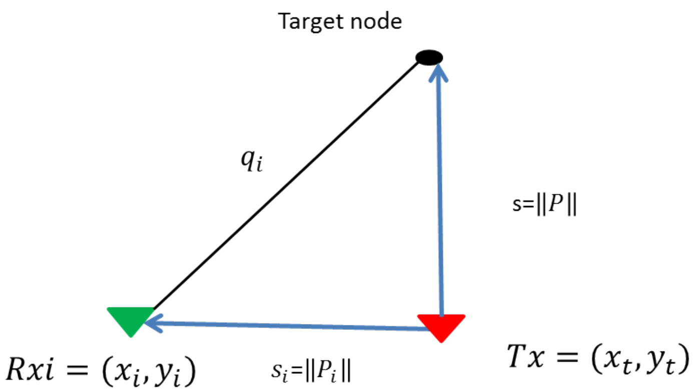


Figure 3.3: Geometric interpretation of spherical interpolation

$$q_i^2 = s^2 + s_i^2 - 2\boldsymbol{p}_i^T\boldsymbol{p}. \tag{3.49}$$

The TOA estimated distances are: $r_i = q_i + s$.

Therefore, the equation (3.49) can be written as:

$$(r_i - s)^2 = s^2 + s_i^2 - 2\boldsymbol{p}_i^T\boldsymbol{p}. \tag{3.50}$$

$$0 = s_i^2 - r_i^2 + 2r_i s - 2\boldsymbol{p}_i^T \boldsymbol{p}. \tag{3.51}$$

As the TOA estimated ranges are noisy measurements, an estimation range error is introduced. Then the equation (3.51) becomes:

$$\varepsilon_i = s_i^2 - r_i^2 + 2r_i s - 2\boldsymbol{p}_i^T \boldsymbol{p}, \qquad i = 1, 2, 3, ...N, \tag{3.52}$$

where (ε_i) is the range estimation error to be minimized.
For $T_x = (x_t, y_t)$, the set of N equations (3.52) can be written in matrix equation form:

$$\boldsymbol{\epsilon} = \boldsymbol{\delta} + 2s\boldsymbol{r} - 2\boldsymbol{A}\boldsymbol{p}, \tag{3.53}$$

where

$$\boldsymbol{\epsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{bmatrix}, \quad \boldsymbol{\delta} = \begin{bmatrix} s_1^2 - r_1^2 \\ s_2^2 - r_2^2 \\ \vdots \\ s_N^2 - r_N^2 \end{bmatrix}, \quad \boldsymbol{r} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix}, \quad \boldsymbol{A} = \begin{bmatrix} x_1 - x_t & y_1 - y_t \\ x_2 - x_t & y_2 - y_t \\ \vdots & \vdots \\ x_N - x_t & y_N - y_t \end{bmatrix}, \quad \boldsymbol{p} = \begin{bmatrix} x - x_t \\ y - y_t \end{bmatrix}.$$

The matrix equation (3.53) is linear in $\boldsymbol{p}$ given s.
The formal least squares solution of (3.53) for $\boldsymbol{p}$ given s when $\boldsymbol{\epsilon} = \boldsymbol{0}$:

$$\boldsymbol{p} = \frac{1}{2}\boldsymbol{A}^{+}(\boldsymbol{\delta} + 2s\boldsymbol{r}), \tag{3.54}$$

where

$$\boldsymbol{A}^{+} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T. \tag{3.55}$$

Substituting the value of $\boldsymbol{p}$ into equation (3.53), we get new equation:

$$\begin{aligned} \tilde{\boldsymbol{\epsilon}} &= \boldsymbol{\delta} + 2s\boldsymbol{r} - \boldsymbol{A}\boldsymbol{A}^{+}(\boldsymbol{\delta} + 2s\boldsymbol{r}) \\ &= (\boldsymbol{I} - \boldsymbol{A}\boldsymbol{A}^{+})(\boldsymbol{\delta} + 2s\boldsymbol{r}). \end{aligned} \tag{3.56}$$

Where $\boldsymbol{I}$ is N by N identity matrix and

$$\boldsymbol{B} = \boldsymbol{A}\boldsymbol{A}^{+}, \tag{3.57}$$

is the projection matrix onto the column space of the matrix $\boldsymbol{A}$.
Then, we can write:

$$\boldsymbol{B}^{\perp} = \boldsymbol{I} - \boldsymbol{B}, \tag{3.58}$$

which is the projection matrix onto the row space of the matrix $\boldsymbol{A}$.
The equation (3.56) can be written then as:

$$\tilde{\boldsymbol{\epsilon}} = \boldsymbol{B}^{\perp}(\boldsymbol{\delta} + 2s\boldsymbol{r}). \tag{3.59}$$

So that the function:

$$J = \boldsymbol{\epsilon}^{T}\boldsymbol{\epsilon} = (\boldsymbol{\delta} + 2s\boldsymbol{r})^{T}\boldsymbol{B}^{\perp}(\boldsymbol{\delta} + 2s\boldsymbol{r}), \tag{3.60}$$

is minimized with respect to s, yield:

$$\tilde{s} = -\frac{1\boldsymbol{r}^{T}\boldsymbol{B}^{\perp}\boldsymbol{\delta}}{2\boldsymbol{r}^{T}\boldsymbol{B}^{\perp}\boldsymbol{r}}. \tag{3.61}$$

Substituting this solution into equation (3.54)

$$\tilde{\boldsymbol{p}} = \frac{1}{2}\boldsymbol{A}^{+}(\boldsymbol{\delta} + 2\tilde{s}\boldsymbol{r}) = \frac{1}{2}\boldsymbol{A}^{+}(\boldsymbol{\delta} - \frac{\boldsymbol{r}^{T}\boldsymbol{B}^{\perp}\boldsymbol{\delta}\boldsymbol{r}}{\boldsymbol{r}^{T}\boldsymbol{B}^{\perp}\boldsymbol{r}}). \tag{3.62}$$

This solution gives the estimated coordinates of the target node.

4

Simulation Results

4.1 Scenario.1

To evaluate the performance of TOA algorithms, we estimate the position of the passive target node T located at $(x = 2.2; y = 1.97)$ by one transmitting antenna T_x with $(x_t = 2; y_t = 3.2)$ and four receiving antennas $(N = 4)$ placed as in table 4.1.

The simulation scenario is illustrated in Fig. 4.1a, where the receiving antennas are

R_x	x (m)	y (m)
1	0	1.78
2	0	2.72
3	5.14	2.4
4	5.14	1.62

Table 4.1: Receiving antennas coordinates

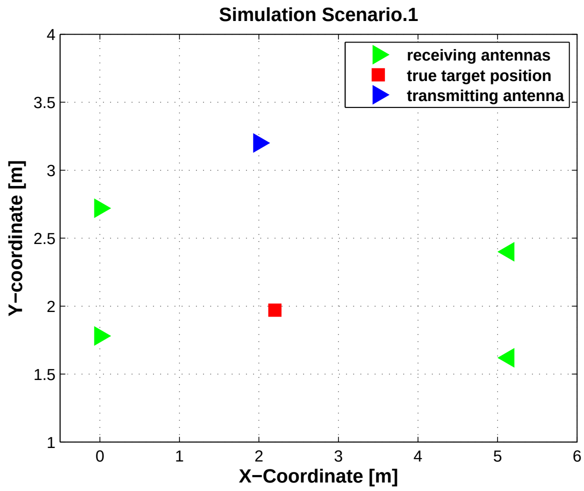
depicted as green arrowheads, the transmitting antenna as blue arrowhead and the passive target node as red dot.The anchor nodes are placed arbitrarily and are not in linear fashion relative to the target node, which makes the position estimation more realistic. Thus, a linearly placed anchor nodes is a special case and simplifies the position estimation.

The position estimation simulation is performed in Additive White Gaussian Noise (AWGN) channel using the known positions of the receiving antennas, transmitting antenna and the resulting noisy TOA range measurements which are corrupted with zero mean Gaussian noise. The localization performance of TOA algorithms are then examined by comparing the estimation result of each algorithm with the actual position and with other algorithms.

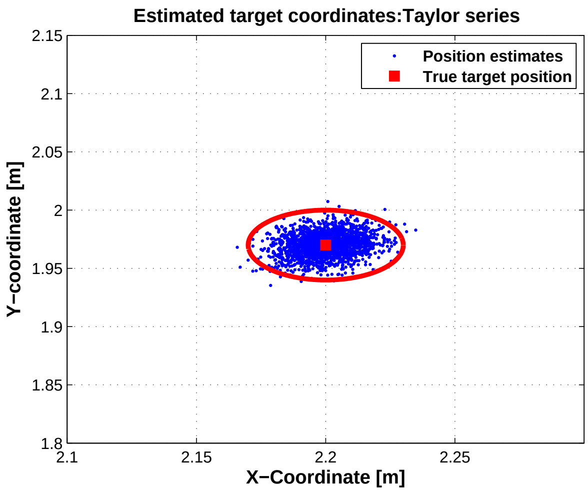
The simulation results of four algorithms are shown in Fig. 4.1b, 4.1c, 4.1d and 4.1e

with TOA range error standard deviation ($\sigma_r = 2cm$) over 2000 measurements. The blue dots are the estimation position of the passive target node in every experiment and the red circle around the true target node position is to compare the accuracy of simulation result of every algorithm with other algorithms.
From Fig. 4.1 we note:

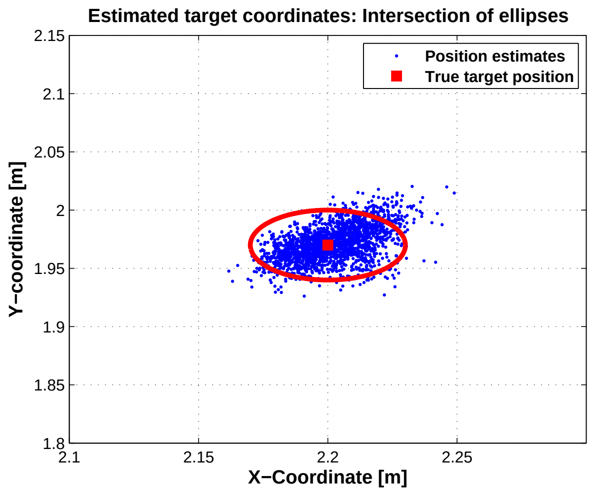
- The first two algorithms, Talyor series method and intersection of ellipses have better positioning performance than other algorithms, where the most of position estimation points are inside the red circle which have radius equal to $r = 3cm$.
- Algorithm.3 and 4 have almost the same positioning performance with higher variation in Y-axis.
- Although, the iterative method is computationally intensive if the initial guess is not close enough to the actual target node position, the iterative algorithm Taylor series has always better positioning performance than algorithm.2; the intersection of ellipses when the initial guess chosen inside the area between the receiving antennas. In this case, the initial guess is $(x_v = 1, y_v = 1)$ with 200 iterations.
- The intersection of ellipses outperform algorithm.3 and 4 and has almost the same positioning performance as Taylor series algorithm, but it also needs prior information about the location of target node to exclude the rest of intersection points between two ellipses.



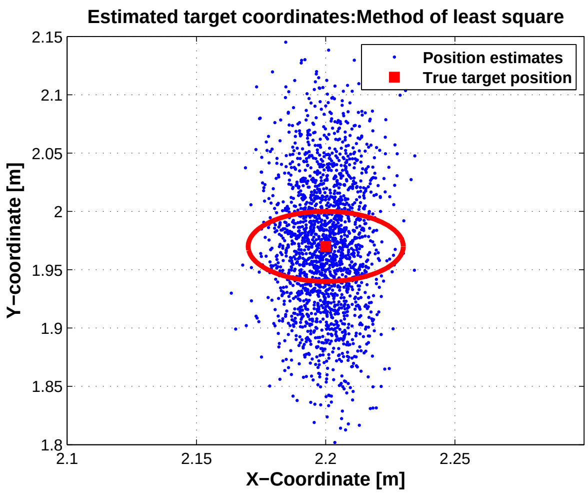
(a) Simulation scenario.1



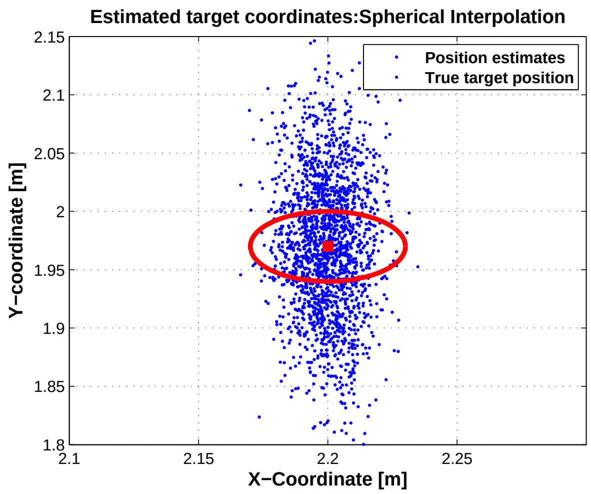
(b) Taylor Series



(c) Intersection of ellipses



(d) Method of least square



(e) Spherical interpolation

Figure 4.1: Position estimation: Scenario.1: $\sigma_r = 2cm$

4.1.1 Precision

For four receiving antennas, the least square estimation of Taylor series is

$$\boldsymbol{\delta} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{D}. \tag{4.1}$$

Where

$$\boldsymbol{\delta} = \begin{bmatrix}\Delta_x & \Delta_y\end{bmatrix}^T, \tag{4.2}$$

is the position estimation errors, and

$$\boldsymbol{D} = \begin{bmatrix}\Delta r_1 & \Delta r_2 & \Delta r_3 & \Delta r_4\end{bmatrix}^T, \tag{4.3}$$

is the range estimation errors.
The covariance matrix which describes the variance of the position estimation is

$$\boldsymbol{C} = cov\{\boldsymbol{\delta}\} = \mathrm{E}\{\boldsymbol{\delta}\boldsymbol{\delta}^T\} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T(\mathrm{E}\{\boldsymbol{D}\boldsymbol{D}^T\})\boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{A})^{-1}. \tag{4.4}$$

Assuming that the range estimation errors are uncorrelated, the range error covariance matrix is

$$cov\{\boldsymbol{D}\} = \mathrm{E}\{\boldsymbol{D}\boldsymbol{D}^T\} = \begin{bmatrix}\sigma_r^2 & 0 & 0 & 0\\ 0 & \sigma_r^2 & 0 & 0\\ 0 & 0 & \sigma_r^2 & 0\\ 0 & 0 & 0 & \sigma_r^2\end{bmatrix} = \boldsymbol{I}\sigma_r^2, \tag{4.5}$$

where $\boldsymbol{I}$ is 4 by 4 identity matrix and σ_r is the range error standard deviation.

The position error covariance matrix

$$\boldsymbol{C} = \mathrm{E}\{\boldsymbol{\delta}\boldsymbol{\delta}^T\} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{I}\sigma_r^2\boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{A})^{-1}. \tag{4.6}$$

$$\boldsymbol{C} = \mathrm{E}\{\boldsymbol{\delta}\boldsymbol{\delta}^T\} = \sigma_r^2(\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{A})^{-1}. \tag{4.7}$$

$$\boldsymbol{C} = \sigma_r^2(\boldsymbol{A}^T\boldsymbol{A})^{-1}. \tag{4.8}$$

The covariance matrix that describes the variance of the position estimation in 2-D-plane can be writing as

$$\boldsymbol{C} = \begin{bmatrix} \sigma_x^2 & \sigma_{x,y} \\ \\ \sigma_{y,x} & \sigma_y^2 \end{bmatrix}, \tag{4.9}$$

where the main diagonal elements σ_x and σ_y represent the standard deviation of the position estimation along X- and Y-axis, and the off diagonal elements $\sigma_{x,y}$ and $\sigma_{y,x}$ describe the correlation between X- and Y coordinates. If both coordinates are uncorrelated then the off diagonal elements $\sigma_{x,y}$ and $\sigma_{y,x}$ are zeros. However, the X- and Y-coordinates are correlated variables.

Figure 4.2 shows the position estimation simulation of a passive target node located at $(x = 2.2; y = 1.97)$, for four algorithms, with the receiving antennas located as in table 4.1, transmitting antenna at $(x_t = 2; y_t = 3.2)$, and range errors standard deviation $\sigma_r = 2cm$.

The standard deviations σ_x and σ_y obtained from covariance matrices of position estimation for all algorithms are depicted as green arrows which are related to $(\pm 3\sigma)$ intervals. Thus, they do not point in the direction of the major and minor axis of ellipses, in which the variances of the position estimates are maximum. This is due to the correlation between position estimates coordinates.

Therefore, to evaluate the precision of positioning estimation, we apply the Principal Component analysis (PCA) of the covariance matrix that gives the orthogonal basis in which the covariance matrix is diagonal .

The the principal component analysis is used to evaluate the performance of certain antenna array for specific scenario. Factorizing the covariance matrix by eigenvalue decomposition

$$\Sigma = \boldsymbol{Q}\Lambda\boldsymbol{Q}^{-1}, \tag{4.10}$$

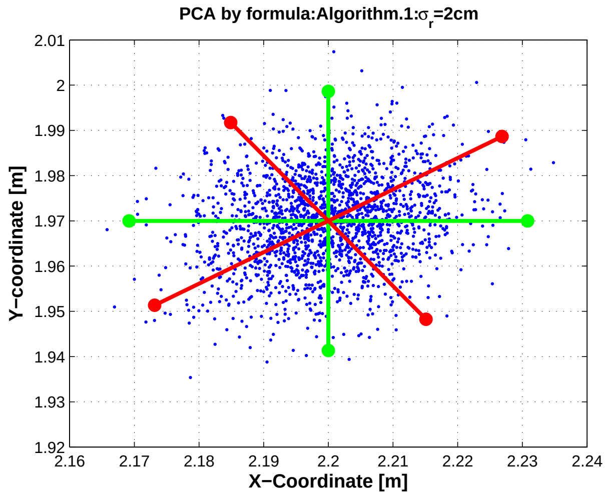
where Λ is a diagonal matrix whose diagonal elements are the corresponding eigenvalues $\lambda_i, i = 1,2$, which represent the successive maximum variances along orthogonal directions given by columns (eigenvectors) of the matrix $\boldsymbol{Q}$.

The red arrows shown in Fig. 4.2 represent the standard deviations $(\sigma_{xr} = \sqrt{\lambda_1},\ \sigma_{yr} = \sqrt{\lambda_2}\ )$ obtained by eigenvalue decomposition of the covariance matrices of the algorithms related to $(\pm 3\sigma)$ intervals, in which in this case the major and minor axis of the ellipses determined by by the intervals ($\pm 3\sqrt{\lambda_i}$) contour the region with 99.7% of position estimates.

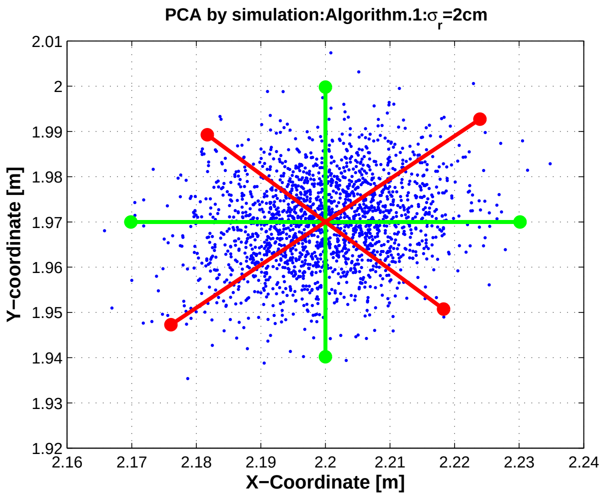
Figure. 4.2a shows the principal component analysis of Taylor series algorithm covariance matrix obtained by equation (4.8), i.e. without simulation results.

Figures. (4.2b), (4.2c), (4.2d),and (4.2e) are the principal component analysis of the covariance matrices obtained from position estimation results. we note that; for Taylor series the PCA obtained by equation formula and simulation results are almost the same. For the other algorithms, the PCA for algorithm.3 and 4 have the same positioning performance with higher variations in Y-axis and less correlation between X and Y-coordinates compared to algorithm.1 and 2.

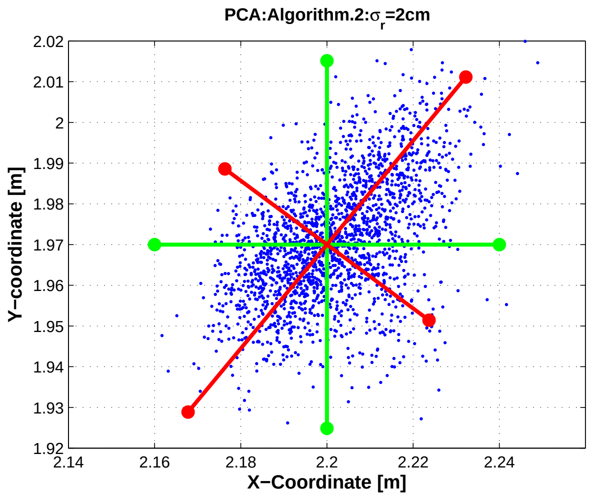
Figures 4.3 and 4.4 show an example of principal component analysis of the covariance matrices of TOA algorithms at different target position in 2-D plane $(x = 0.2 : 5m)$, $(y = 0 : 3.5m)$ for 100 target position. Red stars illustrate the position of the receiving antennas and transmitting antenna. Blue arrows show the successive maximum standard deviations at every target position, which are related to $(\pm 3\sigma)$ of major and minor axis of ellipses. From Fig. 4.4a and 4.4b, its clear that algorithm.3 and algorithm.4 have almost the same positioning performance in the specified area , in which the standard deviations have the same orientation in all estimated target positions. This is not the case for algorithm.1 and 2 , where they have different positioning performance at different target positions. Algorithm.2 has very high variations and poor positioning performance in the upper part of as shown in Fig 4.3b, where algorithm.1 has poor positioning performance in the upper left corner of the simulation area as shown in Fig 4.3a.



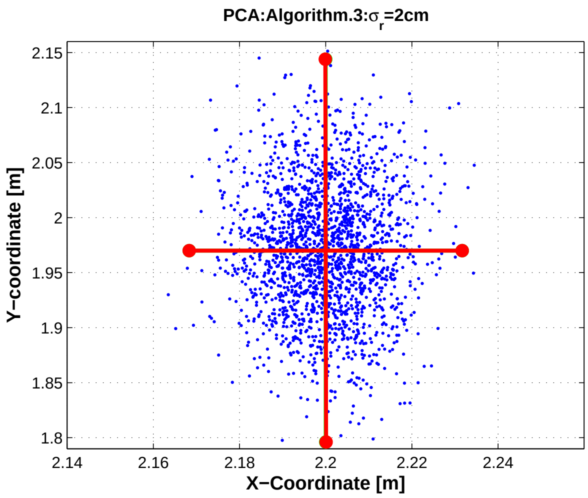
(a) PCA by formula: Taylor Series



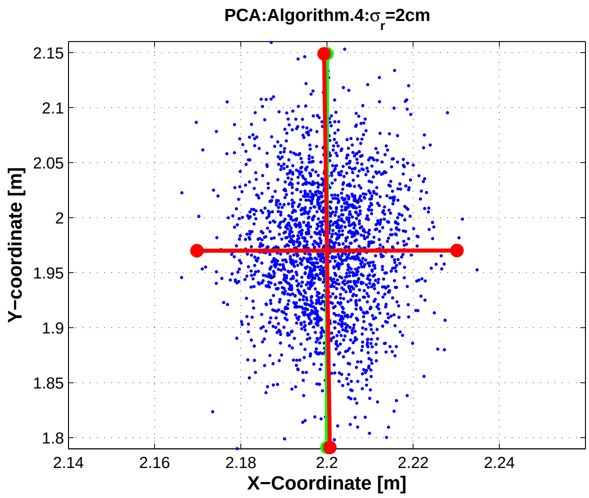
(b) PCA by simulation:Taylor Series



(c) PCA by simulation:Intersection of Ellipses

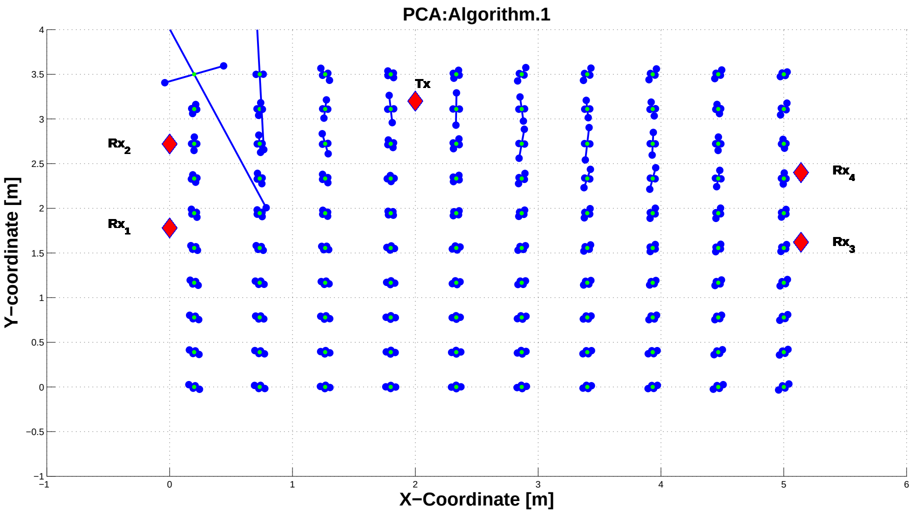


(d) PCA by simulation:Least Square Method

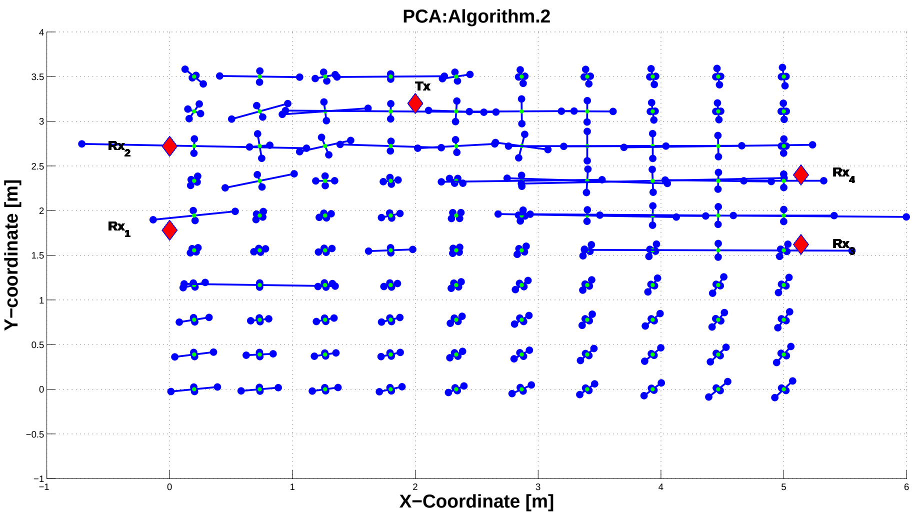


(e) PCA by simulation:Spherical Interpolation

Figure 4.2: Scenario.1: PCA for one target position, $\sigma_r = 2cm$

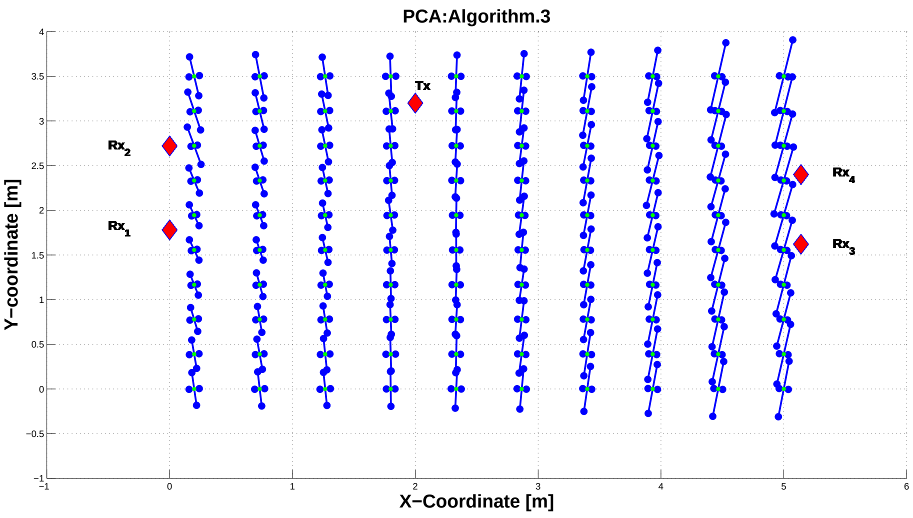


(a) Taylor Series

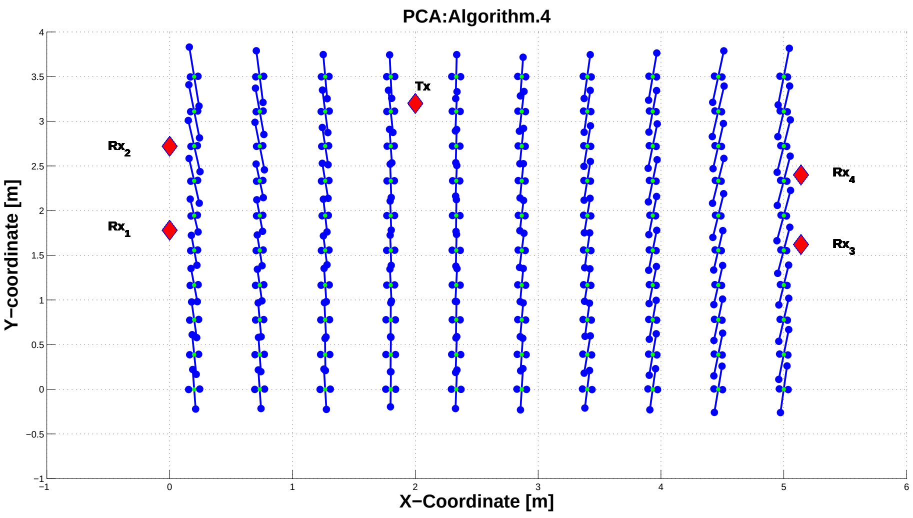


(b) Intersection of ellipses

Figure 4.3: Scenario.1: PCA for different target positions



(a) Method of least square



(b) Spherical interpolation

Figure 4.4: Scenario.1: PCA for different target positions

4.2 Scenario.2

In this case, we assume a different scenario, where we have four nodes and every node comprises of one transmitting antenna and two receiving antennas as shown in table 4.2. The target node is located at the middle of the nodes with $(x = 6, y = 6)$ coordinates. The passive target position estimation then can be done either by TOA's estimated for every node separately, or by TOA's estimated from combining two or more nodes for one chosen transmitting antenna.

		x(m)	y(m)			x(m)	y(m)
N_1	Rx_1	0	4	N_3	Rx_5	12	4
	Tx_1	0	6		Tx_3	12	6
	Rx_2	0	8		Rx_6	12	8
N_2	Rx_3	4	0	N_4	Rx_7	4	12
	Tx_2	6	0		Tx_4	6	12
	Rx_4	8	0		Rx_8	8	12

Table 4.2: Scenario.2. Antennas coordinates

Figure 4.5a shows simulation scenario and four nodes position. The green arrowheads illustrate the receiving antennas, red arrowheads the transmitting antennas and blue dot the true target position. As in the first scenario, the position estimation is performed with TOA range error standard deviation ($\sigma_r = 2cm$) over 2000 experiments. For the intersection of ellipses algorithm, the position estimation is performed for every node,i.e by measuring TOA ranges between the transmitting antenna and the two receiving antennas in every node, then the arithmetic mean is computed for these estimated positions. For algorithm.3 and 4, the target position estimated by measuring the TOA range between at least two node and one transmitting antenna, as for least squares method at least three receiving antennas are needed to estimate the position of the target node.

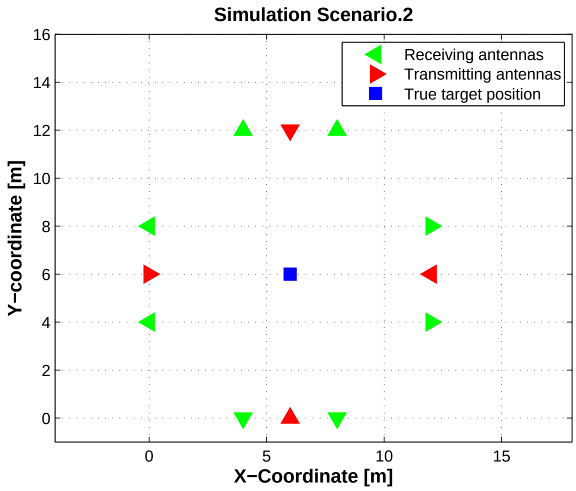
From Fig. 4.5, it is easy to note that algorithm.1; Taylor series method outperforms other algorithms as in the first scenario, where the most of estimation positions inside the red circle. Algorithms 3 and 4 still have the same positioning performance with higher variations in Y- axis.

4.2.1 Precision

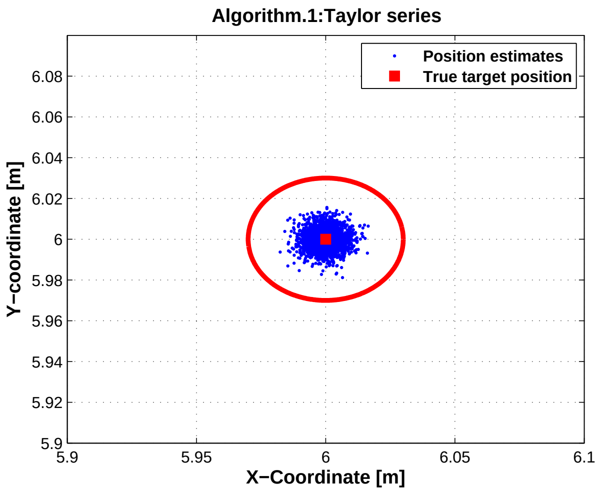
To have deeper insight into the positioning performance of TOA algorithms for the second scenario, principal component analysis of covariance matrices for for all algorithms are applied as shown in Fig. 4.6. The standard deviations σ_x and σ_y obtained from covariance matrices of position estimation for all algorithms are depicted as green arrows related to ($\pm 3\sigma$) intervals. The red arrows represent the standard deviations ($\sigma_{xr} = \sqrt{\lambda_1}$, $\sigma_{yr} = \sqrt{\lambda_2}$) obtained by eigenvalue decomposition of the covariance matrices of the algorithms, in which in this case the major and minor axis of the ellipses determined by the intervals ($\pm 3\sqrt{\lambda_i}$) contour the region with 99.7% of position estimates.

Figure 4.6a shows the PCA for Taylor series algorithm obtained by equation formula (4.8). Compared with PCA of the Taylor series covariance matrix obtained from simulation results in Fig. 4.6b, we note that there is a difference between the orientation of standard deviations obtained by eigenvalue decomposition in both cases. Algorithm.3 and 4 still have very high variations in Y-axis compared with other algorithms. But, unlike the first scenario, there is a high correlation between X and Y-coordinates as shown in Fig. 4.6d and 4.6e. For intersection of ellipses algorithm it is clear from Fig. 4.6c that in this scenario there is no correlation between X and Y- coordinates.

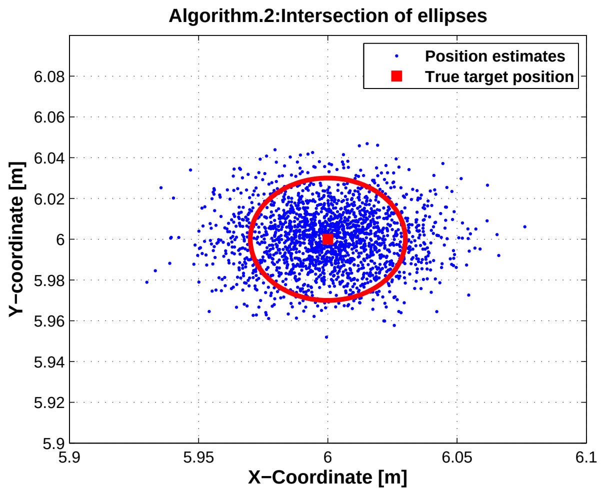
The principal component analysis of the covariance matrices of TOA algorithms at different target position are shown in Fig. 4.7 and 4.8 . Algorithm.3 and algorithm.4 have almost the same positioning performance in the specified area , in which the standard deviations have the same orientation in all estimated target positions, as in the first scenario. Algorithm.1 and 2 have different positioning performance at different target positions.



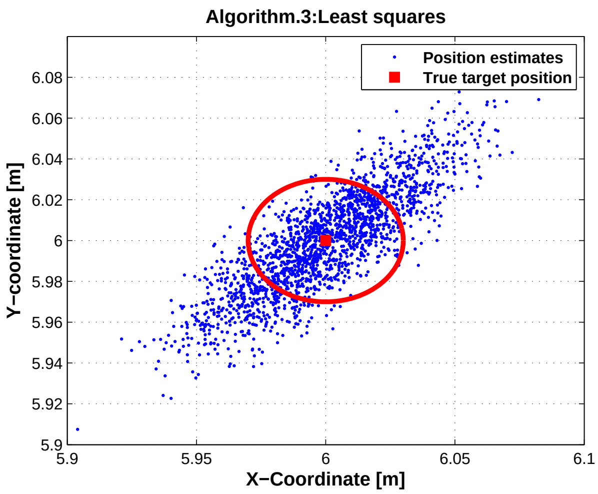
(a) Scenario.2



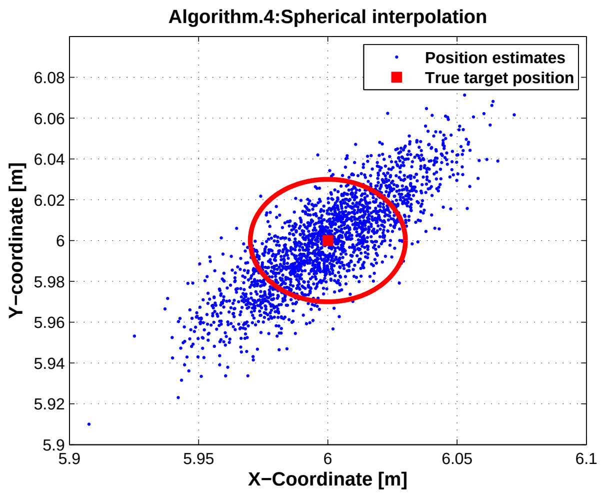
(b) Taylor Series



(c) Intersection of ellipses

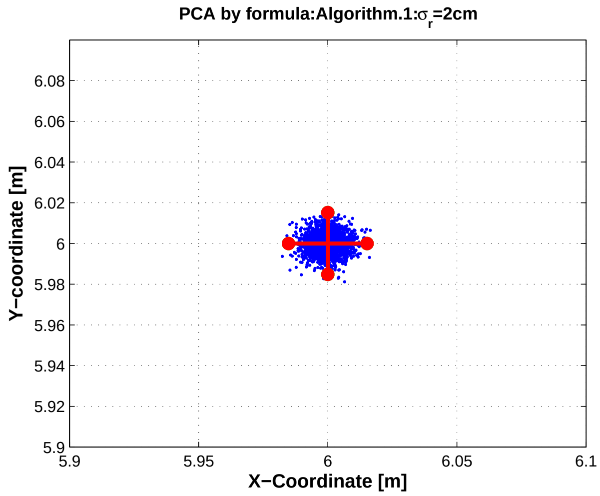


(d) Method of least square

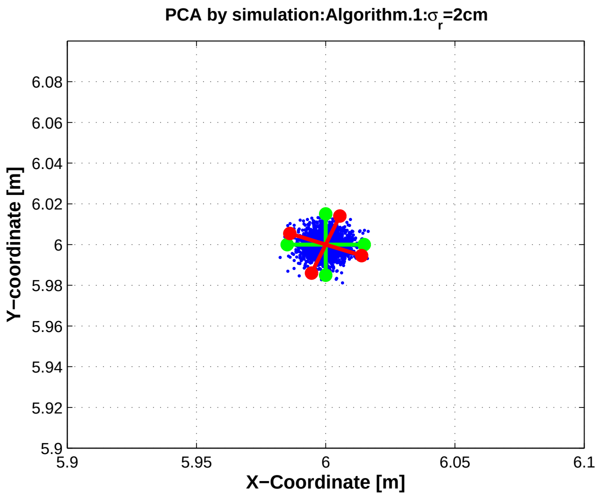


(e) Spherical interpolation

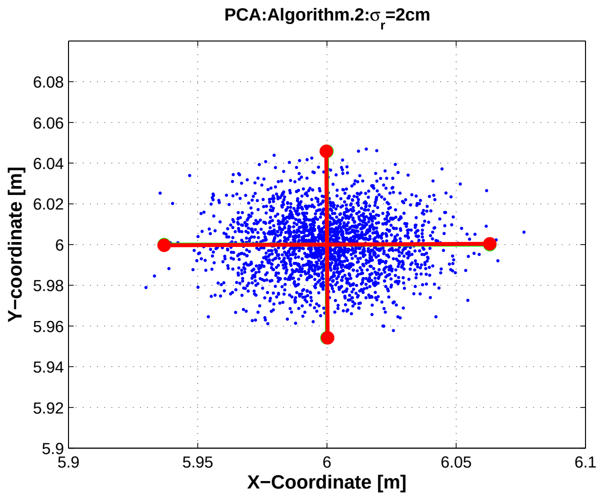
Figure 4.5: Position estimation: Scenario.2: $\sigma_r = 2cm$



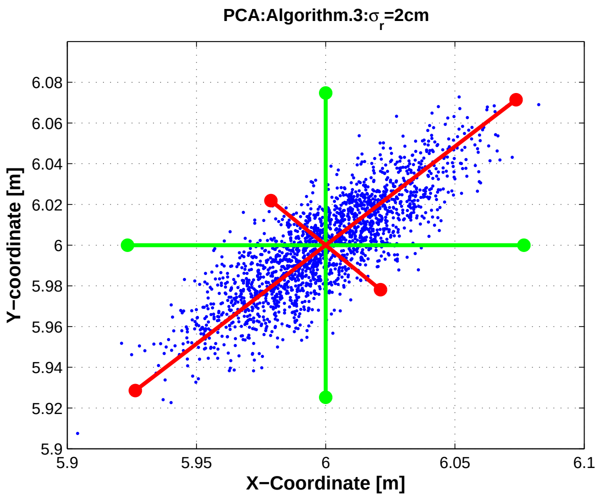
(a) PCA by formula:Taylor Series



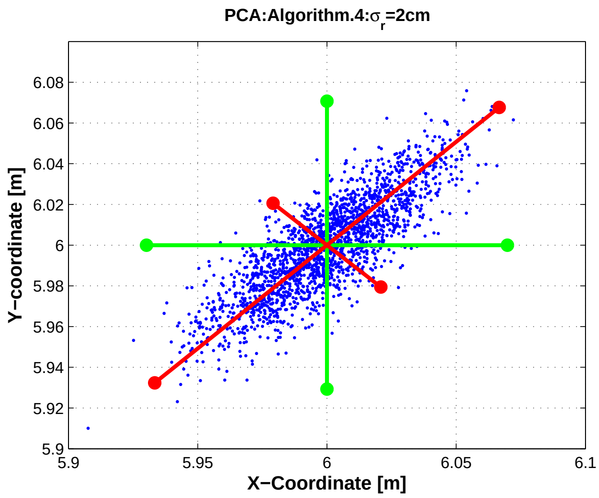
(b) PCA by simulation:Taylor Series



(c) PCA by simulation:Intersection of Ellipses

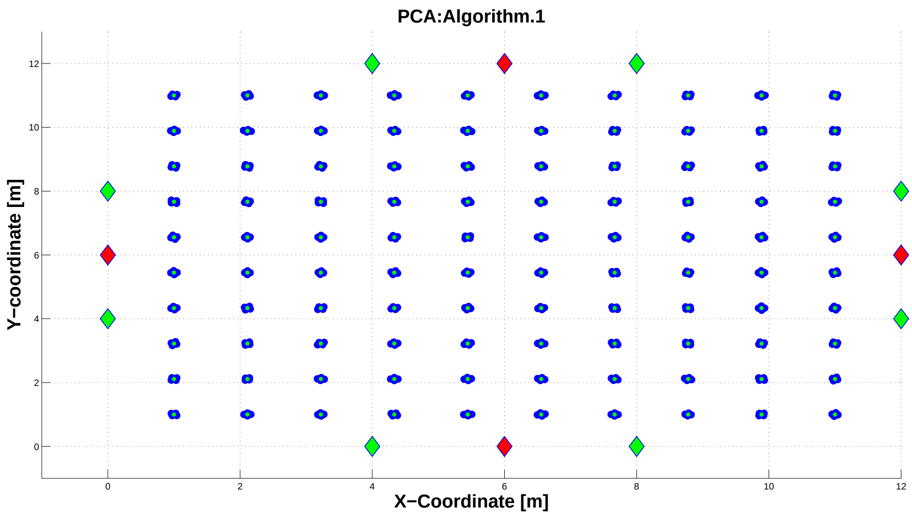


(d) PCA by simulation:Least Square Method

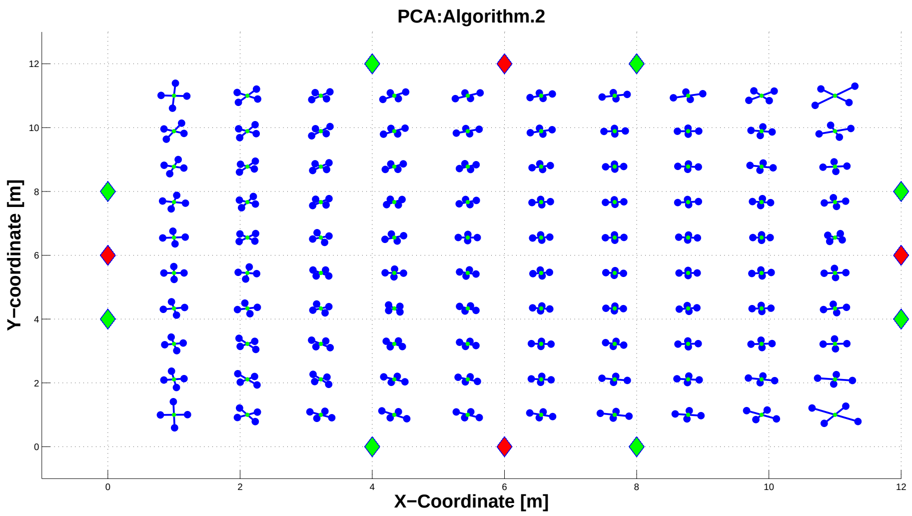


(e) PCA by simulation:Spherical Interpolation

Figure 4.6: Scenario.2: PCA for one target position, $\sigma_r = 2cm$

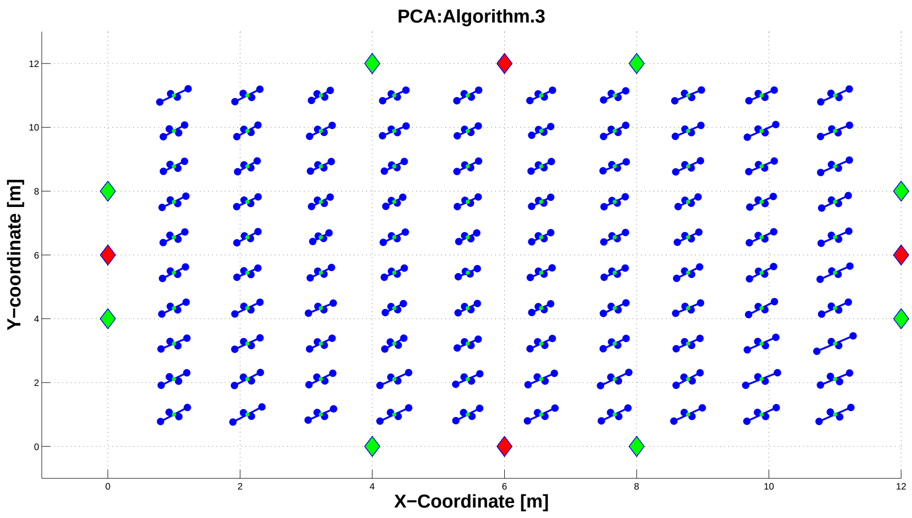


(a) Taylor Series

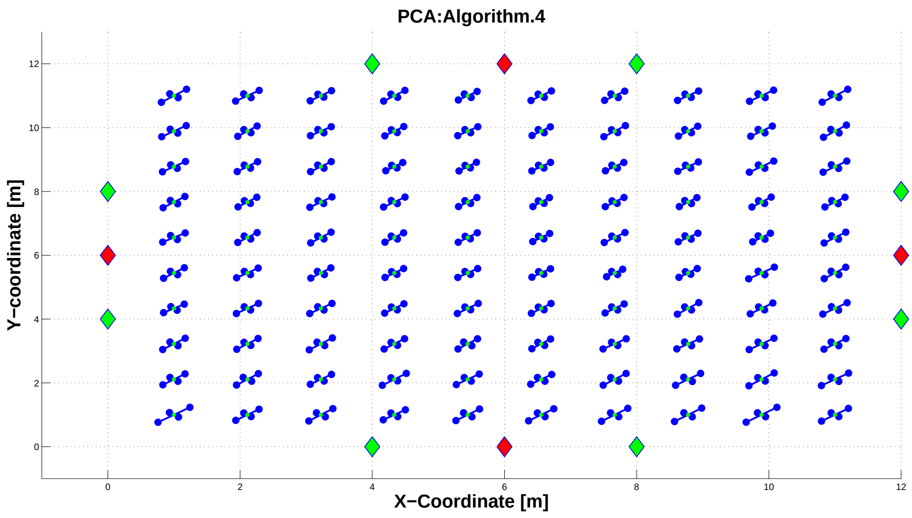


(b) Intersection of ellipses

Figure 4.7: Scenario.2: PCA for different target positions



(a) Method of least square



(b) Spherical interpolation

Figure 4.8: Scenario.2: PCA for different target positions

5

Conclusions and Future Work

5.1 Conclusions

Many localization algorithms have been developed and used to find the position of a passive target objects. These algorithms use different localization approaches.
In chapter 2 some of the localization approaches have been presented. Receiving signal strength (RSS), angel of arrival (AOA) and time of arrival (TOA) approaches have been illustrated, Their Passive target object positioning approaches counterpart are of concern in this theses.
Time of arrival (TOA) range based passive approach is considered to have very high accuracy using UWB technology due to the UWB large bandwidth, high time resolution and low power consumption. In TOA approach, a number of reference nodes carrying receiving antennas measure the time of arrival of signals propagated from at least one reference node carrying transmitting antenna and reflected by the target object that need to be localized. The advantage of passive TOA approach is that the receiving nodes can be synchronized with the transmitting node by acquiring the signal departure time traveling between them.
The estimated TOA ranges define set of nonlinear equations whose solution gives the estimated coordinates of the target node. The TOA algorithm will be responsible for producing an accurate and unambiguous solution to these set of nonlinear equations. Many processing algorithms, with different complexity and restrictions, have been proposed for position location estimation based on TOA approach.
In this thesis, four TOA range based location estimation algorithms are studied in 2-D plane. These TOA algorithms solve the set of nonlinear equations resulting from TOA range measurements in different approaches and utilize least square criteria to solve for the target node position. The mathematical formulas of the algorithms are presented in

chapter 3. Algorithm.1; Taylor series linearizion uses the iterative method and differs from other algorithms that it needs an initial guess a close to the true target node position to guarantee convergence and can be computationally intensive. The other algorithms can be classified under non-iterative methods. Algorithm.2 utilize an intersection between two ellipses that are constituted from two TOA estimated ranges using B*é*zout determinant procedure to find the intersection points. It is also computationally intensive for more than two receiving antennas as there are at least two intersection points between every possible pair of receiving antennas and prior information is needed to remove the ambiguity. Algorithm.3 needs at least three receiving antennas and three TOA ranges measurements to transform the set of non-linear equations into linear equations and solve them by least squares method. The fourth algorithm needs also at least three TOA range measurements to estimate the position of the passive target node by spherical interpolation method which is rely on Pythagoras's theorem. The idea behind the spherical interpolation method is to find a circle that passes the the transmitting antenna and its center is the estimated target node position.
To evaluate and compare the positioning performance of TOA algorithms, two different simulation scenarios in 2-D plane have been suggested in chapter 4. The first scenario is one transmitting antenna and four receiving antennas, in the second scenario, four reference nodes are suggested, with one transmitting antenna and two receiving antennas in every reference node. The position of the passive target node located at the middle of the receiving antennas is estimated by TOA algorithms. The resulting TOA range measurements are corrupted by Gaussian noise with zero mean and a standard deviation of $2cm$. The simulation results in two scenarios have shown that the Taylor series algorithm have the best positioning performance. Algorithm 2 also outperforms algorithms 3 and 4 that have the same positioning performance. Also, Algorithms 1 and 2 are less expensive in terms of number of receiving antennas needed to estimate the position of a target node, as for one transmitting antenna , they need only two receiving antennas.
Principal component analysis (PCA) of the position estimates covariance matrix is implemented to analyzes the performance of TOA algorithms by using an orthogonal transformation to convert the set of correlated target position estimates into a set of values of linearly uncorrelated position estimates.
In conclusion, algorithm.1 (Taylor series linearizion)) and algorithm .2 (intersection of ellipses) have shown better location estimation performance and accuracy in noisy measurements than algorithms 3 and 4. The drawbacks, is that both algorithms (1,2) need prior knowledge about the location of the target node and are computationally more intensive.

5.2 Future Work

The work done in this thesis can be extended in different ways. In the simulations we have assumed a 2-D space simulation scenarios. Investigation can be pursed to see the performance of TOA algorithms in 3-D space. Another way of progress would be to test the performance of these algorithms using time difference of arrival (TDOA) range measurements between the receiving antennas . In this case the synchronization between the transmitting antenna and receiving antennas is not needed.

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