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THE REDEFINITION OF THE AMPERE

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ABSTRACT

The ampere is one of the seven base units of the SI, the international system of units. Its definition is linked to mechanical units, especially the unit of mass, the kilogram. In a future system of units, which will be based on the values of fundamental constants, the ampere will be based on the value of the elementary charge *e*. This paper describes the technical background of the redefinition.

Index Terms – SI units, Josephson effect, quantum Hall effect, single electron effect

1. DEFINITION OF THE AMPERE IN THE SI

The ampere, the unit of electric current, is the SI base unit for electricity and magnetism. Until the middle of the 20th century, the ampere was defined on the basis of electrolytic processes. The currently valid definition is based on electrodynamics, and reads: *The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2 × 10^{-7} newton per metre of length.* Figure 1 illustrates the definition of the base unit "ampere".

This definition exploits the fact that the electrodynamic force F per conductor length l between two straight parallel conductors which are placed at the distance r and through which a current I flows, is $F/l = \mu_0 \cdot (I^2/2\pi r)$ in vacuum. The numerical value of the magnetic constant μ_0 is fixed through the definition of the ampere to $\mu_0 = 4\pi \cdot 10^{-7}$ N/A²

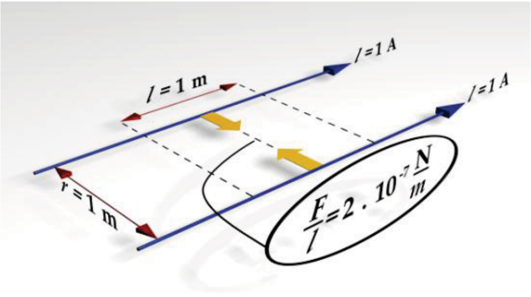

Figure 1 Schematical representation of the ampere definition

Since the numerical value of the speed of light c has been fixed through the definition of the meter, also the numerical value of the electric constant ε_0 is fixed according to the Maxwell relation $\mu_0 \cdot \varepsilon_0 \cdot c^2 = 1$.

The practical implementation of this definition – which, in metrology, is called *realisation* – must be performed with the aid of measuring arrangements that can be realised experimentally. One distinguishes between *direct realisation* – in which the current is related to a mechanical force – and *indirect realisation*. The latter is based on Ohm's law $I = U/R$ and the ampere is realised through the realisation of the electric voltage U and the electric resistance R. The fixing of the numerical value of the electric constant ε_0, which follows from the definition of the ampere, ensures the link between the indirect realisation of the ampere and its definition.

This – still valid – definition of the ampere is based on mechanical quantities. A completely different approach to define the ampere and the electric units is the use of quantum effects in solids. With the aid the Josephson effect [1], the voltage U, and with the aid of the quantum Hall effect [2] the resistance R can be traced back to the elementary charge e and Planck's constant h. Thanks to these effects, the volt, the ohm and, using Ohm's law, also the ampere can be reproduced with highest accuracy.

The new definition of the ampere – which has been suggested by the bodies of the Metre Convention – is based on the elementary charge e. The new definition and the state of the art of research on its practical implementation are described in Section 4. Section 5 deals with consistency tests aiming to ensure the implementation of the new ampere definition. The article closes with an outlook.

2. REALISATION OF THE AMPERE IN THE SI

2.1. Direct realisation

The direct realisation of the ampere, based on the definition, is only of historical interest today. The "conductors of infinite length" are replaced by conductors of a coil. The geometrical difference can be exactly calculated. The force between the coils carrying a current I is measured by balancing it with a gravitational force. The fundamental problem of such an experiment is that the current distribution in the finite diameter wires is not known precisely enough. Therefore only relative uncertainties in the range of a few 10^{-6} were achieved [3] with such a current balance.

2.2. Indirect realization

The indirect realisation of the ampere can be performed with a lower uncertainty than the direct realisation. As mentioned above, the defining equation of an electric resistance, $R = U/I$, is used to realise the ampere via the unit "ohm" and the unit "volt".

The crucial element making it possible to use ε_0 instead of μ_0 is an electrostatic theorem discovered in 1956 [4]. It allows to build capacitors whose capacitance C can be exactly determined from one single length measurement. According to this theorem, a length change Δl results in a capacitance change $\Delta C = \varepsilon_0 \Delta l(\ln 2/\pi)$ for a capacitor of special geometry. Δl is measured interferometrically. With such a "calculable cross capacitor" it is, therefore, possible to realise the unit farad directly. By relating the reactance $1/\omega C$ to an effective resistance R, the resistance unit Ω is finally realised.

For the indirect realisation of the ampere, also the realisation of the unit volt is needed. Similar to the comparison of electrodynamic forces in the "current balance," a "voltage balance" is used to determine an electrostatic force between energised capacitor electrodes. From the relation $(1/2)\Delta CU^2 = mg\Delta z$ (where Δz is a displacement of the two electrodes) the unit volt can be realised, using again the capacitance value derived above.

Such a balance achieved a relative overall uncertainty of 3×10^{-7} for the realisation of the volt [5]. With the indirect method, the ampere can be realised with a similar uncertainty.

Although the indirect realisation of the ampere is the most accurate within the SI, it does in practice not play a role any more since more than 20 years since nowadays electrical units can be traced to fundamental constants with significantly lower uncertainties.

3. REPRODUCTION OF ELECTRIC UNITS

In all sectors of metrology, efforts have been focused on tracing back the units to fundamental constants. Quantum effects ensure that physical quantities – unlike their classical behaviour – take on only certain discrete values. The effects which are relevant for the electric units relate the units volt (via the Josephson effect) and ohm (via the quantum Hall effect) to combinations of the elementary charge e and Planck's constant h.

As in the indirect *realisation* described above, which uses the units ohm and volt, the ampere can also be reproduced indirectly by tracing it back to the h- and e-based units ohm and volt by measuring the resistance and the voltage.

3.1. Josephson effect and the unit "volt"

The Josephson effect [1] occurs in superconductors connected via a very thin insulator. If a DC current I flows through such an arrangement irradiated by microwaves of frequency f, steps of constant voltage appear at values $U_\mathrm{n} = n \cdot K_\mathrm{J}^{-1} \cdot f\,(n = 1, 2, ...)$. K_J is called *Josephson constant* and is, in the underlying theory, given by $K_\mathrm{J} = 2e/h$.

Under typical conditions, voltages of a few 100 µV can be generated with a single Josephson junction, but with series arrays of many junctions, the voltage can be considerably increased. The development of the required, elaborate technology was decisively advanced by PTB. As one of very few national metrology institutes worldwide, PTB is able to manufacture Josephson series arrays which allow to reproduce voltages of up to 15 volt with a relative uncertainty of approx. 10^{-9} [6]. Josephson arrays manufactured with PTB's technology are now being used at numerous national metrology institutes as well as at the BIPM, the Bureau International des Poids et Mesures.

3.2. Quantum Hall effect and the unit "ohm"

Using two-dimensional, i.e. extremely thin conductive layers, *Klaus v. Klitzing* [2] in 1980 discovered a new effect which can be used to realise quantised resistance values. Typically, GaAs-GaAlAs layer crystals are used to provide the 2D electron systems for metrological purposes.

At liquid helium temperatures, a current I is applied to the quantum Hall devices, subject to a strong magnetic field. The Hall resistance R_H (the quotient of Hall voltage and current), exhibits steps of constant resistance with values $R_\mathrm{H} = R_\mathrm{K}/i$ $(i = 1, 2, ...)$. R_K is called the *von Klitzing constant* and is, in the underlying theory, given by $R_\mathrm{K} = h/e^2$. Its value is approx. 26 kΩ. The corresponding resistance values can be reproduced with a relative uncertainty of a few parts in 10^9 [7].

3.3. International conventions

To achieve international uniformity in the dissemination of the electric units volt, ohm, and ampere within the small uncertainties made possible by the Josephson and quantum Hall effect, the International Committee for Weights and Measures, in October 1988, fixed exact values for the *Josephson constant* K_J and for the *von Klitzing constant* R_K and recommended them for the maintenance and dissemination of the units [8]. These recommended values are $K_{J-90} = 483\ 597.9 \times 10^9\ V^{-1}s^{-1}$ and $R_{K-90} = 25\ 812.807\ \Omega$. They were defined such that – according to the most precise measurement results valid at the time – they corresponded to the SI numerical values. Since January 1, 1990, K_{J-90} and R_{K-90} have been used worldwide for the dissemination of the units volt and ohm. Strictly speaking, however, since 1990, the electric units have no longer been SI units but units in a special system based on fundamental constants. The envisaged redefinition of the whole system of units will solve this rather unsatisfactory situation.

4. REDEFINITION OF THE AMPERE

The fact that electric charge is quantised in the form of electrons suggests a definition which takes this into account. Correspondingly, the committees of the Metre Convention advocate a new definition in which the elementary charge e is attributed a fixed numerical value in the unit "ampere times second".

The endorsement is made in a recommendation of the Consultative Committee for Electricity and Magnetism (CCEM) [9] in which different possibilities for the realisation (*"mise en pratique"*) are presented. One possibility of realising the ampere according to the new definition is based on a quantum current source which generates a current corresponding to the relation $I = e{\cdot}f$. As an alternative, it is possible to realise a quantum *ampere meter* which exploits the Josephson and the quantum Hall effects to measure a current I.

The technical fundamentals for the practical implementation of the new definition of the ampere are detailed in the following sections.

4.1. Single-electron current sources

Since the end of the 1980s, it has been possible to manipulate single electrons in small electrical circuits. The basis for this is single-electron tunnelling (SET): metallic or semi-conducting islands, smaller than a µm, and separated by thin insulating barriers, have a very low electric capacitance C. At temperatures in the millikelvin range, quantum-mechanic tunneling through the barriers between the islands is therefore energetically suppressed. This effect is called "coulomb blockade". If, however, the potential energy of the islands is lowered using appropriate gate electrodes, tunneling – and thus the transport of the electrons across the island – becomes possible again. By sequentially triggering, in a chain of such islands, the islands with voltage pulses repeated at a frequency f, it is possible to generate a current $I = e{\cdot}f$ [10]. In principle, such a single-electron pump is precisely that component which allows implementing the new definition of the ampere.

Despite the low error rates of the single-electron pumps, their use is limited to special cases, since only currents of a few pA can be realised precisely. To generate higher currents in the nA range with small error rates, alternative approaches to control the transport of single charges have been developed worldwide, and especially at PTB,.

The most promising approach at the moment for a single-electron device providing currents in the nA range is based on semiconductors. With semiconductors, the barriers are not fixed, but defined by the control gates, and the height of the barriers can be adjusted electrically. In such a device a narrow conducting channel is crossed by electrodes which, when biased, form the barrier for the electrons. With suitably chosen DC voltages at the electrodes, and with an additional AC voltage of frequency f at one electrode, a current $e{\cdot}f$ is generated also in this type of pump. The frequency f and, thus, the current can be considerably higher than in the pumps described above. At a frequency of 1 GHz, a current of 0.16 nA was achieved whose uncertainty was estimated by theoretical modeling to 1 part in 10^7 [11].

In order to ensure that single-electron current sources achieve a reproducibility which is comparable to that of Josephson and quantum Hall elements, a new concept will be employed. This concept implies that in more complex circuits the error rate of the device is monitored during operation at the level of single electrons. The technical possibility for this is also provided by the single-electron tunnelling effect: just as the transport of an electron onto an island can be controlled by applying an electric voltage, the presence of an additional electron on an island causes a measurable change in the transport properties of the device. A concept of such a self-referencing single-electron pump was elaborated at PTB [12][12], where it is presently being implemented.

4.2. The quantum ampere meter

According to one of the suggested *mise-en-pratique* variants, the ampere can be determined in the future SI system also in such a way that – by comparison with the Josephson effect – the voltage drop U along a well-known resistance R carrying a current I is measured. If for R, e.g., a quantum Hall resistance operated at the level $i = 2$ is chosen, then the following applies (for the Josephson voltage level $n = 1$): $I = U/R = K_J^{-1}{\cdot}f/(R_K/2) = 2{\cdot}(K_JR_K)^{-1}{\cdot}f$. If the relations $K_J = 2e/h$ and $R_K = h/e^2$ are exactly valid, this results in the same relation $I = e{\cdot}f$ as in the case of the quantum current source.

For currents in the microampere range and larger, such a measurement can be performed either directly or via a resistance calibrated against the quantum Hall effect. It is, however, important to be able to measure also small currents with low uncertainty. The high resistance values required for this purpose lead to an increased measurement uncertainty, so that another method must be used: The low current is, in a first step, amplified very precisely by means of a so-called cryogenic current comparator (CCC). In the case of a CCC, a small current generates, in a coil with a high number of turns N, a magnetic flux, which is compensated by a flux due to a current exactly N-times as high and flowing in a second coil of only one turn. A superconducting quantum interferometer (SQUID) is used as a sensitive flux null-detector and controls a feedback loop which exactly adjusts the higher current. A quantum ampere meter is composed of a current-amplifying CCC, a temperature stabilized resistance which has been precisely calibrated against a quantum Hall resistance, and a Josephson voltmeter. At present, a quantum ampere meter is being set up at PTB, by which small currents from arbitrary sources can be measured.

5. THE QUANTUM-METROLOGICAL TRIANGLE

The uncertainty of a quantum ampere meter as described above depends on the accuracy of the relations $K_J = 2e/h$ and $R_K = h/e^2$. This is a fundamental issue for the realisation and the dissemination of the electric units in a new, quantum-based, international system of units. The excellent reproducibility of the Josephson [13] and of the quantum Hall effect [7] in various solid state systems only partly contributes to clarify this issue, since this still does not rule out that all experimental realisations could suffer from the same deviation from $K_J = 2e/h$ or $R_K = h/e^2$. A direct experimental check is necessary. In order to substantially improve the present state of knowledge, the uncertainty has to be in the order of 10^{-8} [14]. In the SI as it is now, such an uncertainty cannot be achieved by measuring Josephson voltages or the quantum Hall resistances and by comparison with the SI values of e and h. Therefore, an experimental consistency test was suggested already back in 1985 [15]. It is known as the "quantum-metrological triangle".

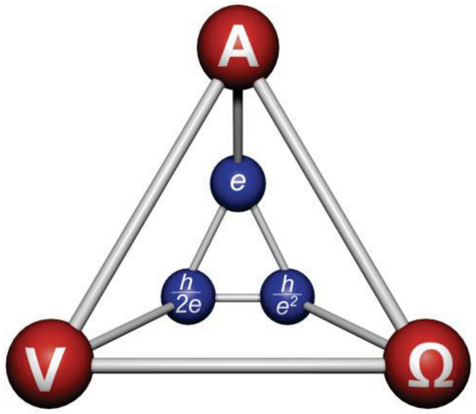

Fig. 2 Schematical representation of the quantum-metrological triangle

Figure 2 is a schematical representation of the so-called quantum-metrological triangle. In the experiment, the voltage drop caused by a single-electron current passing through a quantum Hall resistance is compared with the Josephson voltage which is expected when the relations $K_J = 2e/h$ and $R_K = h/e^2$ are absolutely correct. From this comparison, upper limits can be determined for possible deviations of K_J from $2e/h$, and of R_K from h/e^2 as well as of the quantized charge transported by the single-electron current source from the elementary charge e.

To achieve a relative uncertainty in the order of 10^{-8}, the single-electron current source must supply currents in the order of 1 nA. Furthermore, a CCC must be used – as in the case of the quantum ampere meter. Both set-ups – the direct quantum-metrological triangle and the quantum ampere meter – look rather similar at first sight. There is, however, a fundamental difference between the two: in the case of the quantum ampere meter set-up, the current source is the measurement object whose unknown current is determined, whereas in the case of the direct quantum-metrological triangle, the properties of the single-electron current source must be exactly known. Especially the error rate of the single-electron current source must be determined prior to the experiment by means of independent methods – such as the single-electron detection method described above.

An alternative to this experiment is the co-called "*Indirect Quantum-metrological Triangle*". For this experiment, single-electron current sources can be used which generate currents in the pA range. Using these sources, a capacitor of a capacitance C of approx. 1 pF is charged with a defined charge Q of n electrons. As described in Section 2.2, the capacitance is traceable to the von Klitzing constant R_K by means of an AC bridge. The voltage U across the capacitor which is generated by the n electrons is measured by a Josephson voltmeter. In accordance with the relation $Q = C \cdot U$, the elementary charge e, $R_K = h/e^2$ and $K_J = 2e/h$ can be related to each other. The experiment was performed at NIST, the National Metrology Institute of the USA [16]. The upper limit for a possible deviation of K_J, R_K and e from the expected values could be determined to be 9×10^{-7}

[17]. PTB is working on the verification and improvement of this result [18].

SUMMARY AND OUTLOOK

The history of the ampere spans a wide range of disciplines, from electrochemistry to electrodynamics, and further on to modern solid-state physics. This will eventually lead to abandon the derivation of the ampere from the kilogram. In the new definition, the ampere will be defined on the basis of the elementary charge. This has been made possible by the discovery of macroscopic quantum effects in solids which create a bridge between the electric quantities and the fundamental constants of physics. The development also has an impact on other fields of metrology. The planned redefinition of the kilogram by defining the numerical value of Planck's constant h is motivated by the watt balance [19], an advancement of the ampere balance described in Section 2.1, which is based on the Josephson effect and the quantum Hall effect.

The use of the single-electron transport to realise the quantum ampere is not established yet, but it is a highly topical research subject in the field of electric quantum metrology. The scientific issues in this field are closely related to those of quantum dynamics und quantum data processing in solids, as well as to nano- and superconductor electronics. It is expected that metrological research on the quantum ampere will not only contribute significantly to the new international system of units, but also to the other above-mentioned fields.

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