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PARAMETER ESTIMATION OF MIXTURE DISTRIBUTIONS USING EVOLUTIONARY MODELING FOR EVALUATION OF OPTOMECHATRONIC SYSTEMS

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ABSTRACT

The proposed approach was employed to estimate parameters of mixtures of exponential and Weibull distributions on generated and experimental statistical data. The intervals of mixture's parameters for the generation of an initial population have been chosen according to the method of moments. As target (fitness)-functions the maximum of the absolute value of difference between empirical and theoretical distribution functions (MAE), maximum likelihood (MLE) function and least squares function (LSM) have been chosen.

The comparative analysis of selection, crossover and mutation operators' influence on the algorithm's operating efficiency has been executed. The scheme of a local search embedded into the population algorithm is proposed.

Index Terms - Genetic algorithm, mixture-distribution, exponential distribution, Weibull distribution, MAE, Maximum Likelihood Estimation, Least Squares Method, Method of Moments, LED.

1. INTRODUCTION

Optomechatronical systems consist of mechanical, electronic and software components. Methods to evaluate the reliability of these systems have to take into account different failures mechanisms, which can be difficult to detect or identify.

To enhance the statistical conclusion validity for such experimental data processing, finite mixture-distributions [3,7,15,23,24] and spline-distributions [19] can be used. The mixture of distributions has been established recently in a lot of statistical research papers and its applications are very common in reliability studies.

There are a number of mixture distributions that have been studied. One of the earliest studies in which an analysis of a mixture distribution was attempted was that by Pearson [18], who used the method of moments to estimate the five parameters of a mixture of two univariate normal distributions.

It is well known, that the Weibull distribution has a wide spectrum of applications. This type of distribution is even more useful because multiple causes of failure can be jointly modeled. An extensive review of the Weibull distribution with many different generalizations of this distribution as used by practitioners and possible complications that arise due to this non-uniqueness is presented in [6].

The exponential distribution as a particular case of the Weibull distribution is extensively used to model the behavior of units that have a constant failure rate or units that do not degrade with time or wear out and is a well-behaved simple model.

Therefore, our study in the present work will focus on a mixture of exponential and a mixture of Weibull distributions. The paper includes the estimation of parameters of the mixture distribution using genetic algorithms (GA) and maximum likelihood estimation.

At the present work only the finite mixture-model with k components

$$F(T,\theta)=\sum_{j=1}^{k} p_j F_j(T;\theta_j),$$
$$0<p_j<1, \sum_{j=1}^{k} p_j =1, \tag{1}$$

where $F(T,\theta)$ is a probability distribution function and p_j is a weight of j-component, has been considered.

2. MIXTURE-DISTRIBUTIONS RECOVERY PROBLEM

There are many methods to estimate the parameters of mixture distributions. Both graphical and analytical approaches have been used. The analytical methods started from Pearson's method of moments, the method of quantiles, general curve fitting, the Bayesian approach, etc. The maximum likelihood

(ML) method is one of the most widely used approaches for a statistical estimation, which appeared practical for general mixture problems and the development of the Expectation Maximization (EM) algorithm [2,20]. The result of the work of Schlesinger concerning the monotone convergence of the EM procedure to some possibly local maximum firstly has been studied in [2] and later in full detail in [1,14,16].

But estimation the mixture model parameters with aforementioned algorithms is a sensibly complicated task, because there are no generally accepted methods to define initial values and classic methods require a good initialization in order to converge to the global maximum of the goal function. As an alternativ to standard methods in a recent line of research, methods to avoid using GA have been developed. A review of existing works revealed several interesting strategies in this area. Thus, an integration of the principles of the GA and the ML to recover a Weibull distribution is described in [25]. Tawfick et al. [22] considered an algorithm for solving the maximum mixture likelihood clustering problem using an integer-coded genetic algorithm (IGA-ML) where a fixed length chromosome encodes the object-to-cluster assignment. The algorithm has the advantages of being able to determine the correct number of clusters and to solve the task of decomposition for a mixture of Gaussian distributions. Papers [12,13] present approaches based on GA, Simulated Annealing (SA) and EM to estimate parameters of the mixture of Gaussian model. The described method uses a population of mixture models, rather than a single mixture, interactively in both GA and EM to determine the mixture parameters. In [9] the behavior of a Boolean serial configuration, as a mixture of failures of Weibull distributions in the Boolean system, is analyzed.

In this paper we develop a methodology to analyze failure data of Light-Emitting Diodes (LED). Reliability of LED depends on design and application requirements. Thus, we have to apply a model that takes different failure mechanisms into account. The Weibull failure density function is associated with the time to failure of items.

The Weibull distribution features properties of the most frequently used distributions for reliability analysis. Such are e.g. Exponential, Normal, Log-normal, Gamma and the Rayleigh distributions. Thus, we consider our research on GA application to the estimation of mixture of Weibull distributions. Since the mixture of exponential distributions has one unknown parameter for every component, this type of mixture is also well-behaved.

The mixture of two-parameter Weibull distributions has *2*k+(k-1)* unknown parameters. These are the shape parameters b_j, scale parameters a_j and the mixing parameters p_j for every component *j*:

$$F(T,\theta) = 1 - \sum_{j=1}^{k} p_j e^{-a_j t^{b_j}}, \quad a_j, b_j > 0,\ 0 < p_j < 1,\ \sum_{j=1}^{k} p_j = 1. \tag{2}$$

In the case of an exponential mixture the probability function looks the same with $b_j = 1$.

As stated above, practical application of classical mathematical statistical methods for an estimation of mixture-distribution results in comparatively complicated computational procedures. Here, classical parameter estimation methods strongly depend on the initial values. Therefore, investigations of alternative approaches for an estimation of parameters are highly actual. One of the possible alternatives to solve the assigned task is to develop a suited artificial intelligence technology. In the following, we present how to estimate parameters of a mixture distribution giving examples and using experimental data.

The population of the classic GA consists of binary strings similar to the chromosome structure of biological creatures (binary code GA, BGA). But a binary code can't directly reveal the particular structure of the problem. The real-code GA (RGA) obtains a better solution on function optimizing and features a lesser solution searching time, better calculation precision and overall convergence situation in comparison to the tradition GA effectively [4,5,17]. In this work, we propose a combination of real-code GA with EM to improve the recovery of the referred above mixture distributions.

3. MULTIEXTREMAL OF FITNESS-FUNCTIONS

The solution of many practical problems in several application areas, from engineering to economics, requires the global optimization of a non-linear multimodal objective functional.

The initial focus of this work is to investigate which technology could be used to implement a principle of GA for the task of a mixture distribution recovery. However, before this question can be examined, we have to define target-functions and to examine properties of their multiextremality.

Typically, the function used to determine the optimality of the output is called the target function or evolutionary function or fitness function. We will use the term fitness function in this research since it is typically used in conjunction with genetic algorithms. It will be considered analogous to the evolutionary function. The inputs to the fitness function include a set of statistical observations and a set of unknown parameters. The fitness value can be defined as a numerical value that describes the characteristics of an individual with regard to the current optimum so that different individuals can be compared.

In statistical data analysis the fitness value of each individual in the population can be measured by the following fitness functions (FF):

– the maximum of the absolute value of difference between empirical and theoretical distribution functions (MAE)

$$FF_{MAE}(\Theta)=\left(\max_{i}\left|F_n(t_i)-F(t_i,\Theta)\right|\right),$$

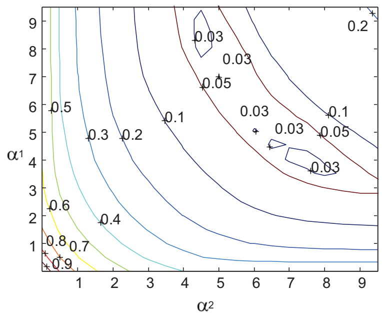
– the maximum likelihood (MLF) function

$$FF_{MLF}(\Theta)=\left(L(\Theta)=\prod_{i=1}^{n}\sum_{j=1}^{k}p_j f_j(t_i,\Theta_j)\right), \quad (3)$$

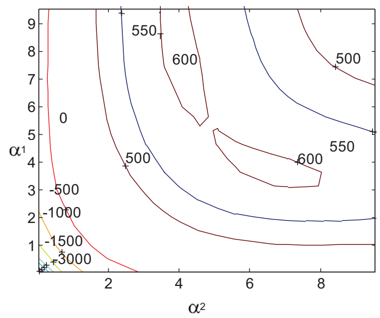
– the least squares function (LSM)

$$FF_{LSM}(\Theta)=\left(\sum_{i=1}^{n}(F_n(t_i)-F(t_i,\Theta))^2\right).$$

We generate the random sample from a mixture of two exponential distributions with parameters $a_1=3, a_2=7, p=0.4$ and limit parameters a_j to the range of 0.1 to 10 and p_j from 0.1 to 0.9 and calculate the MAE and MLE fitness functions for a mixture of two exponential distributions. As a matter of fact MAE and LSM feature extremes in the same points, thus two samples from the different fitness-functions are depicted in figure 1.



MAE



MLE

Figure 1. Fitness-functions

Thus it is evident, that the problem of estimation of mixture-distributions consists of a multiextremal optimization problem. However, in the case of multiextremal functions, this choice suffers from a relevant drawback: depending on the starting point of the search procedure, the algorithm can be trapped into suboptimal or useless false solutions.

4. GENETIC ALGORITHM

GA are adaptive search methods which are independent of initial parameters and can provide an efficient technique to optimize functions in large search spaces [17] based on an adaptive mechanism of biological systems. GA have been applied successfully in many scientific and engineering fields. It has been proved that GA are independent of initialization parameters.

In global real-code optimization terms, each individual in the population is encoded into a floating number, which represents a possible solution to a given problem. For statistical analysis it is important to have not only point estimates of sought-for parameters, but also confidence intervals. As referred above, a famous method to estimate model parameters and to determine confidence intervals is the maximization expectation algorithm (EM). Since GA are independent of initialization parameters they can efficiently optimize functions in large search spaces while the solution obtained by EM is a function of the initial parameters. In this work, we propose a combination of GA with EM to improve the estimation of exponential and Weibull mixture parameters.

We considered the following optimization problem:

$$\begin{aligned} &-\max \; FF_{MAE}(\Theta), \\ &\max \; FF_{MLE}(\Theta), \qquad (4) \\ &-\max \; FF_{LSM}(\Theta), \end{aligned}$$

where FFs are the fitness functions and $\Theta\in\Theta$ is a vector of m unknown parameters of mixture distributions, $\Theta_l\in R, 0\le\Theta_l\le U_l, l=\overline{3,m}$.

As presented above, FFs have several local optimums on a space Θ. Fitness of parents and children can be evaluated by the target-functions (3).

Using a standard GA, an initial population can be generated randomly on the space Θ.

The boundary conditions on parameters are treated using experiential expert knowledge and the method of moments. For our set of parameter we suppose, that

– the shape parameters are $0<b_j\le 9$,

– the scale parameters are $0<a_j$,

– the mixing parameters are $0<p_j<1$.

Furthermore, it is possible to define the constraints of scale-parameters using the method of moments to estimate the exponential distribution

$$0<a_j<2\bar{t},$$

where the overhead constraint is defined according to the method of moments to estimate the exponential recovery.

The selection of individuals from populations to produce successive generations plays an important role. Here, a probabilistic selection based on a ranking of the individual's fitness [17] is carried out.

There are two basic parameters of GA - crossover probability and mutation probability. Crossover probability determines how often a crossover takes place. Crossover is carried out with the objective that new chromosomes might have better properties than old chromosomes and are superior. The methods of crossing two parents θ_m and θ_l is described in [5]. In this paper a simple arithmetic crossover has been used.

Generation of children during a mutation step is performed randomly with a probability of mutation varied within the range of 0.1 to 0.9. Mutation probability means how often parts of chromosome will be mutated. Mutation is carried out to prevent falling GA into a local extreme, though it should not occur too often, because thus GA would change to random search.

In the present work the constrain-based mutation operator was applied using two steps. The first step is that a selected chromosome in the individual is replaced by random values produced from the predefined range by a small probability. Then, these constrains are applied to the individual to produce a valid individual.

This paper focuses on a GA, in which every new individual is optimized locally. Local improvement (LI) procedures have been incorporated into GA in order to improve the algorithm's performance. As a local optimization strategy the EM-algorithm has been used. The EM algorithm provides a general iterative procedure for computing MLE solutions for mixture models. Each iteration consists of two steps: estimation of the missing data by its expectation and maximization of the likelihood.

In the present work the Elitism Selection Strategy has been used. The selection mechanism of this system is the elite method. Sometimes good individuals can be lost when crossover or mutation results in offspring that are weaker than the parents. Elitism involves copying a small part of the fittest individuals unchanged into the next iteration. Each individual is mapped to an individual chromosome.

Further important characteristics of GA are the population size, the sample size and the stopping criterion.

Inspired by the motivation mentioned above, the following steps for the GA are proposed:

Description of the algorithm

Step 1: (Initialization)

- Choice of Fitness Function (FF).
- Define the size G of population P: $|P| = G$.
- Define the stop rule.
- Generation of G individuals as the initial population $\vec{P}_0 = \{\theta^1, \theta^2, ..., \theta^G\}$.

Step 2: (Evaluation)

- Evaluate θ^i in $\vec{P}_0$ by $FF(\theta^i)$.

REPEAT

Step 3: (Parent Selection)

- Sort $FF(\theta^{(1)}) \le FF((\theta^{(2)}) \le ...FF(\theta^{(G)})$.
- Ranking $r_i = \dfrac{Rank_i}{\sum_{j=\overline{1,G}} Rank_j}$, $\sum_{i=\overline{1,G}} r_i = 1$, sort r_i.
- Choose a pair of parents $Par_1, Par_2 \in P$:

  $Par_i \Leftarrow \theta^{(r_i)}$ if $r_i \le Random(0,1) < r_{i+1}$.

Step 4: (Crossover)

- $\xi = Random(0,1)$, $j = \overline{1,2}$,
- $C_j \Leftarrow \xi Par_1 + (1-\xi) Par_2$.

Step 5: (Mutation)

- $\xi = Random(0,1)$, $j = \overline{1,2}$,
- $\widetilde{C}h_j \Leftarrow Mutation(Ch_j)$ if $\xi > Mutation\ rate$.

Step 6: (Local Search)

- $\widetilde{C}h_j{}^* \Leftarrow EM - Algorithm(\widetilde{C}h_j), j = \overline{1,2}$.

Step 7: (Replacement)

- $P \Leftarrow Replace(P \bigcup \widetilde{C}h_j{}^*), j = \overline{1,2}$,
- Delete (2 weakest individuals θ^i, $i = \overline{1, G+1}$).

UNTIL < *one of the stopping criteria is met* >

The algorithm stops when the number of generations reaches the given value of generations or when the value of the fitness for the best point in the current population is less than or equal to the best fitness of the previous generation.

Test functions are important to validate any optimization algorithms and to compare the performance of various algorithms. As test functions De Jong's function and Rastrigin's function have been used.

5. SELECTING GA PARAMETERS

In this section, we demonstrate using GA for estimating parameters of a 2, 3 and 5-component mixture Weibull distribution.

We use different functions to test the optimization performances of RGA for a variety of population sizes and compare these results in table 1. The sample size in the test data set consisted of 30 random variates.

The population size has been chosen from 50 to 200, the crossover and mutation probability from 0.1 to 0.9. The results of these experiments were examined to investigate solution quality as well as computational efficiency. To compare the importance of local optimization, the table lists results for which the local improvement (LI) was used or not used (NLI). For each example, the first column provides the mean functional value found by the GA, the second the number of mixture components, the third the mean of the number of generations it took the GA with LI to find the best solution (not necessarily the optimal), the fourth the crossover probability, the fifth the mutation probability and the two last columns contain the values of the refined Kolmogorov (KM) test [1,16] with a statistical significance of 0.5.

Table 1. Best Solution Quality for statistical modeling data

FF	Comp	Gen	CP	MP	KM,NLI	KM,LI
MAE	2	68	0.3	0.3	0.75	0.75
	2	89	0.3	0.6	0.60	0.71
	2	71	0.6	0.3	0.72	0.89
	2	78	0.6	0.6	0.93	0.99
	3	131	0.3	0.3	0.55	0.75
	3	130	0.3	0.6	0.83	0.83
	3	104	0.6	0.3	0.41	0.81
	3	115	0.6	0.6	0.44	0.82
	5	121	0.3	0.3	0.56	0.76
	5	113	0.3	0.6	0.78	0.78
	5	89	0.6	0.3	0.77	0.77
	5	96	0.6	0.6	0.56	0.85
MLE	2	103	0.3	0.3	0.72	0.72
	2	96	0.3	0.6	0.74	0.86
	2	115	0.6	0.3	0.70	0.74
	2	84	0.6	0.6	0.83	0.97
	3	134	0.3	0.3	0.54	0.85
	3	125	0.3	0.6	0.83	0.83
	3	114	0.6	0.3	0.70	0.71
	3	141	0.6	0.6	0.37	0.85
	5	200	0.3	0.3	0.65	0.77
	5	200	0.3	0.6	0.49	0.79
	5	200	0.6	0.3	0.58	0.75
	5	196	0.6	0.6	0.67	0.80

As table 1 shows, the use of a local improvement procedure improves the quality of the final solution found by GA. A typical example of results of this experiment for mixture of two Weibull distributions for the case $0 < b_1 < 1; 1 < b_1 < 9$ is depicted in figure 2.

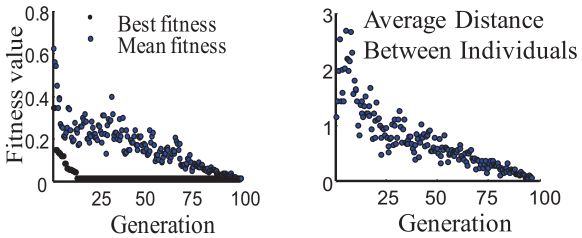


Figure 2. Example of RGA-Results

As can be seen, the genetic algorithm converges to the solution of high degree of certainty. The greatest influence on the speed of convergence and the probability of the best solution finding has the selective type of the genetic operators, which provides a sufficiently great effectiveness of GA work.

6. EXPERIMENTAL SETUP

In this section, we explore the performance of our technology. We trained our model and tested the performance of the algorithm on data obtained from Light-Emitting Diodes (LED). Virtually all LEDs are mounted in a package that provides two electrical leads, a transparent optical window for the light to escape, and, in power packages, a thermal path for heat dissipation [10,11,21]. A low-power package is depicted in fig. 3.

Different types of LEDs have different degradation mechanisms. As a starting point, this study considered a GA-approach where 5 mm epoxy-encapsulated phosphor white LEDs have been used. The subject of inquiry is soldered to the bottom of a reflector-cup in cathode lead. A bond wire connects the top contact to the anode lead.

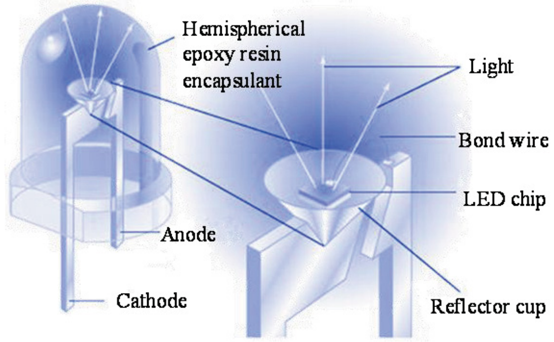
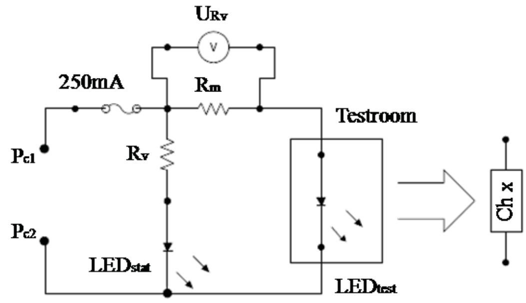
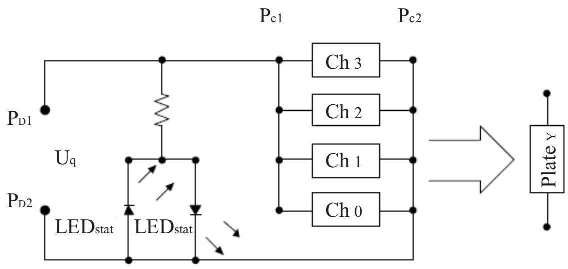


Figure 3. Typical LED-package

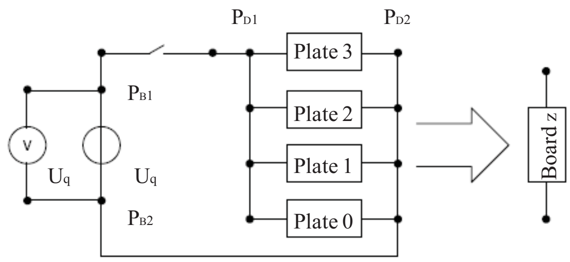
The LEDs life test explored the reduction of light output as a function of time. Figures 4-5 illustrate the experimental setup.



a) The individual life-test channel Ch x



b) The life-test plate consists of 4 channels



c) The life-test board consists of 4 plates

Figure 4. Scheme of the experimental setup

Because each LED has to operate at a particular ambient temperature, all were tested in a specially designed individual life-test channel (figure 4, a). These test channels were designed to keep the temperature constant and placed in a room-temperature box.

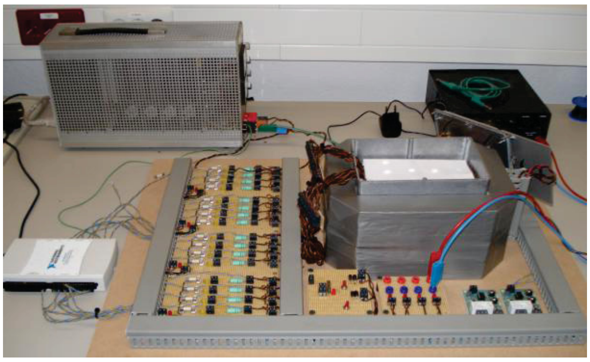


Figure 5. Photo of the life-test setup

Load test and posterior analysis show that via deviation from an ideal current-voltage (I-V) characteristic we can determine our subject of inquiry as a diode with series and parallel resistance and simultaneously with a parasitic diode with lower barrier height and smaller area than the main diode [21]. The observable diode scheme is depicted on figure 6, where D2 denotes a parasitic diode, Rs a series resistances and Rp a parallel resistance.

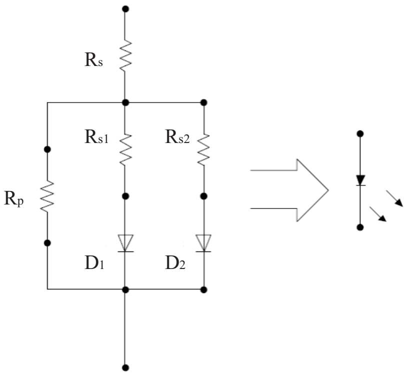


Figure 6. Scheme of a 5 mm LED

The defects occurring on LEDs can be related to different categories which are: the chip as the central element, the internal and the external packaging. Due to different assembly technologies and types of constructions, as well as varying applications, an extended range of failure mechanisms can be observed.

7. RESULTS ANALYSIS

The above equations and GA were used to estimate the parameters for a mixture Weibull distribution. The results were obtained using data for the lifetimes of 28 LEDs that were analysed to indentify the mixture Weibull distribution as given below.

Most LEDs have a natural lifespan that ends in a mechanism of wear. Observable defects within the subject of inquiry are particularly affected by temperature and current. In the present paper we define the reliability as the probability that a product will perform its intended function over a time period *t*. As a defect we define a relative reduction of the current of 25% in load conditions.

The LED-test had been carried out under load conditions of 140 mA and reveals three exterior effects (figure 7): phosphor degradation in a white LEDs causing color shift, minor degradation of the encapsulation and actual degradation of the wire. Phosphor in white LEDs will degrade by time and temperature resulting in a change of the light color, usually to blue [21]. By time the epoxy package can turn yellow under the influence current und temperature.

a) reference LED | b) wire degradation | c) degradation of the encapsulation

Figure 7. Defect demonstration

Figure 8 illustrates the experimental results. The obtained empirical data is plotted on a Weibull distribution graph paper.

The obtained type of the curve corresponds with results of Jiang and Kececioglu [8], which presented a graphic algorithm to identify and estimate the parameters of a Weibull mixture model.

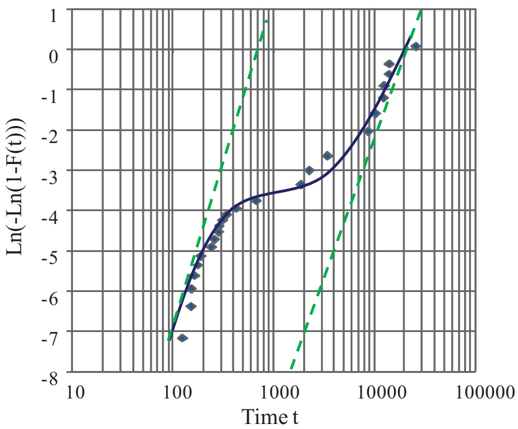


Figure 8. Plot of failure data and fitted curve

Table 2. Solution Quality for experimental data

FF	CP	MP	KM, NLI	KM, LI
MAE	0.6	0.3	0.92	0.92
MAE	0.6	0.6	0.88	0.97
MLE	0.3	0.6	0.84	0.92
MLE	0.6	0.6	0.79	0.94

Table 2 lists the efficiency of the estimation of the mixture of two Weibull distributions with different values of GA-parameters. A size of 100 generations and a crossover and mutation probability according to the best results from table 1 have been chosen.

Fig. 8 depicts the mixture distribution function plot consisting of two component distributions corresponding to two different defect mechanisms. We define these mechanisms as the influence of the diode design (figure 6), resulting in fast destruction and degradation failure of the bond wire and as depicted on figure 7.

8. CONCLUSION AND FUTURE RESEARCH

In this paper, we presented a method to estimate parameters of mixture distributions with different numbers of mixture components based on genetic algorithms.

We have applied this method of failure analysis to 5 mm white LEDs under load conditions of 140 mA. The efficiency of the defined approach is confirmed by the results of statistical modeling and experimental research. The results can be used directly for a reliability analysis and provide a good initialization to accelerate convergence of the EM algorithm.

Possible subsequent research includes two logical directions. The first will be a further analysis and modification of GA-operators and the investigation of a possible Baldwin effect [26]. The second regards investigations of LED failure mechanisms analysis. It should be noted that the reliability of LEDs is very high but depends on design and the applied environment. Results of different failure mechanism analyses will be discussed in a follow-up paper. By testing different commercial LEDs, we can study the degradation mechanisms for different types of LEDs and finally apply a multivariate distribution modeling, which includes simultaneous use of mixture distributions with different types of components.

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