VIBRATION DRIVEN ROBOTS

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ABSTRACT

The design of mobile robots that can move without wheels or legs is an actual engineering and technological problem. Vibration driven robots are locomotion systems that realize a locomotion in a resistive environment without specific propelling devices (wheels, legs, caterpillars, screws) due to oscillatory or undulatory relative motion of their components, part or all of which interact with the environment. Self-propelling mechanisms that consist of a body that has contact with a rough surface and internal masses are considered. Asymmetry in friction that is necessary for the robot to move can be provided in several ways. The robot is equipped with specific contact devises that provide anisotropy for the coefficient of friction, i.e., the coefficient of friction depends on the direction of motion. For example, the contact surface of the robot can be covered with needles. The asymmetry can be provided for isotropic friction by changing the normal pressure of the robot on supporting surface. In this paper we present some basic mathematical models and a prototype of such vibration driven robots.

Index Terms – vibration, micro-robot, resonance, asymptotic method

1. INTRODUCTION

Mobile robots are widely utilized for various operations in environments inaccessible to a human or dangerous for him. Most of these robots move by means of wheels or caterpillars, some of them utilize walking mechanisms. Such robots, however, cannot enter narrow slots (for example, during rescue operations in a zone of wreckage) or move in dense media other than gases or liquids.
Vibration-driven robots are locomotion systems that can move in a resistive environment without specific propelling devices (wheels, legs, caterpillars, screws) due to oscillatory or undulatory relative motion of their components, part or all of which interact with the environment.

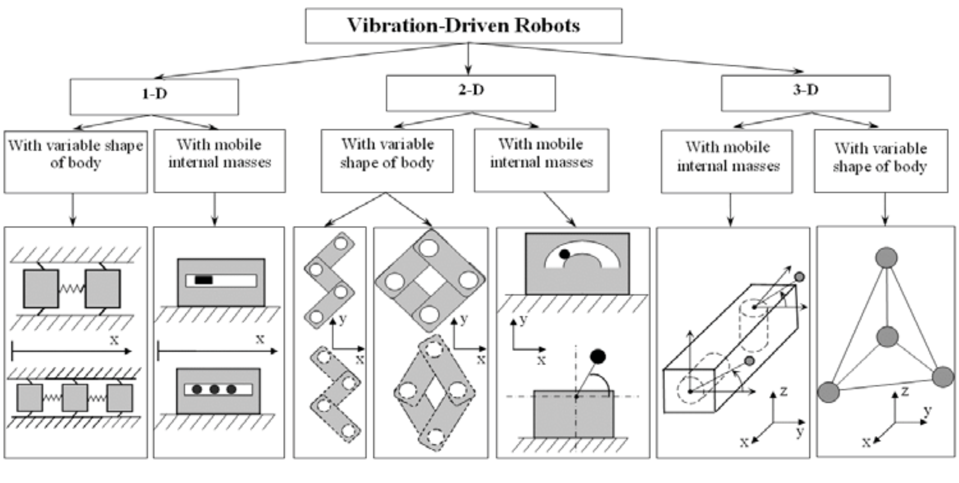

Figure 1: Classification of vibration-driven robots [1]

A classification of vibration-driven robots according to the dimension of the space in which the robots and/or their parts move is proposed (Fig. 1, [1]). The simplest robots are 1-D robots that move along a line. The motion of such robots is provided by either longitudinal redistribution of mass of the robot's body (worm-like motion) or vibratory motion of internal masses in the direction of the motion of the robot [2, 3]. More complicated are 2-D robots that move in a two-dimensional space (along a surface). Such a motion can be performed, for example, by changing the shape of the robot's body (snake-like motion [4]) or by means of internal masses moving in a plane parallel to the plane of motion of the robot. Robots that move along a line but the internal masses move in two dimensions (e.g., along the horizontal and along the vertical) are also classified as 2D robots [1]. An important type of these robots is a robot with rotating internal masses (see section 3) [5]. Most complicated are 3D robots that can move in a three-dimensional space. The motion of 3D robots can be provided by changing the shape of the robot's body or by means of movable internal masses. Vibration driven robots presented in the literature are mostly realized with piezoactuators [6] but also with unbalanced rotors [7]. Some prototypes of mobile robots, resulting from the collaboration between the author's institutes are shown in Fig. 2, [8–10].

Figure 2: Prototypes of mobile robots – 1-D-robot with a piezoactuator (left, [8, 9]), 2-D-robot based on a ferroelastomer (center, [10]), 3-D-robot in form of a tensegrity structure (right, [10])

2. TWO MASS POINTS CONNECTED BY A SPRING

Consider the motion of two mass points (coordinates x_1 and x_2) of mass m, connected by a spring of stiffness c, along the x-axis (Fig. 3).

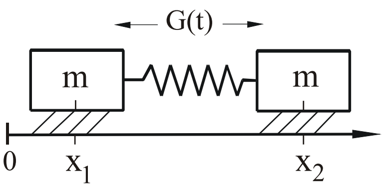

Figure 3: System of two mass points connected by a spring

It is supposed that the mass points are acted upon by a small non-symmetric Coulomb's dry friction force $\varepsilon m F(\dot{x})$, $\varepsilon << 1$, depending on the velocities $\dot{x} = \dot{x}_i \,(i = 1,2)$, where $F(\dot{x}) = F_+$ if $\dot{x} > 0$, $F(\dot{x}) = -F_-$ if $\dot{x} < 0$, and $F(\dot{x}) = F_0$ if $\dot{x} = 0$ while $-F_- < F_0 < F_+$; $F_- \geq F_+ \geq 0$. The system is excited by a small internal force $G(t) = \varepsilon m (b_0 + b\cos\psi)$, $\psi = \nu t$, acting between the mass points. The motion of the system is governed by the equations

$$\begin{aligned} m\ddot{x}_1 + c(x_1 - x_2 + l_0) + \varepsilon m(b_0 + b\cos\psi) + \varepsilon m F(\dot{x}_1) = 0, \\ m\ddot{x}_2 + c(x_2 - x_1 - l_0) - \varepsilon m(b_0 + b\cos\psi) + \varepsilon m F(\dot{x}_2) = 0. \end{aligned} \tag{2.1}$$

Denote $\omega^2 = c/m$ and make the change of variable $x_2 \to x_2 - l_0 - \varepsilon b_0/\omega^2$ to reduce the system of equations (2.1) to the form

$$\begin{aligned} \ddot{x}_1 + \omega^2(x_1 - x_2) = -\varepsilon\left[F(\dot{x}_1) + b\cos\psi\right], \\ \ddot{x}_2 + \omega^2(x_2 - x_1) = -\varepsilon\left[F(\dot{x}_2) - b\cos\psi\right]. \end{aligned} \tag{2.2}$$

To system (2.2) we apply the procedure of averaging according to [11]. For this purpose we introduce new variables: the velocity of the center of mass $V=(\dot{x}_1+\dot{x}_2)/2$ and the displacement of the points relative to the center of mass $z=(x_2-x_1)/2$. For the unperturbed system ($\varepsilon=0$), $V=const$ and the quantity z changes harmonically according to the relations $z=a\cos\varphi$ and $\varphi=\Omega t+\vartheta$, where $\Omega=\sqrt{2}\,\omega$; a and ϑ are arbitrary constants. Use the change of variables $V=(\dot{x}_1+\dot{x}_2)/2$, $z=a\cos(\Omega t+\vartheta)$, and $\dot{z}=-a\Omega\sin(\Omega t+\vartheta)$ to represent system (2.2) in the standard form [11]

$$\begin{aligned}
&\dot{V}=-\frac{\varepsilon}{2}\left[F(V+a\Omega\sin\varphi)+F(V-a\Omega\sin\varphi)\right],\\
&\dot{a}=-\frac{\varepsilon}{2\Omega}\sin\varphi\left[F(V+a\Omega\sin\varphi)-F(V-a\Omega\sin\varphi)+2b\cos\psi\right],\\
&\dot{\varphi}=\Omega-\frac{\varepsilon}{2a\Omega}\cos\varphi\left[F(V+a\Omega\sin\varphi)-F(V-a\Omega\sin\varphi)+2b\cos\psi\right],\\
&\dot{\psi}=\nu,
\end{aligned}\tag{2.3}$$

where V, a, ϑ are slow variables.

We investigate system (2.3) in the vicinity of the main resonance $\nu=\Omega+\varepsilon\Delta$, $\Delta\neq 0$. To that end we introduce the new slow variable $\xi=\psi-\varphi$ and eliminate the fast variable ψ. After averaging with respect to the fast variable φ we obtain

$$\begin{aligned}
&\dot{V}=\begin{cases}-\varepsilon\left(\dfrac{F_-+F_+}{\pi}\arcsin\dfrac{V}{a\Omega}-\dfrac{F_--F_+}{2}\right) & if\ 0\le V<a\Omega,\\ -\varepsilon F_+ \quad if\ V\ge a\Omega,\end{cases}\\
&\dot{a}=\begin{cases}-\dfrac{\varepsilon}{\Omega}\left(\dfrac{F_-+F_+}{\pi}\sqrt{1-\dfrac{V^2}{a^2\Omega^2}}-\dfrac{b}{2}\sin\xi\right) & if\ 0\le V<a\Omega,\\ \varepsilon\dfrac{b}{2\Omega}\sin\xi \quad if\ V\ge a\Omega,\end{cases}\\
&\dot{\xi}=\varepsilon\left(\frac{b}{2a\Omega}\cos\xi+\Delta\right).
\end{aligned}\tag{2.4}$$

We are interested in a velocity-periodic steady-state motion of the entire system. In terms of the averaged model, this motion can be approximated by the steady-state solution of the averaged equations corresponding to $\dot{V}=0$. Then from (2.4) we obtain $\dot{a}=\dot{\xi}=0$ and

$$\begin{aligned}
&V=\frac{\sin\Phi}{|\Delta|}\sqrt{\frac{b^2}{4}-\frac{(F_-+F_+)^2\cos^2\Phi}{\pi^2}}, \qquad a=\frac{V}{\Omega\cdot\sin\Phi},\\
&\xi=\arccos\left[-\frac{\Delta}{|\Delta|}\sqrt{1-\frac{4(F_-+F_+)^2\cos^2\Phi}{\pi^2 b^2}}\right], \quad \Phi=\frac{\pi}{2}\cdot\frac{F_--F_+}{F_-+F_+}.
\end{aligned}\tag{2.5}$$

For the system of two mass points connected by a kinematic drive element, which we use to control the distance between the masses $l(t)$ varies according to the relationship

$$l(t)=l_0+d\sin\omega t, \tag{2.6}$$

a steady state motion with constant velocity V is determined from the expression

$$V=\frac{d\omega}{2}\cos\Phi. \tag{2.7}$$

3. A SYSTEM WITH TWO UNBALANCE ROTORS

Consider the rectilinear motion along a rough plane of two identical bodies of mass M connected by a spring of stiffness c (Fig. 4, left). To each of the bodies an exciter is attached. The exciter is a rigid rotor with a fixed axis of rotation. Both rotors have the same mass m and the same distance l between the center of mass and the axis of rotation. Let x_1 and x_2 denote the coordinates measuring the displacements of the constituent bodies of the system. We assume isotropic Coulomb's dry friction with coefficient k to act between the supporting plane and the bodies. The system is driven by the motion of the exciters, rotating synchronously with the same angular velocity ω and in the same direction with a phase shift φ_0.

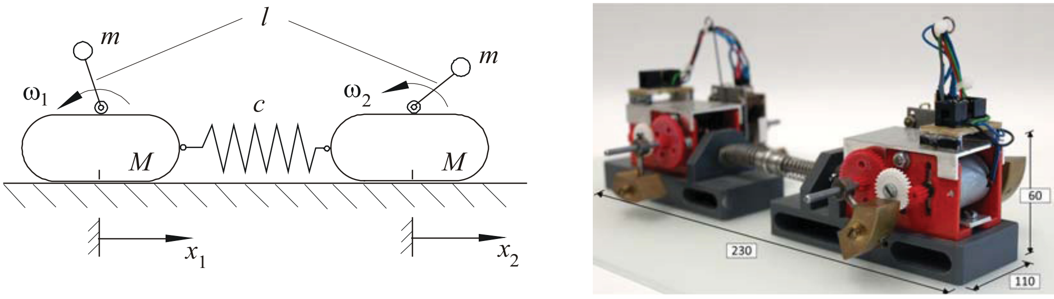

Figure 4: Model of the system with two unbalance rotors (left) and prototype (right)

Introduce the dimensionless variables and parameters (labeled with the asterisk):

$$(x_1, x_2)=(x_1^*, x_2^*)/(2a),\ t=t^*\omega_0,\ \omega_0^2=c/(m+M),\ \nu=\omega/\omega_0,\ \varepsilon=(m+M)k\,g/(2ac),\ \alpha=mcl/\left((m+M)^2 g\right),\ \beta=\alpha/k\,.$$

Here $(m+M)k\,g$ is the maximal value of the friction force acting on each of the bodies, the parameter $2a$ characterizes the maximal elongation of the spring and, accordingly, $2ac$ characterizes the maximal value of the spring force. We assume that $\varepsilon << 1$. The value of the characteristic (steady-state) amplitude is unknown beforehand. For this reason, we can set the value $2a$ arbitrarily, for example $2a=l$. After determining the steady-state amplitude, it should be verified that the elastic force is in fact much larger than the friction force (to justify the hypothesis that $\varepsilon << 1$). The dimensionless equations of motion have the form ($i=1.2$)

$$\begin{array}{l}\ddot{x}_1 + x_1 - x_2 = \varepsilon\beta\nu^2\cos\nu t + r_1,\\ \ddot{x}_2 + x_2 - x_1 = \varepsilon\beta\nu^2\cos(\nu t+\varphi_0) + r_2\end{array},\quad r_i=\begin{cases}-\varepsilon n_i\,\mathrm{sgn}\,\dot{x}_i, & \text{if } \dot{x}_i\neq 0,\\ -f_i, & \text{if } \dot{x}_i=0 \text{ and } |f_i|\le\varepsilon n_i,\\ -\varepsilon n_i\,\mathrm{sgn}\,f_i, & \text{if } \dot{x}_i=0 \text{ and } |f_i|>\varepsilon n_i,\end{cases} \tag{3.1}$$

$$\begin{array}{ll}f_1=\varepsilon\beta\nu^2\cos\nu t - x_1 + x_2, & n_1 = 1-\alpha\nu^2\sin\nu t,\\ f_2=\varepsilon\beta\nu^2\cos(\nu t+\varphi_0) + x_1 - x_2, & n_2 = 1-\alpha\nu^2\sin(\nu t+\varphi_0).\end{array}$$

The quantities n_i represent the normal pressure forces exerted on the body by the supporting plane. Since the plane resists penetration but does not resist separation of the bodies from the plane, these quantities must be nonnegative. To guarantee this condition we assume $\alpha\nu^2 \le 1$. We introduce new variables: the velocity of the center of mass, $V=(\dot{x}_1+\dot{x}_2)/2$, and the displacement of the bodies from their common center of mass, $z=(x_2-x_1)/2=a\cos\varphi$. To investigate the motion of the system in the neighborhood of the main resonance, we assume $\nu=\sqrt{2}+\varepsilon\Delta$, where Δ is a constant quantity that has an order of unity. After transforming system (3.1) to the standard form by introducing the slow variable $\psi=\nu t+\varphi_0/2-\varphi$ and applying the procedure of averaging with respect to the fast variable φ we obtain (the equation for $\dot{\psi}$ is not given)

$$
\dot{V} = \begin{cases} -\varepsilon, & V < -a\sqrt{2}, \\ -\dfrac{2\varepsilon}{\pi}\left[\arcsin\dfrac{V}{a\sqrt{2}} - \right. \\ \left. -2\alpha\sin\dfrac{\varphi_0}{2}\sin\psi\sqrt{1-\dfrac{V^2}{2a^2}} \right], & |V| \le a\sqrt{2}, \\ \varepsilon, & V > a\sqrt{2} \end{cases}
\qquad
\dot{a} = \begin{cases} \dfrac{\varepsilon}{\sqrt{2}}\sin\dfrac{\varphi_0}{2}(\beta\cos\psi + \alpha\sin\psi), \quad |V| < -a\sqrt{2}, \\ -\dfrac{\varepsilon}{\sqrt{2}\,\pi}\left[2\sqrt{1-\dfrac{V^2}{2a^2}} - \pi\beta\sin\dfrac{\varphi_0}{2}\cos\psi - 2\alpha\sin\dfrac{\varphi_0}{2}\sin\psi \cdot \right. \\ \left. \left(\arcsin\dfrac{V}{a\sqrt{2}} - \dfrac{V}{a\sqrt{2}}\sqrt{1-\dfrac{V^2}{2a^2}} \right) \right], \quad |V| \le a\sqrt{2}, \\ -\dfrac{\varepsilon}{\sqrt{2}}\sin\dfrac{\varphi_0}{2}(-\beta\cos\psi + \alpha\sin\psi), \quad |V| > a\sqrt{2}. \end{cases}
\tag{3.2}
$$

We are interested in the velocity-periodic steady-state motion of the system as a whole. The steady-state solution of the averaged equations of motion, corresponding to constant V, can serve as an acceptable approximate model for the steady-state motion of the basic system. If the velocity V according to the averaged equation is constant, the amplitude a is also constant. We introduce the variables $u = \dfrac{V}{a\sqrt{2}}$ and $\gamma = \sin\dfrac{\varphi_0}{2}$ and eliminate ψ from the averaged equations to obtain from (3.2) a system of algebraic equations for the steady-state solution

$$
\begin{gathered}
\frac{\pi^2 \arcsin^2 u}{4\pi^2(1-u^2)} + \frac{4k^2\left[2(1-u^2) - \arcsin u\left(\arcsin u - u\sqrt{1-u^2}\right)\right]^2}{4\pi^2(1-u^2)} = (\gamma\alpha)^2, \\
a = \frac{1}{\Delta}\cdot\frac{1}{\pi\sqrt{2(1-u^2)}}\left\{\frac{\pi}{2k}\arcsin u - \frac{2k}{\pi}\left[(2+u^2)(1-u^2)\arcsin u + 2u(1-u^2)\sqrt{1-u^2} - ars\sin^3 u\right]\right\}.
\end{gathered}
\tag{3.3}
$$

Although the detailed analysis of the nonlinear system of equations (3.3) is complicated, one can draw an important conclusion. Let (u_0, a_0) be a solution of the system of equations (11) for a given set of parameters $(\gamma, \alpha, k, \Delta)$. Then $(-u_0, a_0)$ is a solution for the set of parameters $(\gamma, \alpha, k, -\Delta)$. Hence, the direction of motion can be reversed by changing between the pre-resonant and post-resonant excitation modes, while the speed can be controlled by changing the magnitude of the phase shift between the rotations of the exciters [12]. The prototype (see Fig. 4, right) has the following parameters: $M = 0.12\,kg$, $m = 0.03\,kg$, $l = 0.015\,m$, $c = 130\,N/m$, $k = 0.1$. The natural frequency of the system $\omega_n = \sqrt{2}\,\omega_0 = 44.4\,s^{-1}$, the excitation frequency $\omega = 50\,s^{-1}$. For $\varphi_0 = \pi$, the dimensionless $V = \sqrt{2}\,u\,a$ found from system (3.3) is approximately equal to 0.13, which in dimensional units corresponds to $0.05\,ms^{-1}$.

4. CONCLUTIONS AND OUTLOOK

The motion of mobile systems that consists of two bodies with one inner mass or two bodies connected by a spring and excited by two unbalanced rotors attached to the respective bodies is studied. For the first system the exact equation of motion is integrated numerically considering special control laws. In the second example for small coefficients of friction between the bodies and the rough plane along which the system moves, a system of algebraic equations is obtained for determining an approximate value of the average steady state velocity of the entire system. An experimental model of the vibration driven system designed on the basis of the concept presented in the paper was designed and constructed. Further investigations will be oriented to the two dimensional motion of nonpedal mobile robots and the problem of motion in resistive media. From another point of view it seems to be necessary to investigate also cooperating micro robots, which can solve real practical tasks, i.e. manipulation of objects in al plane.

5. ACKNOWLEDGMENTS

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