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# FATIGUE DAMAGE PARAMETERS AND THEIR USE IN ESTIMATING LIFETIME OF HELICAL COMPRESSION SPRINGS

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## ABSTRACT

The following essay deals with the usability of damage parameters with regard to helical compression springs. Different damage parameters were examined. Of these parameters the usability for helical compressive springs distinguishes decisively.

***Index Terms*** – damage parameter, life time, durability, helical compresson springs

## 1. INTRODUCTION

Helical compression springs are one of the most important mechanical components in engineering products [1]. Very often, the value of springs and of their functioning reliability far exceeds the cost of the springs themselves. It is not currently possible to compute the prospective lifetime of helical compression springs. For this reason, to ensure the springs have been designed safely, dynamic fatigue tests are necessary which run in parallel to the sample manufacturing process and also to the mass production. It is common to design the springs initially in reliance on fatigue damage parameters. These can be established from dynamic fatigue tests on springs made of similar material with the same or similar production parameters.
Damage parameters provide a fatigue limit for a component by subjecting it to cyclical loading, the amount of which will be in direct ratio to the fatigue accumulated in a component from the individual stress reversals and thus to the expected life before failure. The damage parameter most often assessed is the $P_{SWT}$ parameter [2]. This takes into account not only the effects of the range of stress but also the maximum stress when under load.

## 2. THE SMITH, WATSON AND TOPPER FATIGUE PARAMETER ($P_{SWT}$)

In dynamic fatigue tests, the stress amplitude (strain amplitude) and the mean stress (mean strain) both have an effect on the life of the component being tested. This is equally true of helical compression springs.

The relation between the mean stress and the tolerable stress amplitude for a certain component lifetime is shown as a damage parameter by the Smith, Watson and Topper method. This is described by Equation 1, where the product $\sigma_o \cdot \varepsilon_{a,t}$ is what is to be seen as the fatigue damage. It can be interpreted as density of distortion engergy.

$$P_{SWT\sigma} = \sqrt{\sigma_o \cdot \varepsilon_{a,t} \cdot E} = \sqrt{\sigma_o \cdot (\varepsilon_{a,e} + \varepsilon_{a,p}) \cdot E} \qquad (1)$$

The total strain amplitude $\varepsilon_{a,t}$ is composed of the elastic strain amplitude $\varepsilon_{a,e}$ and the plastic strain amplitude $\varepsilon_{a,p}$. If it is assumed that there is no plastic strain during the cyclical loading, Equation 2 will be the result.

$$P_{SWT\sigma} = \sqrt{\sigma_o \cdot \varepsilon_{a,e} \cdot E} = \sqrt{\sigma_o \cdot \frac{\sigma_a}{E} \cdot E} = \sqrt{0{,}5 \cdot \sigma_o \cdot \sigma_h} \qquad (2)$$

The type of stress to which the steel wire of the helical compression spring is loaded is mainly torsion. Shearing stress and bending stress are generally negligible in relation to the degree of torsional stress present. To enable Equation 2 to be applied to helical compression springs, it is necessary to make a transfer from normal stress to tangential stress.

$$P_{SWT\tau} = \sqrt{\tau_o \cdot \gamma_{a,e} \cdot G} = \sqrt{\tau_o \cdot \frac{\tau_a}{G} \cdot G} = \sqrt{0{,}5 \cdot \tau_o \cdot \tau_h} \qquad (3)$$

The relation between the $P_{SWT}$ value for normal stress and torsional shearing stress will be $1/\sqrt{3}$ taking the von-Mises criterion into account. It is possible to establish Wöhler curves as fatigue parameters using dynamic fatigue tests. Here the assumption is that for the same value of the damage parameter in a scatter band the testing points will coincide independently of the mean stress [2].

In the case of helical compression springs, because they are coiled into a tight space, there will be areas in the wire which are under locally increased stress. Generally these points of step-up stress are taken account of by an additional stress correction factor k in that this is multiplied this together with the existing nominal stress values. The step-up stress points are on the inside of the spring coils, with the result that the crack will most probably start there. In the illustrations which follow it will therefore be always the stress on the inside of the coil which is considered.

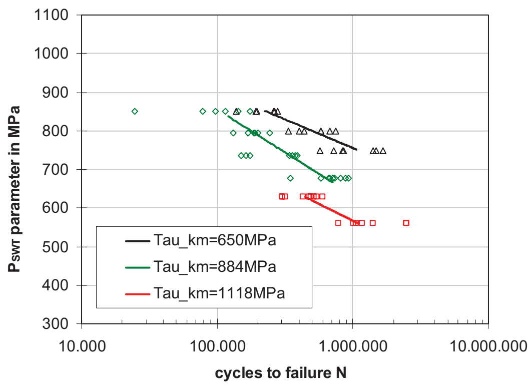


*Figure 1: $P_{SWT}$ parameter: dynamic fatigue test on axle springs of similar shape and manufactured similarly*

Figure 1 shows the results of dynamic fatigue tests on 70 axle springs with the mean stress corrected in different ways. The figure for the fatigue parameter available during the test according to Equation 3 has been plotted against the tolerable number of stress reversals.

As is clear from Figure 1 the influence of the mean stress sensitivity M cannot be adequately taken into account using the $P_{SWT}$ parameter. As Haibach [2] reports, the $P_{SWT}$ figure reflects only mean stress sensitivity of $M \leq 0.4$. The Wöhler curves shown here as fatigue parameters (with survival probability 50%) do not coincide in a scatter band. As the mean stress increases and the tolerable number of stress reversals remains constant, the value for the $P_{SWT}$ parameter drops. This suggests that the mean stress sensitivity of the high strength spring material is underestimated by this damage parameter. If the mean stress is 650 MPa and the $P_{SWT}$ is 800 MPa, the life will be approx. 500,000 stress reversals. If the mean stress is 1118 MPa, the lifetime will be reduced to 50,000 stress reversals, extrapolating from the same fatigue parameter. The lifetime under mean stress of 1118 MPa would be overestimated by a factor of 10 if the $P_{SWT}$ value were applied.

The reasons for this lie in the mathematical description of the $P_{SWT}$ parameter. It takes the mean stress into account using the upper stress level. Furthermore, non-linear elastic distortions of the kind which are known in spring steel wire cannot be represented in the mathematical description.

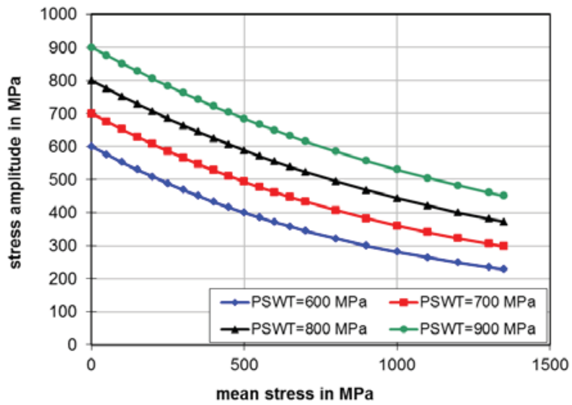


*Figure 2: representation of permissible stress amplitude over and above the mean stress given different values for the damage parameter*

In Figure 2 the permissible stress amplitudes over and above the mean stress for different constant $P_{SWT}$ values are represented. The graph is based on purely linear elastic behaviour of the material. No plastic distortion effects are taken into account. As the mean stress increases the gradient of the curves slackens. The gradient defines the sensitivity to mean stress demonstrated by the $P_{SWT}$ fatigue parameter for a number of different ranges of mean stress.

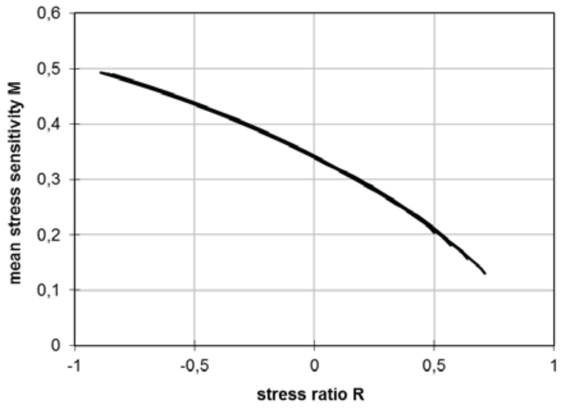


*Figure 3: Mean stress sensitivity as indicated by the $P_{SWT}$ parameter*

If the mean stress sensitivity ($M\tau = 0.52$) established in dynamic fatigue tests on 316 axle springs is compared with the mean stress sensitivity taken into account by the $P_{SWT}$ parameter (see Figure 3), there are clear differences. Above all, in the range which is crucial for helical compression springs, $R > 0$, the influence of mean stress is underestimated. For example, if axle springs manufactured in an identical manner are subjected to dynamic fatigue tests with the same value for the damage parameter and with different mean stress values, the permissible stress amplitude for the springs tested with the higher mean stress will be overestimated. Failure will come early.

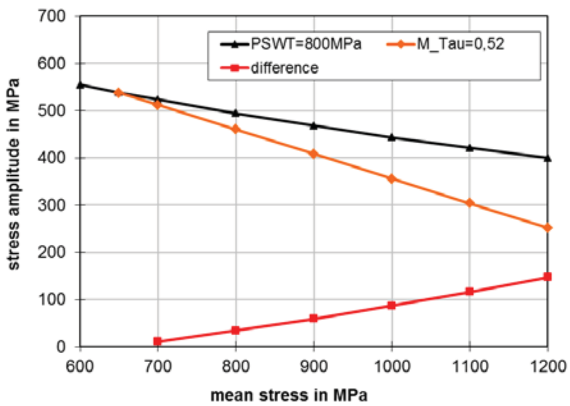


*Figure 4: Permissible stress amplitudes using $P_{SWT}$ value and a prescribed sensitivity to mean stress*

From Figure 4 one can clearly see how great the error can be when working out the stress range if the $P_{SWT}$ parameter is applied. If dynamic fatigue tests are, for instance, carried out to establish experimentally a Wöhler curve for springs and the $P_{SWT}$ value of 800 MPa and mean stress of 650 MPa are applied, the subsequent use of the Wöhler curve to estimate tolerable stress amplitudes for a spring operated (for the same lifetime or fatigue levels) at a mean stress of 1100 MPa will involve an overestimation of the permissible stress amplitude by about 120 MPa. This is equivalent to a stress amplitude error of approx. 30% and an overestimated lifetime by about a factor of 10 (*cf.* experimental results shown in Figure 1). This is not an acceptable deviation. The conclusion is that the $P_{SWT}$ parameter can only be used for a very restricted range of mean stress.

## 3. EXTENDED FATIGUE PARAMETER ACCORDING TO BERGMANN ($P_B$)

Because of the above, Bergmann [5] has suggested extending the original $P_{SWT}$ parameter by means of a nominal value $a_{z/d}$ which will enable the effect of mean stress to be prescribed within the limits $M = 0$ and $M = 1$. Equation 4 provides the general mathematical description:

$$P_B = \sqrt{(\sigma_o + a_{z/d} \cdot \sigma_m) \cdot \varepsilon_{a,t} \cdot E} \tag{4}$$

In this case also, the transfer of Equation 4 into torsional shearing stress is necessary (*cf.* derivation of Equation 3)

$$P_B = \sqrt{(\tau_o + a_s \cdot \tau_m) \cdot \tau_a} \tag{5}$$

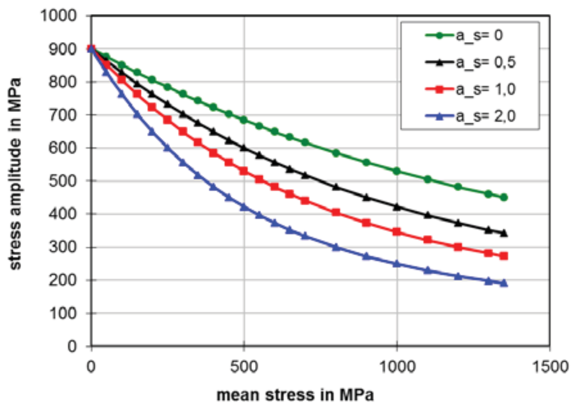


*Figure 5: Graph of permissible stress amplitude above mean stress using different values for the nominal value $a_s$ at a constant fatigue parameter in accordance with Bergmann ($P_B$ = 900 MPa)*

In Figure 5 the permissible stress amplitude above mean stress was deducted for various values of the characteristic $a_s$ with a constant fatigue parameter $P_B = 900$ MPa. The permissible stress amplitudes decrease at different rates independence on the characteristic $a_s$ as the mean stress increases. Taking the definition of sensitivity to mean stress offered by Haibach [2], the characteristic $a_s$ can be a maximum of 2 because that is what produces a maximum possible mean stress sensitivity of M = 1.

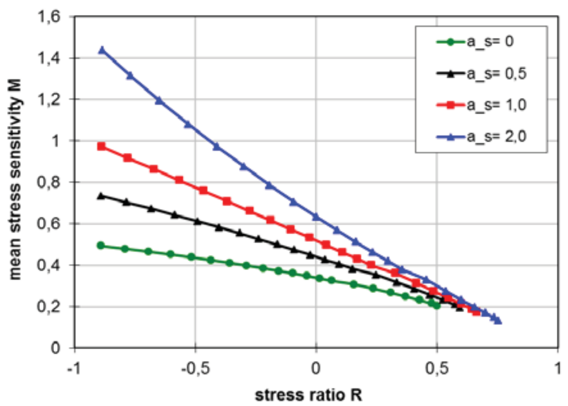


*Figure 6: Mean stress sensitivity shown by means of $P_B$ value*

Different values for mean stress sensitivity can be considered. In contrast to what is possible with the $P_{SWT}$ value, a graph can be made of mean stress sensitivity >0.4 (Figure 6). It must be admitted that even here it is not possible to realise constant mean stress sensitivity over a large range of mean stress because of the mathematical description of the fatigue parameter.

Figure 7, like Figure 1, shows the results of dynamic fatigue test on 70 axle springs with the mean stress $\tau_{km}$ corrected in different ways. The figure for the fatigue parameter available during the test according to Equation 5 has been plotted against the tolerable number of stress reversals.

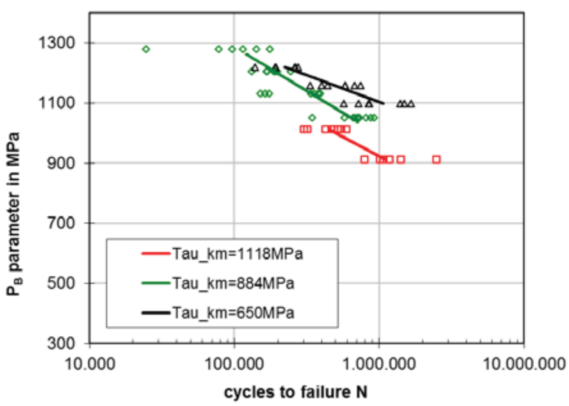


*Figure 7: $P_B$ value with $a_s$ = 2: dynamic fatigue test on axle springs of similar shape and manufactured similarly*

As with the $P_{SWT}$ parameter, the mean stress sensitivity is underestimated in the damage parameter calculated according to Bergmann, though the deviation in the tolerable number of stress reversals becomes smaller the greater the characteristic $a_s$. This means that if the characteristic $a_s$ = 2 and the fatigue parameter $P_B$ is 1200 MPa there is a difference of approx. factor 5 between the results of dynamic fatigue tests at mean stress of 650 MPa and of 1118 MPa.

## 4. FATIGUE PARAMETER INDEPENDENT OF MEAN STRESS ($P_{RKK}$)

To achieve better comparability between dynamic fatigue tests at different mean stress, a new parameter has been developed on the basis of the fundamental idea in Smith, Watson and Topper's damage parameter. This new parameter permits specific adjustment and direct setting of the sensitivity to mean stress. The basic approach will be clear from Figure 8. A plot has been made in Figure 8 of the tolerable stress amplitude over and above mean stress for a constant lifetime. Also, an example of the rise in mean and highest stress has been shown for a sensitivity to torsional mean stress $M_\tau$=0.6.

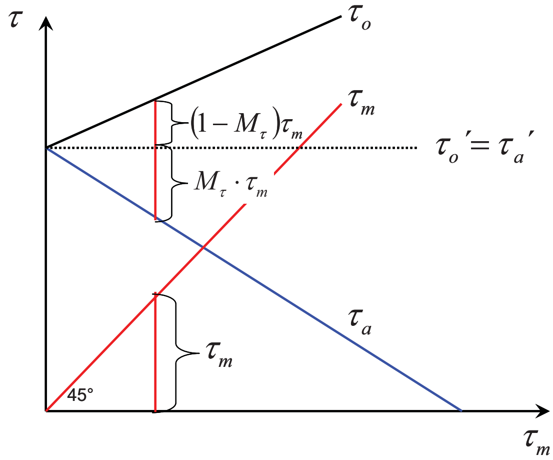


*Figure 8: Phenomenological deviation of fatigue parameter $P_{RKK}$*

The figure for the fatigue parameter at a torsional mean stress $\tau_m$=0 MPa is the starting point for the considerations which now follow. By replacing the upper stress $\tau_o$ and the stress amplitude $\tau_a$ in Equation 6 adapted by means of the mean stress sensitivity and the stress amplitude $\tau_a'$ one obtains Equation 7.

$$P_{SWT} = \sqrt{\tau_o \cdot \tau_a} \tag{6}$$

$$P_{RKK} = \sqrt{\tau_o' \cdot \tau_a'} \tag{7}$$

The intention is to obtain constant upper stress $\tau_o'$ independent of the mean stress and stress amplitude

$\tau_a'$, so that the damage parameter remains constant across the whole mean stress range.

This adaptation is described in Equation 8 and Equation 9 with the aid of the mean stress sensitivity.

$$\tau_o' = \tau_o - (1 - M_\tau) \cdot \tau_m \quad (8)$$

$$\tau_a' = \tau_a + M_\tau \cdot \tau_m \quad (9)$$

When Equation 8 and 9 are substituted in Equation 7, the result is the equation for the damage parameter $P_{RKK}$ which is independent of the mean stress.

$$P_{RKK} = \sqrt{(\tau_o - (1 - M_\tau) \cdot \tau_m) \cdot (\tau_a + M_\tau \cdot \tau_m)} \quad (10)$$

In Figure 9 a plot of the permissible stress amplitude over and above the mean stress has been made for the various mean stress sensitivity values at a constant fatigue parameter $P_{RKK} = 900$ MPa by applying Equation 10. The gradient of the various graphs is constant for the relevant mean stress sensitivity.

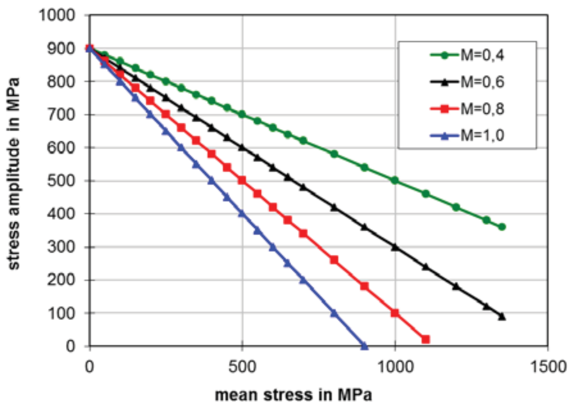


*Figure 9: Graph of permissible stress amplitude over and above mean stress at constant damage parameter $P_{RKK}$*

This is clearly shown in Figure 10 which demonstrates the mean stress sensitivity for various stress ratios R (mean stress values).

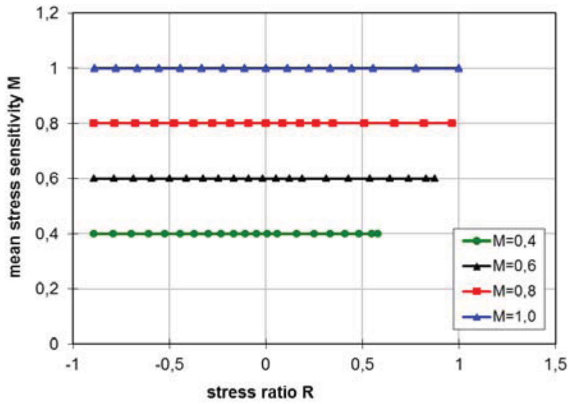


*Figure 10: Mean stress sensitivity represented by the $P_{RKK}$ parameter*

Figure 11, also like Figure 1, shows the results of dynamic fatigue test on 70 axle springs with the mean stress $\tau_{km}$ corrected in different ways. The value for the damage parameter available during the test according to Equation 10 has been plotted against the tolerable number of stress reversals.

The new fatigue parameter according to **R**eich, **K**letzin and **K**obelev ($P_{RKK}$) makes it possible to compare the dynamic fatigue tests on axle springs carried out at different mean stress much better than it has been with the $P_{SWT}$ parameter commonly used to date in the spring manufacturing industry (*cf.* Figure 1*)*. The deviations in the number of stress reversals tolerated for a constant $P_{RKK}$ parameter in the mean stress range given are less than a factor of 2.

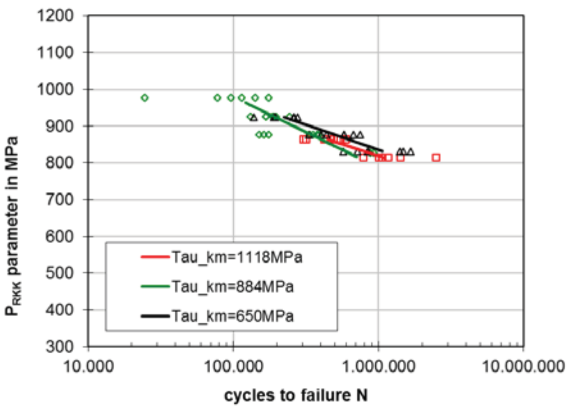


*Figure 11: $P_{RKK}$ parameter: dynamic fatigue test on axle springs of similar shape and manufactured similarly*

For the application for the $P_{RKK}$ parameter it is necessary to know the mean stress sensitivity of the material in question. The means of establishing this are to take it from publications on service strength [2][4], to compute it in accordance with the FKM guideline or to carry out dynamic fatigue tests. The higher the strength becomes, the higher the mean stress sensitivity which is to be expected in principle.

The average mean stress sensitivity found for the experiment on 316 axle springs was $M_\tau = 0.52$ [6]

## 5. FUTURE PROSPECTS

The new $P_{RKK}$ fatigue parameter will make possible significantly better comparability between dynamic fatigue tests on helical compression springs of different mean stress. Wöhler curves for various spring manufacturing methods can be produced on the basis of dynamic fatigue test results already available which have been carried out for publicly funded research projects or within enterprises. On the basis of these the design of helical compression springs under cyclical loads can take place with considerably improved lifetime prediction. By this means the cyclical testing in the course of the sample production process which cost so much in both money and time will be much reduced, because permitted loads for the relevant spring use will be capable of much more accurate estimation. The more data can be created from dynamic fatigue tests on which Wöhler curves can be based, the more accurate and the more universally applicable will be the $P_{RKK}$ parameter.

## 6. ACKNOWLEDGMENT

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