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O. Fütterer / G. Linß

Possibilities of Quality Measurement for Interdependent Product / Process Characteristics

QUALITY MEASUREMENT AND INDUSTRIAL IMAGE PROCESSING

FULL-TEXT DVD-VERSION

1. Introduction to the topic

Process capability indices, also commonly called PCIs, have been an essential tool of quality assurance as far as customer-supplier-interactions and concomitant product controls of every mass production are concerned. These characteristics are usually based on very simple statistics. They give information about every drawing feature and whether the used manufacturing process is able to produce a desired component within the required limits of tolerance or not [1].

Conclusions and statements drawn from the generally known PCI-formulae are correct while they are completely independent of each other of the product characteristics [2]. The product quality in most cases, however, is mainly determined by the interaction and interrelationship of its single parameters, for example the assembly of printed circuit boards, the conveying line systems for pneumatics or hydraulics, form and position tolerances and many more [3]. A lot of such constructions use interdependence criteria as functional optimization and saving of costs. The maximum material principle (MMB), for instance, permits to add the value of a tolerance of a feature, not completely utilized, like the diameter, to other values as the position etc. Nevertheless, such positive features cannot be considered from the point of view of a characteristic in modern statistical quality assurance. Unfortunately, this still leads to scrapping of functional lots. Furthermore, super positions of features of components are known, which might lead to a functional reduction. In that case the customer gets an inoperable product although a sufficient number of PCIs per feature has been verified.

The consideration thus resulted is: to take into account the interaction effects within the quality verification by means of an adaptation or modification of key characteristics. By assuming the definition principles of modern PCIs it was accomplished / there was the decision to design only one single index for the degree of performance of the function of a product lot, called Function Achievement Degree (FAD). The commercial hexagon head screw served as an example for the practical employed study of the formulae that worked theoretically [4]. On the one hand, this component is still examined today by conventional gauging methods because of the strong dependence of the parameters as far as screw thread diameter, pitch and pitch angle are concerned. Due to a lack of a firm proposition and a very high number of parts to be checked, such kind of testing in modern quality assurance should be avoided in the future. On the other hand, this mass product is just offering a provoking package of form and position tolerance when the latest development of norms is considered. Therefore, a more economic production, and, as a consequence, low-priced parts for the customer are produced neither with a modification of the production procedure nor the purchase of special measuring equipment – and what is more important even without any loss of functionality. This potential, however, has been recognized by a few sources but only the present work displays the opportunities how to utilize it.

The procedure presented in this work can be used with all kinds of tolerancing work common in the construction business without any exception and thus does not only enable parameters of one kind e.g. geometrical dimensions, but all dimensions necessary for the function of the whole component to be included in the equipment validation.

2. Steps for the Calculation of Function Achievement Degree (FAD)

In this chapter the building procedure of Function Achievement Degree (FAD) has to be explained. Before the measurement the selection of the most important characteristics of the object has to be determined.

Step 1: Measure, Building of Data Matrix

Starting multivariate methods in a practical way can be initiated by building an *n x m*-data matrix A start of practically all multivariate methods is a building a $n \times m$-data-matrix (observation and design matrix) $\mathbf{X}$ to make a connection between n elements (objects, characteristics, features) and m variables. Single measured data is included in the following matrix:

$$\mathbf{X} = \left(x_{ij}\right)_{\substack{i=1,\dots,n \\ j=1,\dots,m}} = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \dots & \dots & \dots & \dots \\ x_{N1} & x_{N2} & \dots & x_{Nm} \end{pmatrix} \begin{matrix} \leftarrow e_1 \\ \leftarrow e_2 \\ \dots \\ \leftarrow e_N \end{matrix} \atop \begin{matrix} \uparrow & \uparrow & \dots & \uparrow \\ X_1 & X_2 & \dots & X_m \end{matrix}, \quad (1)$$

with X_m for the characteristic has to be analyzed and e_n for the part number. Characteristics for the parts with screw surfaces could be for example the pitch diameter, position tolerances between bold head and screw etc. This depends on each specified case.

Step 2: Consideration of Redundancies

Using data describing the same characteristic can involve the risk of weighing the same. With the using of data describe the same characteristic; we have a risk to weigh the same circumstance more than once. That's why the second step here is to reduce or even not to measure the same characteristics.

Step 3: Consideration of Non-Normality

All measured single characteristics have to be proven by a non-normality test. As one of the reasons of the abnormality, typical for a single characteristic, it is necessary to transform the distribution curves into a normal status by the methods of Johnson, for instance. If the reason for the non-normality typical for the single characteristic it is necessary to transform the distribution curves into normal status with as an example methods of Johnson. At first skewness g_1 and kurtosis g_2 have to be determined:

$$g_1 = \frac{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3\right)}{\sqrt{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^3}}; \quad g_2 = \frac{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4\right)}{\sqrt{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^2}} - 3. \quad (2); (3)$$

With the help of these two distribution parameters it is possible to use a transformation algorithm, see details [4]. If the reason of the non-normal instability of the process, you have to stabilize the system at first.

Step 4: Normalization of Data

Some indices using multivariate methods do not care about the different dimensions of the measured data. This is often a reason for unclear results. That's why it is important to avoid this situation by using the same scale for all dimensions.

Step 5: Generation of multivariate Process Hyper Ellipsoid

A process hyper ellipsoid in n-dimensional space is generally defined as:

$$Q(\underline{\mathbf{x}}) = \sum_{i=1}^{n}\sum_{k=1}^{n} a_{ik} x_i x_k + 2 \cdot \sum_{k=1}^{n} b_k x_k + c = \underline{\mathbf{x}}^{\mathbf{T}} \mathbf{A} \underline{\mathbf{x}} + 2\underline{\mathbf{b}}^{\mathbf{T}} \underline{\mathbf{x}} + c,\ a_{ik},\ b_k,\ c \in \Re \,. \qquad (4, 5)$$

$\mathbf{A}$ is a symmetrical Matrix. Points P with $\overrightarrow{OP} = \underline{\mathbf{x}} = (x_1, x_2, ..., x_n)^T$, fulfilling the equation $Q(\underline{\mathbf{x}}) = 0$, belongs to, mathematically spoken, hyper surface of the second degree or so called quadric in $\Re^n$. For the construction of a hyper ellipsoid the definition of Mahalanobis Distance is very important. Taguchi calls this technique of weighed distances 'the most important quality tools of the future and consequently analogous to the loss function in multivariate space:

$$d_{ik} = \sqrt{(\mathbf{x}_i - \mathbf{x}_k)^T \, \Sigma^{-1} (\mathbf{x}_i - \mathbf{x}_k)} \,. \qquad (6)$$

Mahalanobis Distances take into account the variance and co-variance of the characteristics which have to be searched for. For the calculation of capability indices we use for $\mathbf{x}_k$ the vector average $\boldsymbol{\mu}$. Analog to one dimensional statistics there are 99.73% of all realizations, which have to be positioned inside the limits, to achieve a capability of 1,00. With the help of percentile of central χ^2-distribution:

$$d_{\mu}^2 \leq \chi_{m;1-\alpha}^2 \,. \qquad (7)$$

The position of average is described with vector $\boldsymbol{\mu}$:

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \cdots \\ \mu_m \end{pmatrix} = \frac{1}{n} \mathbf{X}^T \mathbf{1} \qquad (8)$$

For formula (6) of co-variance matrix the following is given as:

$$\Sigma = \frac{1}{n}\mathbf{X}^T\mathbf{X} - \boldsymbol{\mu}\boldsymbol{\mu}^T \text{ or } \Sigma = \begin{pmatrix} s_1^{\;2} & s_{12} & \dots & s_{1m} \\ s_{21} & s_2^{\;2} & \dots & s_{2m} \\ \dots & \dots & \dots & \dots \\ s_{m1} & s_{m2} & \dots & s_m^{\;2} \end{pmatrix}, \tag{9}$$

with equations for the spread of the average diagonal:

$$s_{ii} := s_i^2 = \begin{Bmatrix} \frac{1}{m}\sum_{v=1}^{n}(x_{vi} - \mu_i)^2 \\ \frac{1}{m}\sum_{v=1}^{n} x_{vi}^2 - \mu_i^2 \end{Bmatrix} \tag{10}$$

and co-variance of non-diagonals as:

$$s_{ij} := s_{ji} = \begin{Bmatrix} \frac{1}{m}\sum_{v=1}^{n}(x_{vi} - \mu_i)\cdot(x_{vj} - \mu_j) \\ \frac{1}{m}\sum_{v=1}^{n} x_{vi}x_{vj} - \mu_i\mu_j \end{Bmatrix} \tag{11}$$

Percentiles of $\chi^2_{m;1-\alpha}$ are given in statistic literature, see for more details [4]. For the focus on the target value it is:

$$D = \sqrt{1 + (\boldsymbol{\mu} - \mathbf{ZW})^T\, \Sigma^{-1}(\boldsymbol{\mu} - \mathbf{ZW})}. \tag{12}$$

Step 6: Consideration of Tolerance Limits

Tolerance limits at the contact points of the maximum hyper ellipsoid are equivalent to the one dimensional case, with the formula of tangential (n-1)-dimensional surface:

$$\sum_{i=1}^{n}(X_i - x_i)\left(\frac{\partial f}{\partial X_i}\right)_{x_i} = 0. \tag{13}$$

The normal is defined as:

$$\frac{(X_1 - x_1)}{\left(\frac{\partial f}{\partial X_1}\right)_{x_1}} = \frac{(X_2 - x_2)}{\left(\frac{\partial f}{\partial X_2}\right)_{x_2}} = \dots = \frac{(X_n - x_n)}{\left(\frac{\partial f}{\partial X_n}\right)_{x_n}}. \tag{14}$$

With equation (6) we can find the desired derivatives:

$$Q = (\mathbf{x} - \boldsymbol{\mu})^{\mathrm{T}}\, \boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu}) = \chi^2_{m,1-\alpha} = const. \tag{15}$$

According to a differential rule:

$$\frac{\partial Q}{\partial \mathbf{x}} = 2\boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu}) = 2\mathbf{y}, \tag{16}$$

with $\mathbf{y}$ as a normal vector to the hyper ellipsoid. For partial derivatives that means:

$$\frac{\partial F}{\partial x_1} = 2\left(i_{11}(x_1-\mu_1)+i_{12}(x_2-\mu_2)+\ldots+i_{1n}(x_n-\mu_n)\right),$$

$$\frac{\partial F}{\partial x_2} = 2\left(i_{21}(x_1-\mu_1)+i_{22}(x_2-\mu_2)+\ldots+i_{2n}(x_n-\mu_n)\right),$$

...

$$\frac{\partial F}{\partial x_n} = 2\left(i_{n1}(x_1-\mu_1)+i_{n2}(x_2-\mu_2)+\ldots+i_{nn}(x_n-\mu_n)\right).$$

By equation (16) the tangential surface is given as:

$$\begin{aligned}&\left[i_{11}(x_1-\mu_1)+i_{12}(x_2-\mu_2)+i_{13}(x_3-\mu_3)+\ldots+i_{14}(x_n-\mu_n)\right](X_1-x_1)+\\&+\left[i_{21}(x_1-\mu_1)+i_{22}(x_2-\mu_2)+i_{23}(x_3-\mu_3)+\ldots+i_{2n}(x_n-\mu_n)\right](X_2-x_2)+\\&+\ldots+\left[i_{n1}(x_1-\mu_1)+i_{n2}(x_2-\mu_2)+i_{n3}(x_3-\mu_3)+\ldots+i_{nn}(x_n-\mu_n)\right](X_n-x_n)=0.\end{aligned} \qquad (17)$$

For the surface to axis x_1:

$X_1 = x_1$ and

$$i_{21}(x_1-\mu_1)+i_{22}(x_2-\mu_2)+\ldots+i_{2n}(x_n-\mu_n)=0,$$

...,

$$i_{n1}(x_1-\mu_1)+i_{n2}(x_2-\mu_2)+i_{n3}(x_3-\mu_3)+i_{nn}(x_n-\mu_n)=0.$$

with Gauss algorithm that is:

$$\chi_{n,P} = \frac{X_1-\mu_1}{\sigma_1};\ \chi_{n,P} = \frac{X_2-\mu_2}{\sigma_2},\ \ldots\ \chi_{n,P} = \frac{X_n-\mu_n}{\sigma_n}. \qquad (18)$$

The smallest amount for $\chi^2_{m,1-\alpha}$ is the equivalent to probability.

Step 7: Calculation of FAD (Function Achievement Degree)

For the FAD we have the same limits as for the one dimensional case:

68.2700000 => $\pm 1\sigma$ => 0.33

95.4500000 => $\pm 2\sigma$ => 0.66

99.7300000 => $\pm 3\sigma$ => 1.00

99.9937000 => $\pm 4\sigma$ => 1.33

99.9999400 => $\pm 5\sigma$ => 1.67

99.9999998 => $\pm 6\sigma$ => 2.00 (19)

The formula for FAD is:

$$FAD^m_{pm} := \frac{FAD^m_p}{D} = \frac{FAD^m_k}{D(1-k)} = \frac{1}{3}\frac{\sqrt{f_{m=1}\left(\chi^2_{m,1-\alpha}\right)}}{D(1-k)}$$

with $D = \sqrt{1+(\boldsymbol{\mu}-\mathbf{ZW})^T\,\Sigma^{-1}(\boldsymbol{\mu}-\mathbf{ZW})}$ and $k = 2\frac{\sqrt{(\boldsymbol{\mu}-TM)^T\,\Sigma^{-1}(\boldsymbol{\mu}-TM)}}{\sqrt{(OG-UG)^T\,\Sigma^{-1}(OG-UG)}}$. (20)

4. Example

This is an example for three-dimensional FAD of a screw bold. The three parameters are: position, wrench size, pitch diameter. The whole calculation is given in [4].

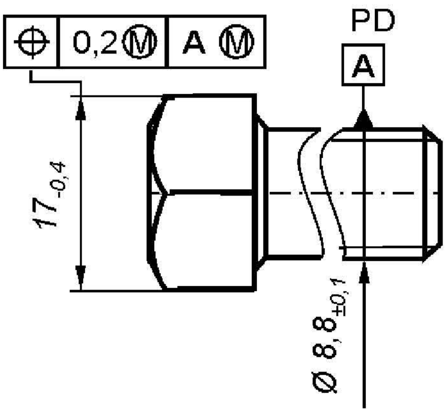

Figure: Three-dimensional case

Table 1 Data overview of the wrench size

1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50
16,7329	16,7997	16,8183	16,8118	16,7329	16,8222	16,7745	16,8010	16,7932	16,7618
16,8213	16,7995	16,8225	16,7997	16,8213	16,7990	16,7884	16,7907	16,7827	16,7760
16,8004	16,8205	16,7897	16,7995	16,8004	16,7642	16,7683	16,7786	16,7929	16,7584
16,7593	16,7613	16,8067	16,8284	16,7593	16,7642	16,7890	16,7947	16,7786	16,7890
16,7525	16,7618	16,8023	16,7314	16,7525	16,7966	16,7934	16,7953	16,7923	16,7645
Mean:		16,7859		Variance:		0,0245			

Table 2 Data overview of the position

1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50
0,1920	0,1277	0,0656	0,1661	0,2411	0,2001	0,2428	0,2270	0,0842	0,2042
0,2320	0,1886	0,1874	0,0524	0,2311	0,1737	0,0388	0,1065	0,1240	0,1221
0,1804	0,1016	0,2100	0,0741	0,1581	0,1665	0,1795	0,2091	0,0719	0,1659
0,0519	0,1677	0,1636	0,2311	0,1330	0,2105	0,0659	0,1534	0,1601	0,2288
0,0387	0,2262	0,2476	0,1122	0,2385	0,0980	0,1744	0,1190	0,0787	0,0933
Mean:		0,0624		Variance:		0,1556			

Table 3 Data Overview of the pitch diameter

1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50
8,8088	8,7997	8,8054	8,8097	8,8077	8,8213	8,8073	8,8183	8,8204	8,8168
8,8113	8,7894	8,7902	8,8176	8,8017	8,8096	8,8158	8,8023	8,7959	8,8105
8,8039	8,7999	8,8154	8,8110	8,8110	8,8173	8,8116	8,8211	8,8092	8,8111
8,8147	8,7961	8,7978	8,8147	8,8094	8,8219	8,8222	8,8101	8,8003	8,8201
8,8110	8,7918	8,8162	8,7977	8,8031	8,8129	8,8354	8,8138	8,8194	8,8121
Mean:		8,8093		Variance:		0,0093			

Three FADs are opposed to conventional indices, see Table 4. You can see clearly that, only in case of considering all important attributes, a correct statement about the quality status of the product is possible. In case of m=3 the product is described by all permitted tolerances and is a capable process. The calculation procedure for higher dimensions is the same.

Table 4: Overview of conventional indices and FADs for three calculated cases

<table>
<tr><th rowspan="2">M</th><th rowspan="2">Attribute</th><th rowspan="2">UT</th><th rowspan="2">LT</th><th rowspan="2">Tar-get</th><th colspan="3">conventional</th><th rowspan="2">Quality feature of attribute</th><th rowspan="2">FAD m</th><th rowspan="2">Quality feature of product</th></tr>
<tr><th>Cp</th><th>Cpk</th><th>Cpm</th></tr>
<tr><td>m=1</td><td>Wrench [mm]</td><td>17,00</td><td>16,60</td><td>17,20</td><td>2,72</td><td>2,52</td><td>2,36</td><td>capable</td><td>2,36</td><td>capable</td></tr>
<tr><td rowspan="2">m=2</td><td>Wrench [mm]</td><td>17,00</td><td>16,60</td><td>17,20</td><td>2,72</td><td>2,52</td><td>2,36</td><td rowspan="2">In-capable</td><td rowspan="2">0,86</td><td rowspan="2">In-capable</td></tr>
<tr><td>Position (without MMC)*</td><td>0,20</td><td>0,00</td><td>0,15</td><td>-</td><td>0,15</td><td>0,32</td></tr>
<tr><td rowspan="3">m=3</td><td>Wrench [mm]</td><td>17,00</td><td>16,60</td><td>17,20</td><td>2,72</td><td>2,52</td><td>2,36</td><td rowspan="3">In-capable</td><td rowspan="3">1,35</td><td rowspan="3">capable</td></tr>
<tr><td>Position (without MMC)*</td><td>0,20</td><td>0,00</td><td>0,15</td><td>-</td><td>0,15</td><td>0,32</td></tr>
<tr><td>pitch-ø [mm]</td><td>8,90</td><td>8,70</td><td>8,80</td><td>3,57</td><td>3,21</td><td>2,46</td></tr>
</table>

5. Conclusion

The most important characteristics of Function Achievement Degree are:

- Replacement for gauging
- Definition of one parameter for the quality feature of complete product
- Consideration of dependent characteristics
- Consideration of interdependent characteristics, if necessary
- For multivariate spaces
- Same quality understanding as for common indices

- Normalized key data

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