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J. Naumenko

## The error control for the numerical computations of the eddy currents

### ENGINEERING DESIGN

A wide range of practical tasks requires numerical computation of magnetic fields in the medium with conducting bodies. Nowadays the theory of these problems is well developed, whereas another important issue - computation of magnetic fields in the presence of multiconnected crack (conducting surface) is paid less attention and provides an area for further study. The main objective of our investigations is to develop a reliable numerical method for these actual problems solution.

The computing of quasi-stationary magnetic fields in presence of the conducting surface (the conducting body with degenerated third dimension) can be reduced to the following operator equations for eddy (Foucault) currents density:

$$\begin{aligned} \mathbf{K}\boldsymbol{\sigma} &= \mathbf{f}_1, \\ \boldsymbol{\delta} &= \lambda \mathbf{K}\boldsymbol{\delta} + \mathbf{f}_2. \end{aligned}$$

The first equation describes eddy currents in case of infinite conductivity of  $S$ , the second equation describes eddy currents in case of finite conductivity of  $S$  in the time-harmonic mode. The multiconnected conducting surface  $S$  and its boundary satisfy the Lipschitz's conditions. Here  $\mathbf{K} = \mathbf{P}\Gamma$ ,  $\Gamma\xi = \frac{1}{4\pi} \iint_S \frac{\xi}{r} dS$ ,  $\lambda = j\mu\gamma h\omega$  is some imaginary

parameter,  $\mu$  is the magnetic permeability of the medium,  $\gamma$  is the conductivity of the surface  $S$ ,  $h$  is the thickness of the  $S$ ,  $\omega$  is the circle frequency of the exiting sources. We suppose  $\mathbf{f}_1, \mathbf{f}_2 \in L^S$ . The space  $L^S$  is the subspace of  $L_2(S)$  ( $L_2(S)$  is the space of two-component complex square-integrable on  $S$  vector functions) and consists of generalized by the Weyl [1] complete solenoidal fields. The definition of orthoprojector  $\mathbf{P}$  is  $\mathbf{P} = \mathbf{P}^L \mathbf{P}^S$  where  $\mathbf{P}^S$  vanishes normal to  $S$  field component and  $\mathbf{P}^L$  is orthoprojector  $L_2(S) \rightarrow L^S$ .

The proof of the existence, uniqueness and numerical stability of the described equations solutions is carried out in [2] in case of  $\mathbf{f}_1 \in W_2^{1/2}(S)$  and  $\mathbf{f}_2 \in L^S$ . Here  $W_2^{1/2}(S)$  is the two-component Sobolev space. The solution  $\boldsymbol{\sigma} \in L_K^S$  and  $\boldsymbol{\delta} \in L^S$ .  $L_K^S$  is the energy space. This proof leads to two inequalities:

$$\begin{aligned} \|\boldsymbol{\sigma}\|_{L_K^S} &\leq \vartheta \|\mathbf{f}_1\|_{W_2^{1/2}(S)}, \\ \|\boldsymbol{\delta}\|_{L^S} &\leq \frac{\|\mathbf{f}_2\|_{L^S}}{1 + |\lambda|/\lambda_1}. \end{aligned}$$

Here  $\lambda_1$  is the first characteristic number of the operator  $\mathbf{K}$ ,  $\vartheta$  is the some constant

independent on  $\mathbf{f}_1$ .

These inequalities open broad possibilities for practice. Since the equations are linear the norms of computational errors of solutions  $\sigma, \delta$  can be a posteriori estimated by the residuals of free members  $\mathbf{f}_1, \mathbf{f}_2$ .

The subprogram for a posteriori estimation of the computational error of the numerical solutions was built on the base of the theory described.

**References:**

- [1] Weyl H. The method of orthogonal projection in potential theory // Duke Math. J. 1940. V. 7. P. 411 – 444.
- [2] Naumenko J. The calculation of sinusoidal in time magnetic in presence of conductive surfaces by integral equations method // Studies of the Southern Scientific Centre of the Russian Academy of Sciences 2007, V. 2. P. 80-93.

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