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E. Jarzębowska

CONTROL ORIENTED DYNAMIC MODELING OF WHEELED MOBILE SYSTEMS

1. INTRODUCTION

Dynamic modeling of nonholonomic systems is an important step in a process of design control algorithms in the nonlinear control theory (NCT). Wheeled mobile systems (WMS) are a large class of nonholonomic systems of a practical significance. From the point of view of mechanics, all the WMS as well as other systems with first order nonholonomic constraints (NC) belong to the same class of systems. Specifically, for all WMS equations of motion may be derived using Lagrange's approach [8,10]. Lagrange's equations contain multipliers that have to be eliminated before a controller is designed, i.e. the reduction procedure is required, before a dynamic control model is developed. A few papers consider dynamics modeling using different approaches, e.g. in [15,17] Kane's equations are used to develop a dynamic control model of a mobile manipulator. In [1] the Boltzmann-Hamel equations are modified to facilitate modeling manipulator systems as well as the WMS. In [9] the NC formulation in quasi-coordinates serves a control algorithm design for a helicopter. In [13] a ski-steering wheeled mining vehicle is modeled by the Boltzmann-Hamel equations. Both Lagrange's and Kane's, and the Boltzmann-Hamel equations serve systems with first order constraints and relations between quasi-velocities and generalized velocities are all linear there.

From the NCT perspective the WMS differ and may not be approached by the same control strategies and algorithms. For instance, some of the WMS may be controlled at the kinematic level and the other may be controlled at the dynamic level only [2,5,7]. Their control properties depend upon the way they are designed and propelled, i.e. how many control inputs are available and whether their wheels are powered or not. They are divided into two control groups, which are approached separately, i.e. the group of fully actuated and the group of underactuated WMS.

A control design process consists of three main steps, i.e. a model building, a controller design and a controller implementation. Most modifications and improvements concern the second and third step of this process. Modeling is a central step of the overall control design process. The NCT uses modeling methods offered by analytical mechanics and it requires dynamic models in symbolic forms for designing control algorithms, see e.g. [8]. There are other reasons for which the selection of a modeling method as well as coordinates that specify motion may be significant, e.g. the control algorithms are nonlinear, their implementation is achieved on-line and the WMS are nonholonomic systems what may cause numerical stability problems [3].

The WMS are designed to perform a variety of tasks. It means that their motion or motion properties may be predefined. Also, their design, operation and control properties may impose other constraints on motion or control. To the best of the author's knowledge, the NCT uses modeling methods of systems with first order constraints and other constraints imposed on systems are satisfied at the step of designing or implementing control algorithms.

Main motivations for the research we present are to unify the WMS modeling and a subsequent control design. The paper deals with the first two steps of the controller design process and it presents a modeling framework that is model-based and control oriented. This modeling framework does not rely on Lagrange's approach. It is not sensitive to the WMS design and constraints put on it, and it facilitates the step of the dynamic controller design. Thus, the dynamic modeling framework serves a unification of the WMS modeling with no regard whether a specific WMS is fully actuated, underactuated, or constrained by additional constraints.

The contribution of the paper is two-folded. First, we demonstrate that a unified model-based control oriented framework may be formulated for the WMS that may be additionally constrained by task-based constraints. The framework is unified in the sense that is suitable for either fully actuated or underactuated WMS modeling and a subsequent controller design for them. Second, we demonstrate that the framework may be developed in suitably selected coordinates like quasi-coordinates and a controller may be designed based on either formulation.

Based on examples of the WMS we demonstrate how to apply this modeling framework to obtain constrained dynamic models and how to reuse them to design tracking control algorithms. Specifically, we select one fully actuated system which is (2,0) type robot and one underactuated system - a roller-racer [2]. We demonstrate that for these systems one modeling framework may be developed using quasi-coordinates. Also, we demonstrate that the specification of all constraints, if possible, at the dynamic model building step, may simplify the control design process. The reason is that many of the task-based or design and control constraints are typical for the WMS. Their inclusion into the constrained dynamics may prevent from a control algorithm modification. Instead, existing control algorithms may be employed. Based on the control-oriented dynamics in quasi-coordinates we design tracking controllers for these systems.

2. CONSTRAINTS ON WHEELED MOBILE SYSTEMS

WMS are assumed to satisfy first order NC that come from the condition of rolling their wheels without slipping. We refer to them as material constraints, since they come from the contact between bodies, i.e. between the wheel and the ground. We do not consider the wheel slip in the paper. Also, the WMS are constrained by other constraints, e.g.:

- design and control constraints that come from the passive wheels and/or underactuation,
- limitations on motors that may be applied to a specific WMS,
- limitations on linear and angular velocities the WMS may reach,
- limitations on accelerations the WMS may reach,
- limitations on a trajectory curvature and its rate of change,
- other limitations caused by the WMS work space.

The constraints listed above are satisfied, of course, in the control design process for the WMS but they are not merged into the constrained dynamics, which is a basis for a nonlinear controller design. Also, one more specification, the most important from the control perspective, which is not recognized as a constraint, is a control goal, i.e. a trajectory to follow in a traditional NCT setup. The only constraints merged into the constrained dynamics are the material first order NC. The constraints and other motion limitations as well as control goals setup in the NCT is significant for the development of a modeling framework presented in the paper.

A modification of the first step of a control-oriented modeling is an introduction of a unified constraint formulation of the form [3,5,6]

$$B(t,q,\dot{q},\ldots,q^{(p-1)})q^{(p)} + s(t,q,\dot{q},\ldots,q^{(p-1)}) = 0\,, \tag{1}$$

where p is the constraint order, q is a n-vector of generalized coordinates, B is a full rank $(k\times n)$ matrix with $n>k$ and s is a k-vector. We assume that (1) are linear in p–th order derivative of coordinates or we can transform them to this form. They may specify both material and non-material constraints on systems. The latter ones are referred to as programmed [3,6]. The type of the constraint equation does not influence the generation of equations of motion of a system subjected to it. The only concern is the constraint order and whether it is ideal. For order $p=0$ we get a position constraint, which may be material and specify for example a constant distance between link ends or be a programmed constraint on a trajectory for a system. When $p=1$ a constraint equation may be material and specify a condition of rolling without slipping. However, it may be a programmed constraint on a desired velocity of a system. Material constraints are of orders $p=0$ or $p=1$, and constraint equations for $p>1$ are of the non-material type. Other examples

of the programmed constraints can be found in [3,5]. For equation (1) we introduce a definition.
Definition 1 [6]: The equations of constraints (1) are completely nonholonomic if they cannot be integrated with respect to time, i.e. we cannot obtain constraint equations of a lower order.
If we can integrate equations (1) *(p-1)* or fewer times, i.e. we can obtain NC of order *1* or order lower than *p*, we say that (1) are partially integrable. If (1) can be integrated completely, we say that they are holonomic. We assume that equations (1) are completely nonholonomic. Our definition is an extension of a definition of completely nonholonomic first order constraints [8,12] and completely nonholonomic second order constraints [14]. Necessary and sufficient integrability conditions for differential equations of arbitrary order such as (1) are proved in [16].

3. MODEL REFERENCE TRACKING CONTROL STRATEGY FOR PROGRAMMED MOTION

In this section we report briefly the development of the control-oriented modeling framework for constrained systems and a strategy for tracking predefined motions. Trajectory tracking which is the most common tracking task is a peculiar case of motion tracking.
A control goal in our control setup is formulated as follows: *Given programmed constraints and material constraints on the WMS specified by equations (1), design a feedback controller to track a desired programmed motion.*

A tracking control strategy dedicated to track programmed motions is designed in [3,4,6]. It is referred to as the model reference tracking control strategy for programmed motion. Architecture of the strategy is presented in figure 1. It is based on two dynamic models both derived in generalized coordinates.

The first one is the reference dynamic model which governs motion equations of a system subjected to NC, either material, programmed or both, specified by (1). This is the reference dynamics block. It consists of the generalized programmed motion equations in the form [3]

$$\begin{aligned} &M(q)\ddot{q} + V(q,\dot{q}) + D(q) = Q(t,q,\dot{q}), \\ &B(t,q,\dot{q},\ldots,q^{(p-1)})q^{(p)} + s(t,q,\dot{q},\ldots,q^{(p-1)}) = 0, \end{aligned} \tag{2}$$

where $M(q)$ is a $(n\text{-}k)\times n$ inertia matrix, $V(q,\dot{q})$ is a $(n\text{-}k)$-velocity dependent vector, $D(q)$ is a $(n\text{-}k)$-vector of gravity forces, and $Q(t,q,\dot{q})$ is a $(n\text{-}k)$-vector of external forces. Equations (2) are referred to as a unified dynamic model of a constrained system since it may be also constrained by non-material constraints, which do not come from the condition of rolling without slipping. They may specify desired trajectories, desired velocities or other system motion properties [3,5].

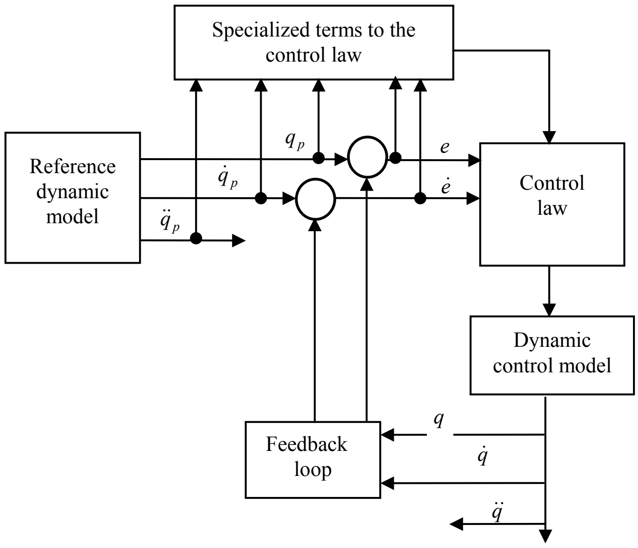

Figure 1. Architecture of the model reference tracking control strategy for programmed motion

Comparing (2) with the equations of motion that classical analytical mechanics offers, it can be seen that any constraints on systems specified by differential equations can be merged into (2). The reference dynamics (2) admits the following properties.

Property 1: Equations (2) are free of constraint reaction forces which are eliminated in the derivation process, i.e. they are in the reduced-state form.

Property 2: Dynamic models of systems with the constraints of order $p=1$ derived by the Lagrange approach and transformed to the reduced-state form are peculiar cases of (2) [3].

The second is the dynamic control model which takes into account only material constraints on the system. This is the control dynamics block. The same framework is used to develop it, for which $p=1$, i.e.

$$\begin{aligned} M(q)\ddot{q} + C(q,\dot{q})\dot{q} + D(q) &= \tau_p, \\ B_1(q)\dot{q} &= 0. \end{aligned} \tag{3}$$

Equations (3) are referred to as a unified dynamic control model. They consist of $(n-k)$ equations of motion and k equations of material constrains. The matrix $M(q)$ is then $(n-k)\times n$ and $B_1(q)$ is a full rank $(k\times n)$ matrix. Since the constraints are material, linear first order, $V(q,\dot{q})$ is replaced by $C(q,\dot{q})\dot{q}$, which quantifies effects of Coriolis and centripetal forces. We assume that other forces or disturbances can be added to the left-hand side of (3).

Outputs of the reference dynamics are inputs to the control law. The control law, in turn, can be plugged into the control dynamics.

The reference dynamics (2) serves programmed motion planning. It is defined as follows.

Definition 2 [3]: Programmed motion planning consists in finding time histories of positions $q_p(t)$ and their time derivatives in motion consistent with the constraints.

Specifically, in this formulation trajectory planning consists in obtaining a solution $q_p(t)$ of equations (2), in which a programmed constraint equation is algebraic. Solutions of (2) also serve verification whether a programmed constraint is reasonable for a system. By "reasonable" we mean that the system is capable of reaching desired positions, velocities and accelerations needed to follow the program, and the programmed constraint does not violate any material constraint.

Equations (3) admit a couple of properties that can be derived from properties 1 and 2 [3,6]. For the WMS two additional properties can be derived.

Property 3: The unified dynamic control model (3) may be applied for underactuated systems. Indeed, let us start a formulation of a control dynamics for an underactuated system from (3) derived for a holonomic system, i.e.

$$M(q)\ddot{q} + C(q,\dot{q}) = E(q)\tau. \tag{4}$$

We assume that q is a $(n\times 1)$-dimensional vector of generalized coordinates that belongs to some configuration manifold Ω, τ are independent control vectors and $\tau \in R^m, m < n$. The matrix $E(q) \in R^{n\times m}$, which means that there is fewer control inputs than degrees of freedom. Now, let us assume that actuated degrees of freedom are represented by elements $q_1 \in R^m$ and unactuated by elements $q_2 \in R^{n-m}$. Partition of the generalized coordinate vector is then $q = (q_1, q_2)$. After some rearrangement of coordinates to obtain vectors q_1 and q_2, the control dynamics (4) can be written as

$$\begin{aligned} M_{11}(q)\ddot{q}_1 + M_{12}(q)\ddot{q}_2 + C_1(q,\dot{q}) &= E_1(q)\tau, \\ M_{21}(q)\ddot{q}_1 + M_{22}(q)\ddot{q}_2 + C_2(q,\dot{q}) &= 0, \end{aligned} \tag{5}$$

where $C_1(q,\dot{q}) \in R^m$, $C_2(q,\dot{q}) \in R^{n-m}$, $M_{ij}, i,j = 1,2,$ are components of the $(n\times n)$ inertia matrix $M(q)$. The matrix $E_1(q) \in R^{m\times m}$ is invertible for all $q\in\Omega$ and it is obtained from $E(q)$ partition, i.e. $E(q) = [E_1(q), 0]^T$. The second equation in (5) can be solved for $\ddot{q}_2$ such that

$$\ddot{q}_2 = -M_{22}^{-1}(q)[M_{21}(q)\ddot{q}_1 + C_2(q,\dot{q})]$$

and substituted to the first one yields

$$\hat{M}(q)\ddot{q}_1 + \hat{C}(q,\dot{q}) = E_1(q)\tau\ , \tag{6}$$

where $\hat{M}(q) = M_{11}(q) - M_{12}(q)M_{22}^{-1}(q)M_{21}(q), \hat{C}(q,\dot{q}) = C_1(q,\dot{q}) - M_{12}(q)M_{22}^{-1}(q)C_2(q,\dot{q}).$

Application of the partial feedback linearizing controller of the form

$$\tau = E_1^{-1}(q)\left[\hat{M}(q)u + \hat{C}(q,\dot{q})\right] \tag{7}$$

transforms equations (6) to

$$\begin{aligned} \ddot{q}_1 &= u, \\ \ddot{q}_2 &= R(q)\ddot{q}_1 + H(q,\dot{q}), \end{aligned} \tag{8}$$

and $R(q) = -M_{22}^{-1}(q)M_{21}(q),\ H(q,\dot{q}) = -M_{22}^{-1}(q)C_2(q,\dot{q})$.

Property 4: The unified dynamic control model (3) may be applied to the underactuated WMS in the same way as to other underactuated systems.

Indeed, let us start from equations (3) and write them as

$$\begin{aligned} M(q)\ddot{q}_1 + C(q,\dot{q}_1) &= E(q)\tau, \\ \ddot{q} &= G(q)\ddot{q}_1 + \dot{G}(q)\dot{q}_1. \end{aligned} \tag{9}$$

Since the constraint equation must be satisfied, it is enough to choose a control law such that $q_1 \to q_p$ for $q_1 \in R^m, q_2 \in R^k$, where the number of degrees of freedom is $m=n-k$. Then $E(q) \in R^{m\times m}$ and $\tau \in R^m, m < n$. Assume now that we have an underactuated system, for which *s*, *s*<*m*, is the number of inputs. We partition q_1 as $q_1 = (q_{1a}, q_{1f})$, where $q_{1a} \in R^s$, $q_{1f} \in R^{m-s}$, and subscripts "*a*" and "*f*" stand for "actuated" and "free", respectively. Then, equations (6) can be presented as

$$\overline{M}(q)\ddot{q} + \overline{C}(q,\dot{q}) = \overline{E}(q)\tau, \tag{10}$$

where now *q* is of the form $q = (q_{1a}, q_{1f}, q_2)$, $\overline{M}$ is a *(n×n)* kind of inertia matrix extended to include terms from the constraint equation, and $\overline{C}$ and $\overline{E}$ are

$$\begin{gathered} \overline{M}(q) = \begin{bmatrix} M_{11}(q) & M_{12}(q) & 0_{s\times k} \\ M_{21}(q) & M_{22}(q) & 0_{(m-s)\times k} \\ G_{3a}(q) & G_{3f}(q) & I_{k\times k} \end{bmatrix}, \\ \overline{C}(q,\dot{q}) = \begin{bmatrix} C_1(q,\dot{q}) \\ C_2(q,\dot{q}) \\ 0 \end{bmatrix}, \qquad \overline{E}(q) = \begin{bmatrix} E_1(q) \\ 0 \\ 0 \end{bmatrix}. \end{gathered} \tag{11}$$

The last row in $\overline{M}$ consists of submatrices that result from the partition of *G(q)*, i.e.

$$G(q) = \begin{bmatrix} I_{s\times s} & 0 \\ 0 & I_{(m-s)\times(m-s)} \\ G_{3a} & G_{3f} \end{bmatrix},$$

The vector $\overline{C}(q,\dot{q})$ components are $C_1 \in R^s, C_2 \in R^{m-s}$ and $C_3 \in R^k$, which is a zero vector. Let us now reorder and rename the coordinates, i.e. $q_{1a} = q_1$, $q_1 \in R^s$ and $(q_{1f}, q_2) = q_2$, $q_2 \in R^{n-s}$.

Then, equations (10) with the coordinate vector $q=(q_1,q_2)$ are equivalent to (5). Consequently, we can apply the partial feedback linearizing controller in the form

$$\tau = E_1^{-1}(q)\left[\overline{M}(q)u+\overline{C}(q,\dot{q})\right] \tag{12}$$

which transforms (10) into (8).

Property 5: Based on properties 1-4, all theoretical control results obtained for the Lagrange based dynamic control models can be applied to the unified dynamic control model (3).

4. THE REFERENCE DYNAMIC MODEL BASED ON THE GENERALIZED FORM OF THE BOLTZMANN-HAMEL EQUATIONS

In this section we present some preliminary results in the area of modeling the WMS dynamics in quasi-coordinates when all constraints are first order. We assume that relations between generalized velocities and quasi-velocities may be nonlinear, i.e.

$$\omega_\beta = \omega_\beta(t,q_\sigma,\dot{q}_\sigma)=0 \qquad \sigma=1,\dots,n\,,\quad \beta=1,\dots,b \tag{13}$$

as opposed to e.g. [1,11] when these relations are linear. Also, we assume that inverse transformations for (13) can be computed in the considered state space, i.e.

$$\dot{q}_\lambda = \dot{q}_\lambda(t,q_\sigma,\omega_r). \qquad \lambda=1,\dots,n \tag{14}$$

Quasi-coordinates can be introduced by the relations

$$d\pi_r = \sum_{\sigma=1}^{n}\frac{\partial\omega_r}{\partial\dot{q}_\sigma}dq_\sigma \qquad r=1,\dots,n \tag{15}$$

and the assumption that (15) are non-integrable holds. Based on (13)–(15), the generalized coordinates are related to the quasi-coordinates as

$$dq_\lambda = \sum_{\mu=1}^{n}\frac{\partial\dot{q}_\lambda}{\partial\omega_\mu}d\pi_\mu\,. \qquad \lambda=1,\dots,n \tag{16}$$

Based on (15) and (16) variations of quasi-coordinates and generalized coordinates are related as

$$\delta\pi_r = \sum_{\sigma=1}^{n}\frac{\partial\omega_r}{\partial\dot{q}_\sigma}\delta q_\sigma, \qquad \delta q_\lambda = \sum_{\mu=1}^{n}\frac{\partial\dot{q}_\lambda}{\partial\omega_\mu}\delta\pi_\mu \qquad r,\lambda=1,\dots,n \tag{17}$$

For the purpose of further development, consider an arbitrary function $\varphi=\varphi(q_\sigma),\sigma=1,\dots,n,$ from C^2 and compute its total differential using (16), i.e.

$$d\varphi = \sum_{\lambda=1}^{n}\frac{\partial\varphi}{\partial q_\lambda}dq_\lambda = \sum_{\lambda=1}^{n}\frac{\partial\varphi}{\partial q_\lambda}\sum_{\mu=1}^{n}\frac{\partial\dot{q}_\lambda}{\partial\omega_\mu}d\pi_\mu = \sum_{\mu=1}^{n}\left(\sum_{\lambda=1}^{n}\frac{\partial\varphi}{\partial q_\lambda}\frac{\partial\dot{q}_\lambda}{\partial\omega_\mu}\right)d\pi_\mu\,.$$

Denoting by

$$\frac{\partial \varphi}{\partial \pi_\mu} = \sum_{\lambda=1}^{n} \frac{\partial \varphi}{\partial q_\lambda} \frac{\partial \dot{q}_\lambda}{\partial \omega_\mu}, \qquad \mu = 1,\dots,n \tag{18}$$

the total differential of $\varphi = \varphi(q_\sigma)$ becomes

$$d\varphi = \sum_{\mu=1}^{n} \frac{\partial \varphi}{\partial \pi_\mu} d\pi_\mu . \tag{19}$$

Relation (19) is the operator formula of the form

$$\frac{\partial \cdot}{\partial \pi_\mu} = \sum_{\lambda=1}^{n} \frac{\partial \cdot}{\partial q_\lambda} \frac{\partial \dot{q}_\lambda}{\partial \omega_\mu} . \qquad \mu = 1,\dots,n \tag{20}$$

Based on (20) the inverse formula is as follows

$$\frac{\partial \cdot}{\partial q_\lambda} = \sum_{r=1}^{n} \frac{\partial \cdot}{\partial \pi_r} \frac{\partial \omega_r}{\partial \dot{q}_\mu} . \qquad \lambda = 1,\dots,n \tag{21}$$

Now, take the principal form of the dynamics motion equation [11]

$$\frac{d}{dt} \sum_{\sigma=1}^{n} p_\sigma \delta q_\sigma = \delta T + \sum_{\sigma=1}^{n} Q_\sigma \delta q_\sigma + \sum_{\sigma=1}^{n} p_\sigma \left[(\delta q_\sigma)^{\cdot} - \delta \dot{q}_\sigma \right] \delta q_\sigma \tag{22}$$

and transform its left hand side using (17), i.e.

$$\sum_{\lambda=1}^{n} p_\lambda \delta q_\lambda = \sum_{\lambda=1}^{n} p_\lambda \sum_{\mu=1}^{n} \frac{\partial \dot{q}_\lambda}{\partial \omega_\mu} \delta \pi_\mu = \sum_{\mu=1}^{n} \left(\sum_{\lambda=1}^{n} p_\lambda \frac{\partial \dot{q}_\lambda}{\partial \omega_\mu} \right) \delta \pi_\mu .$$

Denoting by

$$\tilde{p}_\mu = \sum_{\lambda=1}^{n} p_\lambda \frac{\partial \dot{q}_\lambda}{\partial \omega_\mu}, \qquad \mu = 1,\dots,n \tag{23}$$

the following holds

$$\sum_{\lambda=1}^{n} p_\lambda \delta q_\lambda = \sum_{\mu=1}^{n} \tilde{p}_\mu \delta \pi_\mu . \tag{24}$$

Now, denote by $\tilde{T}$ the kinetic energy of a mechanical system in which the generalized velocities are replaced by the quasi-velocities according to (14), i.e.

$$T = T(t, q_\sigma, \dot{q}_\lambda(t, q_\sigma, \omega_r)) = \tilde{T}(t, q_\sigma, \omega_\sigma) . \qquad \sigma, r, \lambda = 1,\dots,n, \tag{25}$$

Based on (17), the virtual work of external forces $Q_\lambda, \lambda = 1,\dots,n,$ that correspond to virtual displacements δq_λ can be written as

$$\delta A^{'} = \sum_{\lambda=1}^{n} Q_\lambda \delta q_\lambda = \sum_{\lambda=1}^{n} Q_\lambda \sum_{\mu=1}^{n} \frac{\partial \dot{q}_\lambda}{\partial \omega_\mu} \delta \pi_\mu = \sum_{\mu=1}^{n} \left(\sum_{\lambda=1}^{n} Q_\lambda \frac{\partial \dot{q}_\lambda}{\partial \omega_\mu} \right) \delta \pi_\mu .$$

Denoting by

$$\tilde{Q}_\mu = \sum_{\lambda=1}^{n} Q_\lambda \frac{\partial \dot{q}_\lambda}{\partial \omega_\mu} \qquad \mu = 1,\dots,n \qquad (26)$$

we obtain that

$$\sum_{\lambda=1}^{n} Q_\lambda \delta q_\lambda = \sum_{\mu=1}^{n} \tilde{Q}_\mu \delta \pi_\mu \ . \qquad (27)$$

Based on (13) and (17) we have that

$$\delta\omega_r = \sum_{\lambda=1}^{n} \frac{\partial \omega_r}{\partial q_\lambda}\delta q_\lambda + \sum_{\lambda=1}^{n} \frac{\partial \omega_r}{\partial \dot{q}_\lambda}\delta \dot{q}_\lambda \ ,$$

$$(\delta\pi_r)^{\cdot} = \sum_{\lambda=1}^{n}\left(\frac{\partial \omega_r}{\partial \dot{q}_\lambda}\delta q_\lambda\right)^{\cdot} = \sum_{\lambda=1}^{n} \frac{\partial \omega_r}{\partial \dot{q}_\lambda}(\delta q_\lambda)^{\cdot} + \sum_{\lambda=1}^{n}\frac{d}{dt}\left(\frac{\partial \omega_r}{\partial \dot{q}_\lambda}\right)\delta q_\lambda \ , \qquad \lambda = 1,\dots,n$$

and hence

$$(\delta\pi_r)^{\cdot} - \delta\omega_r = \sum_{\lambda=1}^{n} \frac{\partial \omega_r}{\partial \dot{q}_\lambda}\left[(\delta q_\lambda)^{\cdot} - \delta\dot{q}_\lambda\right] + \sum_{\lambda=1}^{n}\left[\frac{d}{dt}\left(\frac{\partial \omega_r}{\partial \dot{q}_\lambda} - \frac{\partial \omega_r}{\partial q_\lambda}\right)\right]\delta q_\lambda \ ,$$

what may be written as

$$\sum_{\lambda=1}^{n} \frac{\partial \omega_r}{\partial \dot{q}_\lambda}\left[(\delta q_\lambda)^{\cdot} - \delta\dot{q}_\lambda\right] = (\delta\pi_r)^{\cdot} - \delta\omega_r - \sum_{\lambda=1}^{n}\left[\frac{d}{dt}\left(\frac{\partial \omega_r}{\partial \dot{q}_\lambda} - \frac{\partial \omega_r}{\partial q_\lambda}\right)\right]\delta q_\lambda \ . \quad r = 1,\dots,n \qquad (28)$$

Multiplying both sides of (28) by $\tilde{p}_r$, summing it over *n* and using the second of relations (17) yields

$$\sum_{r=1}^{n}\tilde{p}_r \sum_{\lambda=1}^{n} \frac{\partial \omega_r}{\partial \dot{q}_\lambda}\left[(\delta q_\lambda)^{\cdot} - \delta\dot{q}_\lambda\right] = \sum_{r=1}^{n}\tilde{p}_r\left[(\delta\pi_r)^{\cdot} - \delta\omega_r\right] - \sum_{r=1}^{n}\sum_{\mu=1}^{n}\sum_{\lambda=1}^{n}\tilde{p}_r \frac{\partial \dot{q}_\lambda}{\partial \omega_\mu}\left[\frac{d}{dt}\left(\frac{\partial \omega_r}{\partial \dot{q}_\lambda} - \frac{\partial \omega_r}{\partial q_\lambda}\right)\right]\delta\pi_\mu \ .$$

Based on (23) $p_\lambda = \sum_{r=1}^{n}\tilde{p}_r \frac{\partial \omega_r}{\partial \dot{q}_\lambda}$, $\lambda = 1,\dots,n$ and hence we get

$$\sum_{\lambda=1}^{n} p_\lambda\left[(\delta q_\lambda)^{\cdot} - \delta\dot{q}_\lambda\right] = \sum_{r=1}^{n}\tilde{p}_r\left[(\delta\pi_r)^{\cdot} - \delta\omega_r\right] - \sum_{r=1}^{n}\sum_{\mu=1}^{n}\sum_{\lambda=1}^{n}\tilde{p}_r \frac{\partial \dot{q}_\lambda}{\partial \omega_\mu}\left[\frac{d}{dt}\left(\frac{\partial \omega_r}{\partial \dot{q}_\lambda}\right) - \frac{\partial \omega_r}{\partial q_\lambda}\right]\delta\pi_\mu \ . \qquad (29)$$

By introducing the following notation

$$W_\mu^r = \sum_{\lambda=1}^{n}\frac{\partial \dot{q}_\lambda}{\partial \omega_\mu}\left[\frac{d}{dt}\left(\frac{\partial \omega_r}{\partial \dot{q}_\lambda}\right) - \frac{\partial \omega_r}{\partial q_\lambda}\right], \qquad r,\mu = 1,\dots,n \ , \qquad (30)$$

(29) may be transformed to the form

$$\sum_{\lambda=1}^{n} p_\lambda\left[(\delta q_\lambda)^{\cdot} - \delta\dot{q}_\lambda\right] = \sum_{r=1}^{n}\tilde{p}_r\left\{\left[(\delta\pi_r)^{\cdot} - \delta\omega_r\right] - \sum_{\mu=1}^{n} W_r^\mu \delta\pi_\mu\right\}. \qquad (31)$$

Inserting (24), (25), (27) and (31) into (22) we finally obtain

$$\frac{d}{dt}\sum_{\mu=1}^{n}\tilde{p}_\mu\delta\pi_\mu = \delta\tilde{T} + \sum_{\mu=1}^{n}\tilde{Q}_\mu\delta\pi_\mu + \sum_{r=1}^{n}\tilde{p}_r\left[(\delta\pi_r)^{\cdot} - \delta\omega_r\right] - \sum_{r=1}^{n}\tilde{p}_r\sum_{\mu=1}^{n}W_\mu^r\delta\pi_\mu \ . \qquad (32)$$

Equation (32) is the principal form of the equation of motion of a mechanical system developed in quasi-coordinates for nonlinear relations between quasi-velocities and generalized velocities. When these relations are linear, equation (32) becomes the equation obtained in [11]. Symbols W_μ^r may be regarded as the generalized Boltzmann symbols for the nonlinear relations between the quasi-velocities and generalized velocities.

To develop the motion equations in the non-inertial coordinates we can start from equation (32). Based on (25) and (23), we have

$$\frac{\partial \tilde{T}}{\partial \omega_\mu} = \sum_{\lambda=1}^{n} \frac{\partial T}{\partial \dot{q}_\lambda} \frac{\partial \dot{q}_\lambda}{\partial \omega_\mu} = \tilde{p}_\mu . \tag{33}$$

Using (33), the left hand side term of (32) can be developed as

$$\frac{d}{dt} \sum_{\mu=1}^{n} \tilde{p}_\mu \delta\pi_\mu = \sum_{\mu=1}^{n} \frac{d}{dt}\left(\frac{\partial \tilde{T}}{\partial \omega_\mu}\right)\delta\pi_\mu + \sum_{\mu=1}^{n} \tilde{p}_\mu \left(\delta\pi_\mu\right)^{\cdot}. \tag{34}$$

First, third and fourth terms on the right hand side of (32) can be transformed as

$$\delta\tilde{T} = \sum_{\mu=1}^{n} \frac{\partial \tilde{T}}{\partial q_\mu}\delta q_\mu + \sum_{\mu=1}^{n} \frac{\partial \tilde{T}}{\partial \omega_\mu}\delta\omega_\mu = \sum_{\alpha=1}^{n}\left(\frac{\partial \tilde{T}}{\partial q_\mu}\frac{\partial \dot{q}_\mu}{\partial \omega_\alpha}\right)\delta\pi_\alpha + \sum_{\mu=1}^{n} \frac{\partial \tilde{T}}{\partial \omega_\mu}\delta\omega_\mu$$

and due to (20)

$$\delta\tilde{T} = \sum_{\mu=1}^{n} \frac{\partial \tilde{T}}{\partial \pi_\mu}\delta\pi_\mu + \sum_{\mu=1}^{n} \frac{\partial \tilde{T}}{\partial \omega_\mu}\delta\omega_\mu = \sum_{\mu=1}^{n}\left(\frac{\partial \tilde{T}}{\partial \pi_\mu}\delta\pi_\mu + \frac{\partial \tilde{T}}{\partial \omega_\mu}\delta\omega_\mu\right), \tag{35}$$

where the subscript α is replaced by μ in the first sum on the right hand side. Next

$$\sum_{r=1}^{n} \tilde{p}_r\left[(\delta\pi_r)^{\cdot} - \delta\omega_r - \sum_{\mu=1}^{n} W_\mu^r \delta\pi_\mu\right] = \sum_{r=1}^{n} \tilde{p}_r (\delta\pi_r)^{\cdot} - \sum_{r=1}^{n} \frac{\partial \tilde{T}}{\partial \omega_r}\delta\omega_r - \sum_{r=1}^{n}\sum_{\mu=1}^{n} \frac{\partial \tilde{T}}{\partial \omega_r} W_\mu^r \delta\pi_\mu .$$

Replacing the subscript r by μ in the first and second sums on the right hand side of the last relation, we obtain

$$\sum_{r=1}^{n} \tilde{p}_r\left[(\delta\pi_r)^{\cdot} - \delta\omega_r - \sum_{\mu=1}^{n} W_\mu^r \delta\pi_\mu\right] = \sum_{\mu=1}^{n} \tilde{p}_\mu \left(\delta\pi_\mu\right)^{\cdot} - \sum_{\mu=1}^{n} \frac{\partial \tilde{T}}{\partial \omega_\mu}\delta\omega_\mu - \sum_{r=1}^{n}\sum_{\mu=1}^{n} \frac{\partial \tilde{T}}{\partial \omega_r} W_\mu^r \delta\pi_\mu . \tag{36}$$

Relations (34)-(36) inserted into (32) and after terms rearrangement yield

$$\sum_{\mu=1}^{n}\left[\frac{d}{dt}\left(\frac{\partial \tilde{T}}{\partial \omega_\mu}\right) - \frac{\partial \tilde{T}}{\partial \pi_\mu} + \sum_{r=1}^{n} \frac{\partial \tilde{T}}{\partial \omega_r} W_\mu^r - \tilde{Q}_\mu\right]\delta\pi_\mu = 0. \tag{37}$$

For a holonomic system, variations $\delta\pi_\mu, \mu = 1,...,n$, are independent and equations of motion are

$$\frac{d}{dt}\left(\frac{\partial \tilde{T}}{\partial \omega_\mu}\right) - \frac{\partial \tilde{T}}{\partial \pi_\mu} + \sum_{r=1}^{n} \frac{\partial \tilde{T}}{\partial \omega_r} W_\mu^r = \tilde{Q}_\mu . \qquad \mu = 1,...,n \tag{38}$$

Equations (38) are the generalized Boltzmann-Hamel equations for the holonomic system.

Relations between the quasi-velocities and generalized velocities may be nonlinear as (13). For linear relations we obtain the Boltzmann-Hamel equations derived e.g. in [11]. It can be verified easily that when quasi-coordinates are equivalent to generalized coordinates, i.e. $\pi_r = q_r, r = 1,...,n$, and quasi-velocities are equivalent to generalized velocities, i.e. $\omega_r = \dot{q}_r, r = 1,...,n$, then (38) are Lagrange's equations with $W^r_\mu = \gamma^r_{\alpha\mu} = 0,\ \alpha, \mu, r = 1,...,n$.

Consider now a system subjected to material or programmed NC. Assume that the constraints are of the first order and have the form (13), i.e.

$$\omega_\beta = \omega_\beta(t, q_\sigma, \dot{q}_\sigma) = 0 . \qquad \beta = 1,...,b \qquad (39)$$

Based on (17), the relations

$$\delta\pi_\beta = \sum_{\sigma=1}^{n} \frac{\partial \omega_\beta}{\partial \dot{q}_\sigma} \delta q_\sigma = 0, \qquad \beta = 1,...,b \qquad (40)$$

hold for all ω_β and hence equations (37) can be written as

$$\sum_{\mu=b+1}^{n} \left[\frac{d}{dt}\left(\frac{\partial \widetilde{T}}{\partial \omega_\mu} \right) - \frac{\partial \widetilde{T}}{\partial \pi_\mu} + \sum_{r=1}^{n} \frac{\partial \widetilde{T}}{\partial \omega_r} W^r_\mu - \widetilde{Q}_\mu \right] \delta\pi_\mu = 0 . \qquad (41)$$

The system possesses *(n-b)* degrees of freedom and the variations $\delta\pi_{b+1},...,\delta\pi_n$ are independent. We obtain then *(n-b)* equations of motion in the form

$$\frac{d}{dt}\left(\frac{\partial \widetilde{T}}{\partial \omega_\mu} \right) - \frac{\partial \widetilde{T}}{\partial \pi_\mu} + \sum_{r=1}^{n} \frac{\partial \widetilde{T}}{\partial \omega_r} W^r_\mu = \widetilde{Q}_\mu \qquad \mu = b+1,...,n \qquad (42)$$

to which *n* kinematic relations

$$\dot{q}_\lambda = \dot{q}_\lambda(t, q_\sigma, \omega_r) \qquad \sigma, \lambda = 1,...,n, \quad r = b+1,...,n \qquad (43)$$

have to be added. Equations (42) are the generalized Boltzmann-Hamel equations for a nonholonomic system. Equations (42) and (43) consist of *(2n-b)* equations for *n* unknown *q's* and *(n-b) ω's*. Notice that *b* of *ω's* are satisfied based on the specification of the constraint equations (39). The rest of quasi-velocities are selected arbitrarily by a designer or a control engineer. We may say that the first order constraints are "swallowed" by these *b ω's* and the rest of *(n-b) ω's* are selected arbitrarily. These are the main advantages of the introduction of the quasi-velocities. Equations (42) and (43) can be presented as first order differential equations in *ω's*

$$\begin{gathered} M(q)\dot{\omega} + C(q,\omega) + D(q) = \widetilde{Q}, \\ B(q,\omega) = 0. \end{gathered} \qquad (44)$$

Numerical simulation advantages, mostly no need to numerically stabilize the constraint equations (39) due to the introduction of the quasi-velocities, will be demonstrated in examples.

5. THE MODEL REFERENCE TRACKING CONTROL STRATEGY FOR PROGRAMMED MOTION IN A QUASI-COORDINATES DESCRIPTION

Basically, architecture of the tracking strategy is designed in such a way that it separates the non-material and material constraints. They are merged into separate models. It gives rise to an idea of a derivation of both dynamic models using other set of coordinates. The theoretical modeling framework presented in section 3 may be employed to generate the blocks of the tracking control strategy for programmed motion in any set of coordinates. Before we develop the control strategy with the aid of the dynamic models in quasi-coordinates, we introduce the following definition.

Definition 3: Programmed motion planning for a system subjected to the constraints (39) consists in finding time histories of positions $q_p(t)$, quasi-velocities $\omega_p(t)$ and their time derivatives in motion consistent with the constraints.

The control goal is formulated as follows: *Given programmed constraints and material constraints specified by the constraints (39), design a feedback controller to track a desired programmed motion.*

The reference dynamics has the form (44) where all constraints are merged into it.

The unified dynamic control model also has the form (44) with the control inputs added, i.e.

$$\begin{gathered} M(q)\dot{\omega} + C(q,\omega) + D(q) = \tau, \\ B(q,\omega) = 0. \end{gathered} \tag{45}$$

Equations (45) are the unified dynamic control model in quasi-coordinates and p=1 for them. They consist of *(n-b)* equations of motion and *n* equations for unknown *q's*.

Architecture of the model reference tracking control strategy for programmed motion remains unchanged as shown in figure 1.

6. EXAMPLES

6.1. 2-wheeled mobile robot

To illustrate the application of the modeling framework, in which a desired motion is treated as a programmed constraint, and the application of the strategy for programmed motion tracking, select a 2-wheeled robot, which is kinematically equivalent to a unicycle, i.e. the coordinate vector for it is $q = (x, y, \theta, \varphi)$ with θ and φ being the rolling and heading angles, respectively. By *r* we denote its wheel radius, *m* is its body mass, and I_θ, I_φ are moments of inertia. Select the programmed constraint to be

$$x^2 + y^2 - \phi(t) = 0 = \Phi(t) \tag{46a}$$

with $\varphi(t)$=0.2t+1. Material constraints for the robot are

$$\dot{x} = r\dot{\theta}\cos\varphi \qquad \text{and} \qquad \dot{y} = r\dot{\theta}\sin\varphi\,. \tag{46b}$$

For simulation, only the programmed constraint has to be numerically stabilized, i.e. the relation $\dot{\Phi}(t) + \alpha\Phi(t) = 0$ is used. Quasi-velocities for the reference motion are selected as

$$\begin{aligned}
\omega_1 &= \dot{x} - r\dot{\theta}\cos\varphi = 0, \\
\omega_2 &= \dot{y} - r\dot{\theta}\sin\varphi = 0, \\
\omega_3 &= \dot{\Phi}(t) + \alpha\Phi(t) = 0, \\
\omega_4 &= \dot{\varphi}.
\end{aligned} \tag{47}$$

and for the controlled motion ω_3 differs, i.e. $\omega_3 = \dot{\theta}$.

Equations of the reference motion (44) for the robot in quasi-coordinates are as follows

$$\begin{aligned}
&\dot{x} = r\dot{\theta}\cos\varphi, \\
&\dot{y} = r\dot{\theta}\sin\varphi, \\
&\dot{\theta} = \frac{\dot{\Phi}(t)}{2r(x\cos\varphi + y\sin\varphi)}, \\
&\dot{\varphi} = \omega_4, \\
&I_\varphi \dot{\omega}_4 = \frac{-\dot{\Phi}(t)(mr^2 + I_\theta)(x\sin\varphi + y\cos\varphi)}{2r^2(x\cos\varphi + y\sin\varphi)}
\end{aligned} \tag{48}$$

The control dynamics model according to (45) is

$$\begin{aligned}
&\dot{x} = r\omega_3\cos\varphi, \\
&\dot{y} = r\omega_3\sin\varphi, \\
&\dot{\theta} = \omega_3, \\
&\dot{\varphi} = \omega_4, \\
&(mr^2 + I_\theta)\dot{\omega}_3 = \tau_3, \\
&I_\varphi \dot{\omega}_4 = \tau_4.
\end{aligned} \tag{49}$$

To track the program (46a) we use the computed torque controller whose components are

$$\tilde{\tau}_3 = (mr^2 + I_\theta)\tilde{u}_3 \qquad \text{and} \qquad \tilde{\tau}_4 = I_\varphi \tilde{u}_4 \tag{50}$$

with $\tilde{u}_3 = \dot{\omega}_{3p} + 2\sigma_1(\omega_{3p} - \omega_3) + \sigma_1^2(\theta_p - \theta)$, $\tilde{u}_4 = \dot{\omega}_{4p} + 2\sigma_2(\omega_{4p} - \omega_4) + \sigma_2^2(\varphi_p - \varphi)$.

The subscript "p" stands for the programmed variable of the reference block, σ_1 and σ_2 are the convergence rates selected for the specific program. For our simulation they are both set equal to 20. Simulation results are presented in figures 2 and 3.

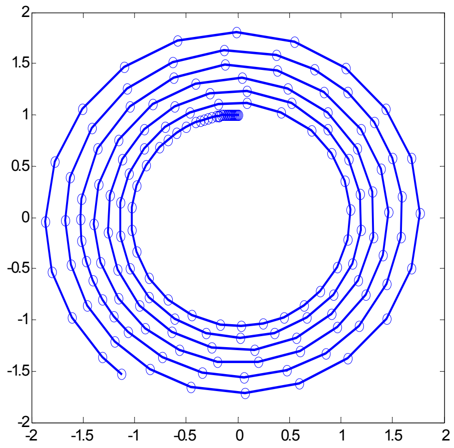

Figure 2. Programmed motion tracking: reference (—), controlled motion (ooo)

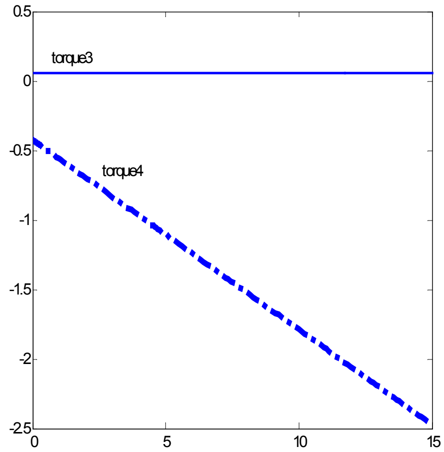

Figure 3. Control torques: Torque3 – heading angle control, Torque4 – rolling angle control

6.2. Roller-racer

The selection of the quasi-coordinates for the roller-racer modeling is due to its design and the way of propulsion [2,7]. Firstly, this is the underactuated system for which control has to be executed at the dynamics level. Secondly, the propulsion and steering come from a rotary motion at a joint that connects two platforms the roller-racer consists of, see fig. 4.

A roller-racer rider learns what value of a forward velocity is enough to ride quite a smooth undulatory motion that permits maneuvering. Also, the rider learns how fast his rotary motion at the rotary joint should be to execute this maneuvering velocity. Then, it is straightforward to think about the forward velocity and the angular velocity of the roller-racer to be the "natural coordinates" from the rider perspective. They are also natural from the tracking control perspective. They are good candidates to be selected as quasi-velocities.

Motion specification in quasi-velocities enables designing a controller in such a natural manner, i.e. the quasi-velocities may be selected as follows:

$$\begin{aligned} \omega_1 &= v = \dot{x}\cos\theta + \dot{y}\sin\theta, \\ \omega_2 &= \dot{\theta} \end{aligned} \tag{51}$$

for the maneuvering and

$$\begin{aligned} \omega_3 &= \dot{x}\sin\theta - \dot{y}\cos\theta = 0, \\ \omega_4 &= -\dot{x}\sin\psi + \dot{y}\cos\psi + l_1\dot{\theta}\cos(\psi-\theta) + l_2\dot{\psi} = 0 \end{aligned} \tag{52}$$

for the NC satisfaction.

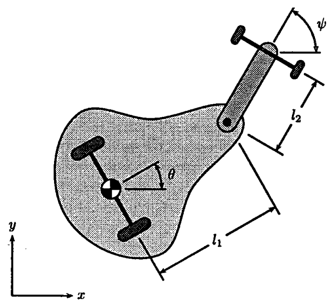

Fig.4. The model of the roller-racer

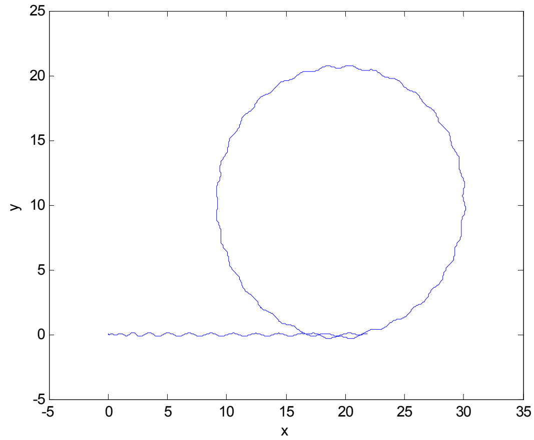

Fig. 5. Desired trajectory tracking

The control dynamics for the roller-racer is specified by

$$M(\varphi)\dot{\omega} + C(\varphi,\dot{\varphi},\omega)\omega + D\omega = E(\varphi)\tau, \qquad \dot{q} = B(q)\omega, \tag{53}$$

$$\dot{\varphi} = \omega_1 e_1 + \omega_2(e_2 - 1). \tag{53a}$$

where now $\omega = (\omega_1, \omega_2)$ is a (2×1) vector and ω_3, ω_4 are set equal to zero since they satisfy the constraints as indicated in (52). We additionally introduced a relative orientation angle $\varphi = \psi - \theta$. We solved the first of equations (53) for $\dot{\omega}_1$ and substituted into the second one to obtain

$$\dot{\omega}_2 = f_2(\varphi,\omega) + b_2(\varphi)\tau, \tag{54}$$

where τ is a single input and

$$f_2 = \frac{(m_{11}+d_1)a_2 - m_{21}a_1}{m_{12}m_{21} - (m_{11}+d_1)m_{22}}, \qquad b_2 = \frac{e_1 m_{21} - e_2(m_{11}+d_1)}{m_{12}m_{21} - (m_{11}+d_1)m_{22}},$$

with m_{ij} being the components of the inertia matrix [2].

For illustrative purposes, the desired trajectory for the roller-racer is parameterized as $\theta_d = 0.2(t - t_1)$ where t_1 is the time at which a maneuver may start. A controller is designed with respect to the quasi-velocity ω_2. The control dynamics for the roller-racer is derived using (44) and the traditional nonlinear control theory approach is used. In this approach a desired motion is not treated as a programmed constraint and it is not reflected in the selection of $\omega's$. It is taken into account at the second step of the control design process, i.e. at the step of a controller design. In [2] we use a partial feedback linearizing controller and design a composite computed torque controller that can track a desired motion. The sample motion, which is a circular trajectory as presented in [2] is illustrated in fig. 5.

It can be seen that the Boltzmann-Hamel equations are first order differential equations, they are in the reduced-state form and may offer a fast way to obtain equations of motion either for the reference or control blocks. Also, it is convenient to select the quasi-velocities, since they satisfy both the material and programmed constraints.

7. CONCLUSIONS

In the paper we develop the theoretic model-based control oriented modeling framework. It yields equations of motion for a nonholonomic system in quasi-coordinates. We demonstrate that the framework may offer a fast way to obtain equations of motion for a system either for the dynamic analysis or control. Also, it is convenient to use quasi-velocities for the derivation of equations of motion for the NS with the programmed constraints since they may be selected to satisfy both material and programmed constraints, and the two kinds of constraints are treated in the same way in the control oriented modelling and the controller design.

Simulation results confirm that model-based control oriented modeling in quasi-coordinates is efficient and it supports numerical stabilization of the constraint equations. Future research is needed in the area of designing controllers using quasi-coordinates description to fully exploit properties of the control dynamics developed in the quasi-coordinates.

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