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Vis-à-vis vs. mixed dock door assignment: A comparison of different cross dock layouts

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Abstract
An important decision problem during the layout phase of a cross docking terminal is whether (vis-à-vis policy) or not (mix policy) separating the terminal into disjunct inbound and outbound areas. The latter policy promises more degrees of freedom, when assigning doors to truck destinations during operational planning, but comes for the price of ambiguous material flows and congestions inside the terminal. Both policies are compared with regard to their impact on operational planning, where deterministic, stochastic and unknown inbound loads are differentiated. Our results show the mix policy being superior in most cases except for unknown inbound loads.

Keywords: Logistics; Cross Docking; Door-Layout

1 Introduction

A cross docking terminal serves as an intermediate node in a distribution network, where inbound truck loads arriving from multiple destinations can be sorted and consolidated, so that full outbound truckloads head to their final destinations. This way economies of transportation can be realized and it is not astounding that cross docking receives increased attention in today's globalized competition with its increasing volume of transported goods. Success stories on cross docking, which resulted in considerable competitive advantages are reported for several industries with high proportions of distribution cost, such as retail chains (Wal Mart; Stalk et al., 1992), mailing companies (UPS; Forger, 1995), automobile manufacturers (Toyota; Witt, 1998) and less-than-truckload logistics providers (Gue, 1999; Kim et al., 2008).

On the negative side, cross docking comes for the price of double handling and extended delivery times. Thus, to not jeopardize positive consolidation effects by inefficient material handling processes and unsatisfied customers suited planning procedures to synchronize inbound and outbound flows are required. In the recent years the operational planning problems of (i) destination assignment and (ii) trucks scheduling received a lot of attention in the research community (see Boysen and Fliedner (2010) for a detailed review):

- Destination assignment, which covers the mid-term assignment of dock doors to inbound and outbound destinations, is treated by Tsui and Chang (1990, 1992), Gue (1999), Bartholdi and Gue (2000), Bermudez and Cole (2001), Oh et al. (2006) as well as Bozer and Carlo (2008). In real-world operations such an assignment is fixed over a longer planning horizon, e.g., a month, and all trucks serving the respective destination are processed at the same door. It is the objective to assign destinations in such a manner, that the sum of transshipment distances weighed with representative average loads are minimized, so that some quadratic-assignment-like problem (e.g., see Finke et al., 1978) is to be solved.
- Truck scheduling decides on the succession of trucks at the dock doors over a short-term, e.g., daily, planning horizon given the restrictions of assigned destinations. Multiple scheduling procedures inspired by traditional machine scheduling are presented, e.g., by Chen and Lee (2007), Miao et al. (2007), Yu and Egbelu (2008), Boysen (2010), Boysen et al. (2010).

However, it is a well-known phenomenon in supply chain management that the better part of operational costs is already irreversibly fixed during the product design phase (e.g., see Blanchard, 1978; Michaels and Wood, 1989). Analogously, the gains achievable in a cross dock by suited optimization approaches for operational planning problems are restricted by the decisions made when determining a terminal's layout. Up to now, layout planning was only treated by Bartholdi and Gue (2004), who investigate the best shape of a cross dock. They compare I, L, T, H and X-shapes for terminals and determine which layout is especially suited for what number of doors with regard to the resulting travel distances inside a terminal.

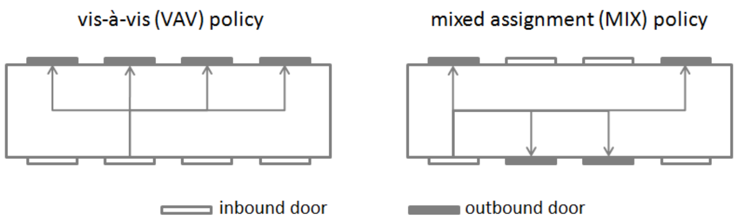

Figure 1: Schematic representation of alternative door layouts

However, there remains an important unexplored question with regard to the layout. In real-world settings, it is a common policy to assign all dock doors belonging to one side of the terminal exclusively to inbound trucks and the doors of the other side to outbound trucks (see Boysen and Fliedner, 2010). This policy, which we denote as the vis-à-vis (VAV) policy, ensures unidirectional material flows from one dock side to the other, so that congestions of forklifts inside the terminal are reduced and supervision of material flows is alleviated. The VAV-policy also facilitates controlling the traffic flow in the surrounding area of a terminal, so that obstructions of inbound and outbound trucks can be reduced. On the other hand, the VAV-policy implies, that any load is moved across the dock – a distance, which might be reduced, if loads are allowed to be interchanged between adjacent doors of the same dock side. The alternative policy, which allows inbound and outbound doors to be assigned in an intermixed succession all around the terminal, is denoted as the mixed assignment (MIX) policy. Clearly, the MIX-policy leaves more degrees of freedom in assigning truck destinations to dock doors during operational planning, but comes for the price of ambiguous material and traffic flows inside and outside the terminal. Figure 1 depicts both alternative policies.

It is the aim of this paper to compare both layout policies with regard to their impact on operational planning. Therefore, some analytical findings are presented and a simulation study is executed, where for both policies optimized destination assignments are determined and resulting material flows inside a terminal are compared. This way, the additional degrees of freedom promised by the MIX-policy are quantified and an experienced terminal operator can weigh up these gains against the hardly quantifiable effects of more or less ambiguous material and traffic flows.

The remainder of the paper is structured as follows. Section 2 specifies our research question and defines some assumptions with regard to the cross docking terminals investigated. Then, Section 3 describes the setup of a comprehensive computational study, in which the two policies are compared in different simulation settings to quantify resulting operational effects. Then, analytical and simulation results are presented for three different forms of information availability with regard to the shipments arriving at a cross dock: deterministic (Section 4), stochastic (Section 5) and unknown (Section 6) inbound loads. Finally, conclusions are drawn in Section 7.

2 Problem description and basic assumptions

This study compares two different layout policies for cross docking terminals: the VAV-policy, which assigns inbound and outbound doors to separate sides of the terminal, and the MIX-policy, where intermixed successions of inbound and outbound doors are allowed. The latter policy is less restrictive and, thus, promises advantageous destination assignments during operational planning. Specifically, the MIX-policy should enable destination assignments, which reduce the distances to be covered during transshipment and, thus, reduce processing times inside the terminal. It is the aim of this study to quantify the influence of both policies on operational destination assignment. For given loads to be processed and a fixed terminal layout the destination assignment problem (DAP) is defined as follows:

DAP: Given sets I, O and V ($I \cup O = V = \{1, \ldots, |V|\}$ of destinations, where set I (O) contains all inbound (outbound) destinations, a flow matrix $\mathbf{b}$ with b_{io} defining the number of items to be transshipped between destinations $i \in I$ and $o \in O$, and a set $D = \{1, \dots, |V|\}$ of dock doors with d_{jk} being the distance between doors j and k. Assign destinations to dock doors, such that

$$Z(\phi) = \sum_{i \in I} \sum_{o \in O} b_{io} \cdot d_{\phi(i),\phi(o)} \rightarrow \min, \tag{1}$$

with ϕ being a permutation of V defining the assignment of destination j to door $\phi(j)$.

If the MIX-policy is applied, inbound and outbound destinations can be assigned arbitrarily, so that DAP faces no additional restrictions. On the other hand, if the VAV-policy holds, door set D is partitioned into disjunct subsets $\Theta = \{1, \dots, |I|\}$ (inbound doors) and $\Psi = \{|I| + 1, \dots, |D|\}$ (outbound doors). Inbound and outbound destinations may only be assigned to their respective subset of doors, so that the following additional restrictions must hold: $\phi(i) \in \Theta \, \forall \, i \in I$ and $\phi(o) \in \Psi \, \forall \, o \in O$. The positive impact promised by the MIX-policy can now be quantified by comparing both objective values $Z(\phi)$ resulting from both versions of DAP for given flow matrix $\mathbf{b}$ and distance matrix $\mathbf{d}$. This impact is quantified by a simulation study where different flow matrices and terminal layouts are systematically compared.

However, to realize the gains promised by the additional degrees of freedom of the MIX-policy, DAP is heavily influenced by the information availability of inbound loads:

- Especially, if high valued and/or express shipments are transported, it is a common policy to inform the cross dock about prospective loads arriving, e.g., the same day. In this case, inbound loads are *deterministic* and known with certainty. For instance, in the cross docks of automobile industry this kind of information availability is especially important to avoid stock-out of just-in-time materials. Thus, if dock destinations are fixed only over a shorter planning horizon, e.g., for a few days, the information on any inbound load arriving during the respective time span might be deterministic, so that solving DAP (and the additional degrees of freedom when applying the MIX-policy) seems especially advantageous for reducing transshipment effort.

- Typically, destinations are fixed over a longer time span, e.g., a month, so that shipments arriving at the terminal during the planning horizon can only be forecasted. In this case, only *stochastic* information on truck loads might be available. Whether planned gains of optimal DAP solutions can in deed be realized depends on the realizations of daily truck loads. This information availability is typical for mailing and postal services, where only representative mail volumes for any pair of destinations are known for previous time periods.

- Finally, we consider the case of *unknown* inbound loads, where the terminal operator is unaware of incoming shipments prior to opening the trailer. In this case, solving DAP cannot be executed in a reasonable fashion and destinations can only be assigned by chance, which, in the real-world, is often realized by applying a first-come-first-serve policy. This policy is especially applied by smaller less-than-truckload logistics providers, which dread investment cost in information technology for improving data availability.

These three cases are distinguished, when investigating the influence on operational transsshipment efficiency of both layout policies. Finally, we restrict our research to a special kind of cross docking terminal, which, however, represents the most widespread terminal setting in real-world cross docking operations:

- Only I-shaped cross docks are considered, which means that terminals are rectangular and dock doors are located along the longer sides of the building.
- Furthermore, it is assumed that both sides have the same number of dock doors and doors are equispaced, i.e., any pair of neighboring doors of the same side shows identical distance δ.
- It is assumed that a terminal has the same number of inbound and outbound doors (and destinations). Note that a large discrepancy in the number of inbound and outbound doors required is a knock-out criterion for reasonably applying the VAV-policy in the real-world, since large parts of the terminal would remain unused. Therefore, a comparison of both policies seems only reasonable, if this premise holds.
- Shipments arrive on pallets and are moved by forklift (or by a worker on foot) inside the terminal. Terminal layout where shipments are moved by a conveyor belt (e.g., see McWilliams et al., 2005) are not considered. Consequently, distances between doors are measured between their centers applying a rectilinear (manhattan) metric.

With these restrictions on hand, the following section summarizes the general setup of our simulation study.

3 Setup of simulation study

3.1 Generating cross dock settings

To derive test instances for simulating cross dock operations some assumptions on the terminal layout and the shipments to be processed are required. These assumptions are described in the following.

Terminal layout: To determine the influence of a terminal's size the number of doors $|D|$ (with $|I| = |O| = \frac{|D|}{2}$) and width W, which covers the straight distance (in meters) from one side of the terminal to the other, are varied as follows: $|D| \in \{24, 48, 96\}$ and $W \in \{18, 27, 36\}$. Furthermore, to determine rectilinear distances d_{jk} between any pair of doors j and k a representative real-world distance of $\delta = 4$ meters between neighboring doors of the same side is assumed. In case of the MIX-policy and adjacent doors of the same side, distances also depend on the way w a forklift drives into the terminal before turning into the main aisle dedicated to horizontal movement along the spread of the terminal. A typical storage layout with a middle aisle, so that $w = \frac{1}{2}W$, is depicted on the left hand side (part (a)) of Figure 2. Here, a forklift starting at door 1 has to travel to and back the middle of the terminal, even if a door of the same side is to be visited, so that it makes no difference (in travel distance) whether door 2 or 3 is to be visited. Clearly, the potential advantage of the MIX-policy increases, if alternative aisle layouts (as the one depicted in Figure 2(b)) are applied. Therefore, within our simulation distance w from door to horizontal aisle is varied as follows: $w \in \{\frac{1}{4}W, \frac{1}{3}W, \frac{1}{2}W\}$. Note that parameter w has no influence on distances between doors of different sides, so that the location of aisles has no impact when applying the VAV-layout.

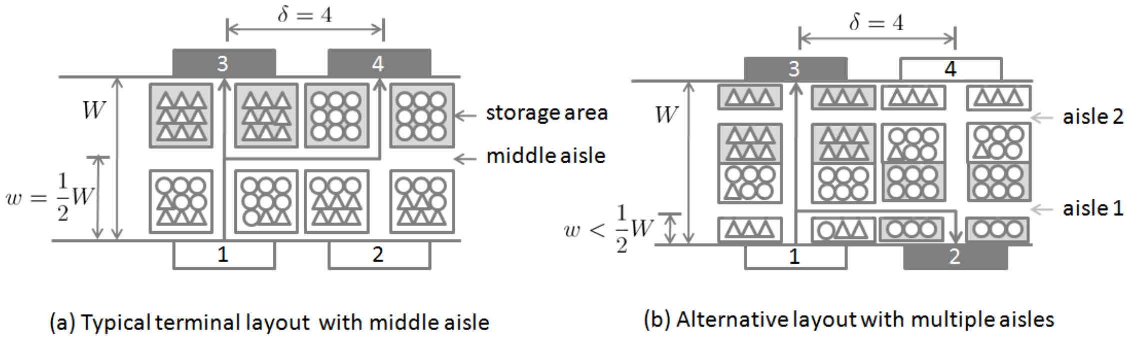

Figure 2: Alternative storage and aisle layouts

Shipments: In our simulation, we differentiate whether each inbound destination serves few, many or a mixed number of outbound destinations. If only few number of destinations are served, then, the number of outbound destinations with $b_{io} > 0$ per inbound destination i is randomly determined by drawing an equally distributed random number from interval $[1; \frac{|O|}{4}]$, whereas for many and a mixed number of destinations intervals $[\frac{3|O|}{4}; |O|]$ and $[1; |O|]$ are applied, respectively. Then, for each outbound destination with $b_{io} > 0$ the actual number of shipments is determined by drawing an equally distributed random number out of interval $[100; 500]$.

Table 1 summarizes the simulation parameters, which were varied during instance generation. All parameters are combined in a full factorial design and instance generation per parameter constellation is repeated 100 times, so that in total 8,100 instances are generated. For any of these instances and both layout policies the respective DAP problem is to be solved.

symbol	description	values
$\|D\|$	number of doors	24, 48, 96
W	width of terminal (in meters)	18, 27, 36
w	distance from door to horizontal aisle	$\frac{1}{4}W, \frac{1}{3}W, \frac{1}{2}W$
–	number of outbound destination served per inbound	few, many, mixed

Table 1: Parameters for instance generation

3.2 Solving the destination assignment problem

Since DAP is an NP-hard optimization problem (see Sahni and Gonzalez, 1976, for the quadratic assignment problem) it seems impossible to solve instances of real-world size to optimality. Therefore, we apply a straightforward simulated annealing (SA) procedure, which proved successful in a previous study of Bozer and Carlo (2008). SA is a stochastic meta-heuristic that is able to overcome local optima. It is based on the probabilistic acceptance of modified neighboring solutions inspired by thermal processes for obtaining low-energy states in heat baths (e.g., Kirkpatrick et al., 1983; Aarts et al., 1997).

Our SA-approach directly operates on the assignment vector ϕ, with $\phi(j)$ being door assigned to destination j as defined for DAP (see Section 2). First a random vector is determined, where in case of the VAV-policy the separation of inbound and outbound doors is to be respected, and evaluated by objective function (1). Neighboring solutions are determined by randomly drawing a dock door and swapping destinations with its next neighboring (left, right and vis-à-vis) door (MIX-policy) or the next neighboring (left and right) door of the same terminal side (VAV-policy) leading to the best objective value. Whether or not a neighboring solution ϕ' obtained by a swap move is accepted is decided according to traditional probability schemes (Aarts et al., 1997):

$$Prob(\phi' \text{ replacing } \phi) = \begin{cases} 1, & \text{if } Z(\phi') \leq Z(\phi) \\ exp\left(\frac{Z(\phi)-Z(\phi')}{C}\right), & \text{otherwise} \end{cases} \tag{2}$$

Our SA is steered by a simple static cooling schedule (see Kirkpatrick et al., 1983). Control parameter C is initialize as 10% of the objective value of the initial solution vector and then continuously decreased by multiplying it with factor $f = \sqrt[IT]{\frac{C^{stop}}{C}}$ in each iteration. The procedure is stopped when reaching given stop parameter $C^{stop} = 1.0$ after $IT = 50,000$ iterations or after 500 iterations without accepting a neighboring solution. Finally, SA terminates after three re-initialized repetitions in direct succession without improving the objective value and returns the solution with the minimum objective function value $Z(\phi)$ found. In our computational study, we have invariably used control parameter values as described above. Note that preliminary studies have indicated that this parameter constellation outperforms other settings and has obtained promising results.

4 Deterministic inbound loads

In case of deterministic inbound loads all shipments arriving at a terminal over the planning horizon, i.e., the time span a respective assignment of destination to dock doors is fixed, are know with certainty. With regard to DAP this means that real loads materialize as given by flow matrix **b** and operational differences between both policies actually result as determined when solving DAP. Clearly, the MIX-policy leaves more degrees of freedom to DAP, so that the following straightforward proposition can be stated:

Proposition: In case of deterministic inbound loads $Z^*(\text{MIX}) \leq Z^*(\text{VAV})$ holds, where $Z^*(\cdot)$ denotes the optimal solution value of the respective policy.

The proof directly follows from the VAV-policy facing the additional restriction of inbound and outbound sides when solving DAP. Therefore, the MIX-policy will at least deliver the same or an even better operational performance than the VAV-policy. To quantify this advantage, Table 2 lists the results of our simulation study. Here, the relative increase in operational performance ($\frac{Z(VAV)-Z(MIX)}{Z(VAV)}$) of the MIX-policy over the VAV-policy is averaged over all instances of the respective parameter constellation (denoted as avg rel).

		$\|D\| = 24$			$\|D\| = 48$			$\|D\| = 96$		
W	w	few	mixed	many	few	mixed	many	few	mixed	many
18	$\frac{1}{4}W$	27.4	19.9	16.5	21.7	15.1	11.6	14.9	11.0	7.5
	$\frac{1}{3}W$	17.8	13.5	11.0	15.4	11.1	7.9	12.2	8.9	5.3
	$\frac{1}{2}W$	2.5	1.4	0.4	5.5	3.2	1.0	6.7	4.1	1.3
27	$\frac{1}{4}W$	30.6	22.6	19.1	24.4	17.6	14.3	17.2	13.1	9.5
	$\frac{1}{3}W$	19.8	15.1	12.7	17.3	12.5	9.7	13.0	10.0	6.7
	$\frac{1}{2}W$	2.0	1.1	0.3	4.2	2.5	0.9	5.9	3.8	1.1
36	$\frac{1}{4}W$	32.9	24.3	20.8	26.4	19.4	16.1	18.9	14.7	11.1
	$\frac{1}{3}W$	21.0	16.2	13.8	18.1	13.4	10.9	14.1	10.7	7.7
	$\frac{1}{2}W$	1.5	1.0	0.3	3.8	2.2	0.7	5.2	3.4	1.0

legend: avg rel [%]

Table 2: Relative increase of operational performance of MIX- over VAV-policy depending on W, w, $|D|$ and the pattern of material flow

The results of Table 2 reveal the following influence of the parameters of instance generation:

- The smaller w the higher is avg rel. Clearly, if the distance w from an inbound door to the horizontal aisle decreases, the distance between neighboring doors of the same side reduces, too. In case of the MIX-policy, DAP can assign neighboring doors to busy inbound-outbound relations and therefore, reduce travel distances as well as increase operational efficiency compared to the VAV-policy.

- With regard to cross dock width W two effects need to be distinguished: If $w = \frac{1}{2}W$ it makes no difference in distance whether a neighboring door of the same side or the opposite side is to be visited, so that the MIX-policy only profits from additional degrees of freedom in the assignment of destinations to doors. This effect diminishes with increasing W, since absolute distances to be bridged increase and, thus, the relative increase reduces. If $w < \frac{1}{2}W$ the additional advantage of reduced distances between adjacent doors becomes relevant. This effect becomes the higher the wider the terminal is.

- The fewer the destinations to be served per inbound door the higher is the advantage of the MIX-policy. If only a few destinations are served per inbound door, then DAP can assign doors more targeted and only a few loads need to be exchanged between non-neighboring doors. In this case, the additional degrees of freedom of the MIX-policy are especially valuable and avg rel. turns out higher.

- Again, the two different effects of the MIX-policy need to be distinguished, if the influence of the number $|D|$ of dock doors is to be specified. If $w = \frac{1}{2}W$ only increasing degrees of freedom are relevant. Clearly, this effect adds up the more doors are existent. On the other hand, if $w < \frac{1}{2}W$ additionally reduced distances need to be considered. With increasing number of doors, some transshipment between far distant doors occur, which lifts the objective value on a higher absolute level and, in turn, reduces the average advantage of the MIX-policy.

With these dependencies on hand, the operational advantage of the MIX-policy over the VAV-policy ranges between 0.3% (wide dock, middle aisle, few doors and many destinations per inbound) and 32.9% (wide dock, short distance to horizontal aisle, few doors and few destinations per inbound). Especially, if distance w up to the horizontal aisle is reduced, these gains become considerable. If, however, horizontal movements are processed via a middle aisle, the operational advantage of the MIX-policy seems not distinct enough to surmount the negative effects of ambiguous material and traffic flows.

5 Stochastic inbound loads

In case of stochastic inbound loads only a forecasted number of items to be transshipped between any inbound destination i and outbound destination o over the planning horizon is available. Therefore, DAP is to be solved by filling flow matrix $\mathbf{b}$ with forecasted loads per inbound outbound relation. The resulting destination assignment is applied and the real transshipment effort results from the actual number of shipments to be exchanged between i and o and their assigned doors. To emulate different levels of forecast accuracy, we vary standard deviation $\sigma \in \{0.2\mu, 0.4\mu\, 0.6\mu\}$, when randomly drawing the forecasted number of shipments according to a normal distribution $\mathcal{N}(\mu, \sigma^2)$, with expected value μ being fixed to the respective deterministic value (see Section 3.1). The results of this experiment are summarized in Table 3. Note that $\sigma = 0.0\mu = 0$ represents deterministic loads.

These results reveal that the relative increase of operational performance of the MIX- over the VAV-policy is astonishingly stable against inaccurate forecasts of material flows. Clearly, absolute transshipment distances increase the higher σ. However, with regard to the relative gap both policy seem to suffer from inaccurate forecasts to a similar extent. Only a slight decrease of the MIX-policy's performance gains with increasing forecast error (rising σ) can be stated. Thus, it can be concluded that the findings of the deterministic case, e.g., especially wide docks with a short distance to horizontal aisle, few doors and few destinations per inbound truck promise high operational gains for the MIX-policy, also hold for the case of stochastic loads and are very robust against forecast errors.

6 Unknown inbound loads

Finally, this section considers unknown inbound loads. It is assumed that the terminal operator is unaware of arriving shipments prior to opening a trailer. In this case, solving the DAP is not possible since flow matrix $\mathbf{b}$ remains unknown. The only information available in addition to matrix $\mathbf{d}$ defining the distances between dock doors is whether a truck arriving at the terminal is inbound or outbound truck. Facing such an information availability, the decision problem on how to allocate inbound and outbound destinations around the terminal can only aim at minimizing the average distance between any pair of inbound and outbound doors. In this context, we will prove that applying the VAV-policy leads to an optimal door layout with regard to average distance, if a terminal layout with a middle aisle ($w = \frac{1}{2}W$) is applied.

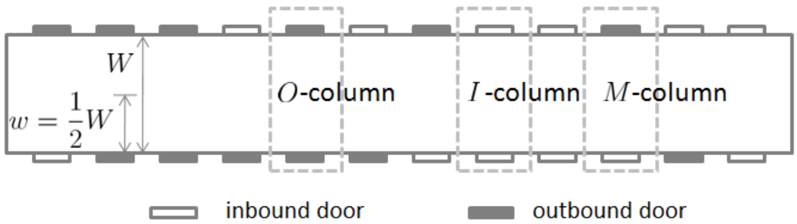

Figure 3: Example terminal and definitions

Definition: Two opposite dock doors of different sides are denoted as a (door) column. A column, which is assigned two inbound (outbound) destinations, is labeled an I-column (O-column). A column consisting of one inbound and one outbound destination is denoted as M-column.

Note that due to $w = \frac{1}{2}W$ average distances to any other column of doors are not influenced by the sides inbound and outbound destinations are assigned in an M-column, so that no further differentiation is required. A specific terminal layout consisting of a sequence of I-, O- and M-columns is denoted as a string, where strings are interpreted from left to right. For the example terminal of Figure 3 the respective string is

$$s_1 = (M, O, O, M, O, M, M, I, I, M, M, I). \tag{3}$$

Lemma 1: In any string the number of I-columns is equal to the number of O-columns.

Proof: Immediately follows from defining the total number of inbound doors being equal to the number of outbound doors in Section 2. $\square$

Lemma 2: Any string containing substrings, where multiple O- (or I-) columns follow each other only (potentially) interrupted by M-columns, shows equal or higher average distance than an identical string, where for the left most O- (I-) succession the last O- (I-) column is interchanged with the first I- (O-) column following the succession.

Proof: Without loss of generality we proof the lemma for a succession of O-columns. Otherwise, the string can be transformed without changing average distances by simply inverting the meaning of I- and O-columns. For the proof, a string can be subdivided into three blocks B_1, B_2 and B_3 as is depicted for example string s_1:

$$s_1 = (\underbrace{M, O, O, M}_{B_1}, O, \underbrace{M, M}_{B_2}, I, \underbrace{I, M, M, I}_{B_3}). \tag{4}$$

While block B_2, which can also be empty, contains all M-columns between both swap columns, Blocks B_1 and B_3 consist of all columns left and right these two columns, respectively. Due to the definition of both swap columns, block B_1 has at least as many O-columns as I-columns, so that in relation to block B_1 the reductions in average distance gained by shifting the selected I-column to the left at least outweighs the additional distance caused by shifting the selected O-column to the right. Analogously, the same argument holds in relation to block B_3. Furthermore, distances in relation to block B_2 are not altered by the swap, since any additional distance in one direction is accompanied by a reduction of the same amount into the other direction. Finally, remaining distances between blocks (B_1 and B_2, B_1 and B_3, B_2 and B_3) and the two swap columns are not affected, which completes the proof. $\square$

For our example string s_1 the respective swap leads to an absolute distance reduction of $3\delta(8+8) = 48\delta$ or a reduction of $\frac{48\delta}{12^2} = \frac{1}{3}\delta$ in the average distance and results in string s_2:

$$s_2 = (M, O, O, M, I, M, M, O, I, M, M, I). \tag{5}$$

Lemma 3: Any string where O- and I-columns follow each other rotationally only (potentially) interrupted by M-columns, shows a higher average distance than an identical string, where the left most O- and I-columns are interchanged with two M-columns.

Proof: The selected O- and I-column subdivide the string into three blocks C_1, C_2 and C_3 as depicted for example string s_3:

$$s_3 = (\underbrace{M}_{C_1}, O, \underbrace{M}_{C_2}, I, \underbrace{O, I, M, M, O, M, I, M}_{C_3}). \tag{6}$$

Blocks C_1 and C_2 contain all M-columns prior and between the O- and I-column to be interchanged, respectively. Block C_3 contains all remaining columns after the swap columns. Note that any block can also be empty. Due to Lemma 1, block C_3 shows an identical number of O- and I-columns, so that any block shows an identical number of O- and I-columns. Therefore, in relation between swap pair and all three blocks each increase in distance is necessarily accompanied by a reduction of the same size and distances between blocks remain unaffected. Only, distances of the swap pair are reduced, where the reduction becomes the higher the longer the C_2-block is. $\square$

For our example string s_3 the swap leads to string s_4 and an absolute distance reduction of 4δ or a reduction in average distance of $\frac{4\delta}{12^2} = \frac{1}{36}\delta$:

$$s_4 = (M, M, M, M, O, I, M, M, O, M, I, M). \tag{7}$$

Theorem 1: With regard to the average distance between all inbound and outbound doors and $w = \frac{1}{2}W$ a door layout exclusively consisting of M-columns is optimal.

Proof: The proof is by contradiction. Consider an optimal string, i.e., a door layout with minimum average distance, which does not exclusively contain M-columns. According to Lemma 1 and Lemma 2 this string can be altered by a series of swap moves, so that a string results where (apart from intermediate M-columns) O- and I-columns occur in rotational succession. None of these swap moves worsens average distance. Then, according to Lemma 3 any pair of successive O- and I-columns can be interchanged by two M-columns and average distance only reduces with any change move. Thus, the initial solution cannot be optimal and the theorem holds. $\square$

Clearly, one possibility to translate an optimal string exclusively consisting of M-columns into a real-world door layout is to assign all doors of one side to inbound and the other to outbound operations. Therefore, the VAV-policy leads to a minimum average distance between all inbound and outbound doors if a middle aisle exists ($w = \frac{1}{2}W$).

Remark: In the unrealistic case of $w > \frac{1}{2}W$ the VAV-policy also leads to an optimal door layout. However, for $w < \frac{1}{2}W$ the proof needs to be extended. An optimal door layout still exclusively consists of M-columns, because Lemma 1, 2 and 3 are independent of w and W. Only the swap move in the proof of Lemma 3 has to be concretized. To ensure that the average distance does not increase, we have to swap doors between the selected O- and I-column on the same terminal side. Thus, an optimal door layout with $w < \frac{1}{2}W$ exclusively consists of M-columns and, additionally, it holds that the number of inbound and outbound doors per terminal side is either $\lceil \frac{1}{4}|D| \rceil$ or $\lfloor \frac{1}{4}|D| \rfloor$. Figure 4 displays an optimal door layout for our example and $w < \frac{1}{2}W$.

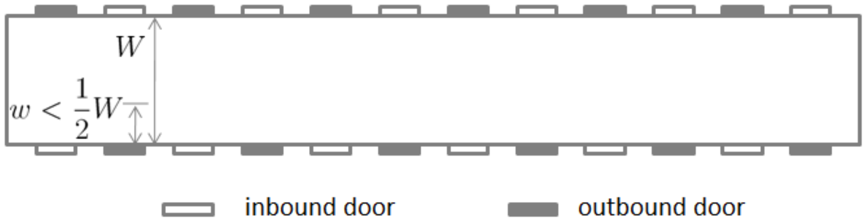

Figure 4: Optimal door layout for unknown inbound loads and $w < \frac{1}{2}W$

If instead of the VAV-policy a MIX-policy is applied, then inbound and outbound trailers can only be assigned by pure chance all around the terminal. Recall that in the real-world this case is often realized by assigning trailers according to a first-come-first-serve policy. As the VAV-policy was shown to be optimal for $w = \frac{1}{2}W$, we aim at quantifying this disadvantage. On the other hand, the MIX-policy should be advantageous if $w < \frac{1}{2}W$ holds, so that again quantifying the gap is desirable. For the VAV-policy the average distance AVD_k^{VAV} of an inbound door being located in column k (with columns $k = 1, \ldots, n$ and $n = \frac{|D|}{2}$ being numbered from left to right) to all outbound doors amounts to:

$$AVD_k^{\text{VAV}} \qquad = W + \frac{\delta}{n}\left(\left(k - \frac{n+1}{2}\right)^2 + \frac{n^2-1}{4}\right) \quad \forall\, k = 1, \ldots, n. \tag{8}$$

Clearly, the mean distance from an inbound door to all outbound doors grows quadratically in the offset from the middle of the dock. Thus, destinations delivering or receiving a multitude of shipments should be served in the center of the cross dock. Unfortunately, with unknown inbound loads information on the number of shipments is not available. In the case of a random assignment of inbound and outbound destinations (MIX-policy), the expected mean distance $AVD_k^{\text{MIX}}(w)$ from an inbound door in column k with distance w to horizontal aisle changes to:

$$AVD_k^{\text{MIX}}(w) = \frac{nW + (n-1)2w}{2n-1} + \frac{\delta}{n - \frac{1}{2}}\left(\left(k - \frac{n+1}{2}\right)^2 + \frac{n^2-1}{4}\right) \quad \forall\, k = 1, \ldots, n. \tag{9}$$

Note that, since exact locations of the inbound doors are unknown, $AVD_k^{\text{MIX}}(w)$ only denotes an expected value. With these formulas on hand the expected gap in average distance between both policies can be quantified for different terminal widths W, numbers of doors $|D|$ and distances between adjacent doors δ. Table 4 lists the influence of these parameters (for $\delta = 4$) on the absolute and relative gap between MIX- and VAV-policy, where each single deviation is calculated by $Z(VAV) - Z(MIX)$ and $\frac{Z(VAV)-Z(MIX)}{Z(VAV)}$, respectively.

The results of Table 4 reveal a moderate disadvantage between 0.7% and 2% of the MIX-policy if a middle aisle is applied ($w = \frac{1}{2}W$). On the other hand, if $w < \frac{1}{2}W$ the MIX-policy shows an relative advantage over the VAV-policy up to 15.3%. Note that these analytical findings make an additional simulation study superfluous. However, it is reassuring that after 10,000 simulation runs these results are confirmed up to the first position after decimal point. Additionally, equations (8) and (9) allow to calculate $w^*(W, n, \delta)$, which denotes the distance to horizontal aisle where both policies break even. By equating $\sum_{k=1}^{n} AVD_k^{\text{VAV}}$ and $\sum_{k=1}^{n} AVD_k^{\text{MIX}}(w)$ and solving for w^* we retrieve:

$$w^*(W, n, \delta) = \frac{1}{2}W - \frac{\delta}{2n^2(n-1)} \sum_{k=1}^{n} \left(\left(k - \frac{n+1}{2} \right)^2 + \frac{n^2 - 1}{4} \right). \tag{10}$$

Thus, if the distance to horizontal aisle in a terminal is larger than w^*, then the VAV-policy shows superior with regard to expected average distance. Otherwise the MIX-policy will turn out more efficient. Table 5 lists the results of $w^*(W, n, \delta)$ for different terminal widths W and number of doors $|D|$ for $\delta = 4$. For instance, in a cross dock with 48 dock doors and the MIX-policy being applied aisles for horizontal movement along the terminal length need to be located at least 0.694 meters prior to the middle of the terminal. Otherwise, the VAV-policy outperforms the MIX-policy.

It can be concluded that for unknown inbound loads the VAV-policy seems best suited. Even if aisles for horizontal movement are shifted closer to the terminal sides, operational gains promised by the MIX-policy are small and have to additionally outweigh the MIX-policy's disadvantage of ambiguous material and traffic flows.

7 Conclusions

This paper compares two basic policies for designing the door layout of an I-shaped cross docking terminal. The vis-à-vis (VAV) policy assigns one side of the terminal exclusively to inbound and the other to outbound operations. The mixed assignment (MIX) policy allows for a facultative succession of intermixed inbound and outbound doors all around the terminal. When comparing both policies with regard to their operational efficiency for different information availabilities, the following conclusions are drawn:

- If inbound loads arriving at the terminal are known with certainty, the MIX-policy shows a considerable advantage only if the distance up to the horizontal aisle to reach doors of the same terminal side are located close to the terminal sides. Otherwise, this effect diminishes, so that it becomes hard for the MIX-policy to outweigh its additional disadvantage of ambiguous material and traffic flows.

- In case of stochastic inbound loads, where the number of shipments to be exchanged between inbound and outbound destinations need to be estimated, different levels of forecasting errors have only negligible influence on the gap between both policies' operational performance. The findings for the deterministic case are still valid: Especially wide docks with a short distance to horizontal aisle, few dock doors and few destinations per inbound truck promise high operational gains for the MIX-policy.

- If inbound loads are unknown prior to opening a trailer and a middle aisle is applied, then – counter to intuition – the VAV-policy was proven to be advantageous over the MIX-policy. Even for horizontal aisles close to terminal sides operational gains promised by the MIX-policy are shown to be small, so that the VAV-policy seems the better choice.

Future research should aim to quantify congestions among transshipment vehicles, e.g., forklifts, resulting from a specific door layout. If a valid anticipation for these potential obstructions can be determined, then trading off operational gains against congestions needs not be left over to the experience of a terminal operator, but can directly be quantified in a joint measure of operational performance.

Appendix: Deduction of equations in Section 6

If the VAV-policy is applied, the average distance of the inbound door in column k (with columns $k = 1, \dots, n$ being numbered from left to right and n being the number of doors of one terminal side) to any outbound door can be split into the distance W across the dock and horizontal movement:

$$AVD_k^{\text{VAV}} = W + \frac{1}{n}\left(\sum_{o=1}^{k-1} o\delta + 0\delta + \sum_{o=k+1}^{n}(o-k)\delta\right) \quad \forall\, k = 1, \dots, n. \tag{11}$$

Here, the horizontal movement consists of three parts: The distances from inbound door k to all outbound doors to the left of k ($\sum_{o=1}^{k-1} o\delta$), the distance to the opposite outbound door (0δ) and distances to all outbound doors to the right of k ($\sum_{o=k+1}^{n}(o-k)\delta$). Simplifying this equation leads to:

$$\begin{aligned}
AVD_k^{\text{VAV}} &= W + \frac{\delta}{n}\left(\sum_{o=1}^{k-1} o + \sum_{o=k+1}^{n}(o-k)\right) \\
&= W + \frac{\delta}{n}\left(\frac{k(k-1)}{2} + \frac{(n-k+1)(n-k)}{2}\right) \\
&= W + \frac{\delta}{n}\left(k^2 - k - nk + \frac{n^2+n}{2}\right) \\
&= W + \frac{\delta}{n}\left(\left(k - \frac{n+1}{2}\right)^2 + \frac{n^2-1}{4}\right) \quad \forall\, k = 1, \dots, n. \quad \forall\, k = 1, \dots, n.
\end{aligned} \tag{12}$$

If the MIX-policy is applied, expected average distance can be split into the distance across the dock and horizontal distance:

$$AVD_k^{\mathrm{MIX}}(w) = \frac{nW + (n-1)2w}{2n-1} + \frac{2}{2n-1}\left(\sum_{o=1}^{k-1} o\delta + \sum_{o=k+1}^{n} (o-k)\delta\right) + \frac{1}{2n-1}0\delta \qquad \forall\, k = 1, \ldots, n. \tag{13}$$

Since there is an inbound door in location k, there are $2n-1$ doors remaining. The probability that a particular outbound door is located at the same or at the opposing terminal side is $\frac{n-1}{2n-1}$ or $\frac{n}{2n-1}$, respectively. Thus, the expected vertical distance across the dock is $\frac{nW+(n-1)2w}{2n-1}$, whereas horizontal distances are calculated analogously to AVD_k^{VAV}. Here, the probability for a particular outbound door being in a particular column is $\frac{1}{2n-1}$ or $\frac{2}{2n-1}$ depending on k being even or not. $AVD_k^{\mathrm{MIX}}(w)$ can be simplified as follows:

$$\begin{aligned} AVD_k^{\mathrm{MIX}}(w) &= \frac{nW + (n-1)2w}{2n-1} + \frac{2}{2n-1}\left(\sum_{o=1}^{k-1} o\delta + \sum_{o=k+1}^{n} (o-k)\delta\right) + \frac{1}{2n-1}0\delta \\ &= \frac{nW + (n-1)2w}{2n-1} + \frac{2\delta}{2n-1}\left(\sum_{o=1}^{k-1} o + \sum_{o=k+1}^{n} (o-k)\right) \\ &= \frac{nW + (n-1)2w}{2n-1} + \frac{\delta}{n-\frac{1}{2}}\left(\left(k - \frac{n+1}{2}\right)^2 + \frac{n^2-1}{4}\right) \\ &\qquad \forall\, k = 1, \ldots, n. \end{aligned} \tag{14}$$

When furthermore w equals $\frac{1}{2}W$, $\frac{nW+(n-1)2w}{2n-1}$ changes to W, so that, clearly, the MIX-policy results in a higher average distance than VAV.

In order to calculate break even distance $w^*(W, n, \delta)$ up to horizontal aisle, for which VAV- and MIX-policy lead to identical average distances, we equate:

$$\sum_{k=1}^{n} AVD_k^{\mathrm{MIX}}(w) = \sum_{k=1}^{n} AVD_k^{\mathrm{VAV}}, \tag{15}$$

which leads to:

$$n\frac{nW + (n-1)2w}{2n-1} + \frac{\delta}{n-\frac{1}{2}}(*) = nW + \frac{\delta}{n}(*) \tag{16}$$

with

$$(*) = \sum_{k=1}^{n} \left(k - \frac{n+1}{2}\right)^2 + \frac{n^2-1}{4} \tag{17}$$

This equation has to be solved for w:

$$n\frac{nW+(n-1)2w}{2n-1} = nW + \frac{\delta(2n-1-2n)}{(2n-1)n}(*) \tag{18}$$

$$nW+(n-1)2w = (2n-1)W - \frac{\delta}{n^2}(*) \tag{19}$$

$$w = \frac{1}{2}W - \frac{\delta}{2n^2(n-1)}(*) \tag{20}$$

Altogether, we obtain:

$$w^*(W,n,\delta) = \frac{1}{2}W - \frac{\delta}{2n^2(n-1)}\left(\sum_{k=1}^{n}\left(\left(k-\frac{n+1}{2}\right)^2+\frac{n^2-1}{4}\right)\right) \tag{21}$$

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W	w	σ	$\|D\|=24$ few	mixed	many	$\|D\|=48$ few	mixed	many	$\|D\|=96$ few	mixed	many
18	$\frac{1}{4}W$	0.0μ	27.4	19.9	16.5	21.7	15.1	11.6	14.9	11.0	7.5
		0.2μ	26.9	19.9	16.2	21.3	14.9	11.3	15.1	11.0	7.3
		0.4μ	26.8	19.1	15.6	20.7	14.6	11.0	14.5	10.8	7.1
		0.6μ	26.5	19.0	15.2	20.4	14.2	10.9	14.4	10.7	6.9
	$\frac{1}{3}W$	0.0μ	17.8	13.5	11.0	15.4	11.1	7.9	12.2	8.9	5.3
		0.2μ	18.1	13.5	10.8	16.1	10.7	7.9	11.9	8.9	5.3
		0.4μ	17.5	13.0	10.4	15.1	10.6	7.5	11.5	8.6	5.0
		0.6μ	17.4	12.8	10.3	14.7	10.6	7.3	11.7	8.3	5.0
	$\frac{1}{2}W$	0.0μ	2.5	1.4	0.4	5.5	3.2	1.0	6.7	4.1	1.3
		0.2μ	2.8	1.5	0.3	5.4	3.1	1.0	6.8	4.2	1.3
		0.4μ	3.1	1.6	0.2	4.9	2.8	0.9	6.3	4.1	1.0
		0.6μ	2.2	1.2	0.1	4.7	2.5	0.8	6.0	3.8	1.0
27	$\frac{1}{4}W$	0.0μ	30.6	22.6	19.1	24.4	17.6	14.3	17.2	13.1	9.5
		0.2μ	30.5	22.6	18.8	24.1	17.5	14.0	17.2	13.0	9.3
		0.4μ	29.7	21.8	18.3	23.3	16.9	13.6	16.8	12.6	9.1
		0.6μ	29.1	21.2	18.0	23.2	16.8	13.5	16.3	12.5	8.9
	$\frac{1}{3}W$	0.0μ	19.8	15.1	12.7	17.3	12.5	9.7	13.0	10.0	6.7
		0.2μ	19.8	15.0	12.6	17.1	12.1	9.5	13.0	9.8	6.5
		0.4μ	19.5	14.7	12.3	16.6	11.9	9.1	12.9	9.8	6.4
		0.6μ	19.5	14.4	12.0	16.1	11.9	9.1	12.1	9.5	6.2
	$\frac{1}{2}W$	0.0μ	2.0	1.1	0.3	4.2	2.5	0.9	5.9	3.8	1.1
		0.2μ	2.2	1.1	0.4	4.6	2.6	0.8	5.9	3.9	1.1
		0.4μ	2.0	1.1	0.3	4.2	2.5	0.7	5.7	3.5	0.9
		0.6μ	1.9	1.1	0.1	4.4	2.4	0.7	5.0	3.5	0.8
36	$\frac{1}{4}W$	0.0μ	32.9	24.3	20.8	26.4	19.4	16.1	18.9	14.7	11.1
		0.2μ	32.6	24.1	20.6	26.0	19.2	15.9	18.8	14.6	11.0
		0.4μ	31.9	23.3	20.1	25.1	18.6	15.5	18.4	14.2	10.7
		0.6μ	31.3	22.9	19.5	25.1	18.5	15.3	17.9	13.9	10.6
	$\frac{1}{3}W$	0.0μ	21.0	16.2	13.8	18.1	13.4	10.9	14.1	10.7	7.7
		0.2μ	21.3	16.2	13.7	18.3	13.3	10.7	13.9	10.7	7.6
		0.4μ	20.6	15.7	13.4	18.0	13.2	10.6	13.6	10.6	7.5
		0.6μ	20.3	15.6	13.2	17.0	12.8	10.3	13.5	10.4	7.2
	$\frac{1}{2}W$	0.0μ	1.5	1.0	0.3	3.8	2.2	0.7	5.2	3.4	1.0
		0.2μ	1.7	1.0	0.3	3.8	2.2	0.7	5.1	3.3	1.0
		0.4μ	1.4	0.8	0.2	3.1	2.1	0.5	4.9	3.2	0.9
		0.6μ	1.2	0.9	0.2	3.3	1.9	0.5	4.5	3.2	0.7

legend: avg rel [%]

Table 3: Relative increase of operational performance of MIX- over VAV-policy depending on σ, W, w, $|D|$ and the pattern of material flow

|D|	W	$w = \frac{1}{4}W$	$w = \frac{1}{3}W$	$w = \frac{1}{2}W$
	18	43,4 / 10.7	26.1 / 6.4	-8.3 / -2.0
24	27	69.2 / 13.4	43.4 / 8.4	-8.3 / -1.6
	36	95.0 / 15.3	60.6 / 9.7	-8.3 / -1.3
	18	89.4 / 7.5	54.2 / 4.5	-16.3 / -1.4
48	27	142.2 / 10.1	89.4 / 6.3	-16.3 / -1.2
	36	195.1 / 12.0	124.6 / 7.6	-16.3 / -1.0
	18	181.4 / 4.6	110.2 / 2.8	-32.3 / -0.8
96	27	288.3 / 6.6	181.4 / 4.2	-32.3 / -0.7
	36	395.1 / 8.2	252.6 / 5.3	-32.3 / -0.7

legend: avg abs [meters]/avg rel [%]

Table 4: Absolute and relative expected increase in average distance of the MIX-policy over the VAV-policy for $\delta = 4$

	$\|D\| = 24$			$\|D\| = 48$			$\|D\| = 96$		
	18	27	36	18	27	36	18	27	36
$w^*(W, n, \delta)$	8.28	12.78	17.28	8.31	12.81	17.31	8.32	12.82	17.32
$\frac{1}{2}W - w^*(W, n, \delta)$		0.722			0.694			0.681	

$w^*(n, W, \delta)$ [meters], $\frac{1}{2}W - w^*(n, W, \delta)$ [meters]

Table 5: $w^*(n, W, \delta)$ for $\delta = 4$ and different parameter values for W and $|D|$