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## Torque generation of flat micromotors

### ABSTRACT

The range of applications for micromotors covers a wide spectrum of life. Micromotors can be found in ATMs, self phones, or micro actuators, to name just a few. Several type of micromotors has been developed to decrease dimensions, power consumption and increase the magnitude of torque. The present work explores, how the torque generation depends on the properties of the coils.

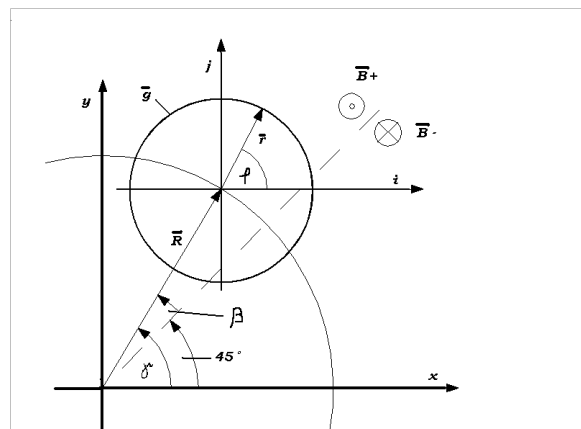
We started at the physical basis of torque generation; and derived theoretically the torque as a function of the rotation angle for various shape of coils: circle, ellipse, rectangle and ring-segment. In case of ellipse it was very difficult to describe the right analytical function; it's required advanced mathematical processes. Due to the non centrisymmetric attribute of the ellipse curve, the settlement of the elliptic coils affects the magnitude of torque fluctuation. The function determines this effect, too. We proved our analytical results by FEM models, wherin we applied our measurement data of the surface electromagnetic induction allocation on the magnet. These functions allows to find the best coil shape for a given motor dimension, to maximize the magnitude of torque while the fluctuation can be hold on the minimum.

### INTRODUCTION

Flat micromotors are a special kind of electromotors. In These motors the vector of magnetic induction is parallel to the axis of the rotation. Because this placement, the torque as the function of the rotation angle, strongly depends on the shape and the placement of the coils. The function of torque is a very important property of the micromotor. It's determines the range of applications of the motor, the lifetime of the bearings, and other parts, etc.

### CIRCLE SHAPE COIL

Circle shape coils can not be found in actual motors. The reason, why does worth examining this shape is the simplicity of the geometry. First of all, let us suppose the magnetic induction, is homogeneous, and the direction of the vector changes along a straight line. We will leave any other magnetic fields out of consideration.



**Fig. 1:** Circle shape coil and the magnetic induction vector

Equation of a rotating circle is:

$$\bar{g} = \begin{bmatrix} r \cdot \cos \varphi + R \cdot \cos \gamma \\ r \cdot \cos \varphi + R \cdot \sin \gamma \\ 0 \end{bmatrix} \quad (1)$$

Vector of magnetic induction is given:

$$\bar{B} = \begin{bmatrix} 0 \\ 0 \\ \text{sig}(y-x) \cdot B \end{bmatrix} \quad (2)$$

The signum function determines the switching of the magnetic induction vector by the slanting line.

Elementary vector of magnetic force is Lorentz force which can be described in next form:

$$\bar{f} = \frac{d\bar{g}}{d\varphi} I \times \bar{B}, \quad \frac{d\bar{g}}{d\varphi} = \begin{bmatrix} -r \cdot \sin \varphi \\ r \cdot \cos \varphi \\ 0 \end{bmatrix}, \quad (3),(4),(5)$$

$$\bar{f} = \begin{bmatrix} r \cdot \sin \varphi \cdot \text{sig}(r \cdot \sin \varphi + R \cdot \sin \gamma - r \cdot \cos \varphi - R \cdot \cos \gamma) \\ r \cdot \cos \varphi \cdot \text{sig}(r \cdot \sin \varphi + R \cdot \sin \gamma - r \cdot \cos \varphi - R \cdot \cos \gamma) \\ 0 \end{bmatrix} \cdot B \cdot I$$

The vector of resultant force is the sum of elementary magnetic force vectors along the curve:

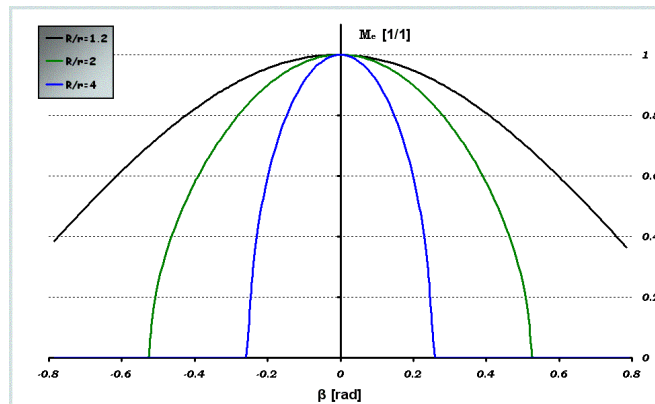
$$\bar{F}_e = 4I \int_0^\alpha B \cos \varphi d\varphi = I [B \sin \varphi]_0^\alpha = 4BI \sin \left( \arccos \left( 1 - \frac{h}{r} \right) \right) \quad \alpha = \arccos \left( 1 - \frac{h}{r} \right) \quad (6) (7)$$

After the execution of the integration, the resultant torque is:

$$\bar{M}_e = k(\gamma) \bar{F}_e = \bar{M}_e = 4NBIR \left( \cos 45^\circ - \gamma \right) * \sin \left( \arccos \left( 1 - \frac{R}{r} \sin(45^\circ - \gamma) \right) \right) \quad (8)$$

$$\text{where } N \text{ is the number of turns and } k = R \cos \beta \quad (9)$$

Amplitude of the torque depends on the rotation radius (R) of the coils. Slope of the torque function depends on the R/r ratio (**Fig. 2**).



**Fig. 2.** Generated torque by various size of circle shape coil



These points will be used for the limits of the integration of elementary force vectors.

The torque function is:

$$\overline{M}|_{x,y} = \oint_{\overline{g}} \overline{H}|_{x,y} \times d\overline{f} = 2 \int_{\zeta_2}^{\zeta_1} \overline{H}|_{x,y} \times d\overline{f} \quad \text{where } \overline{g} \text{ is the ellipse curve}$$

$$\overline{H} \text{ is the location vector of the magnetic unit force} \quad (14)$$

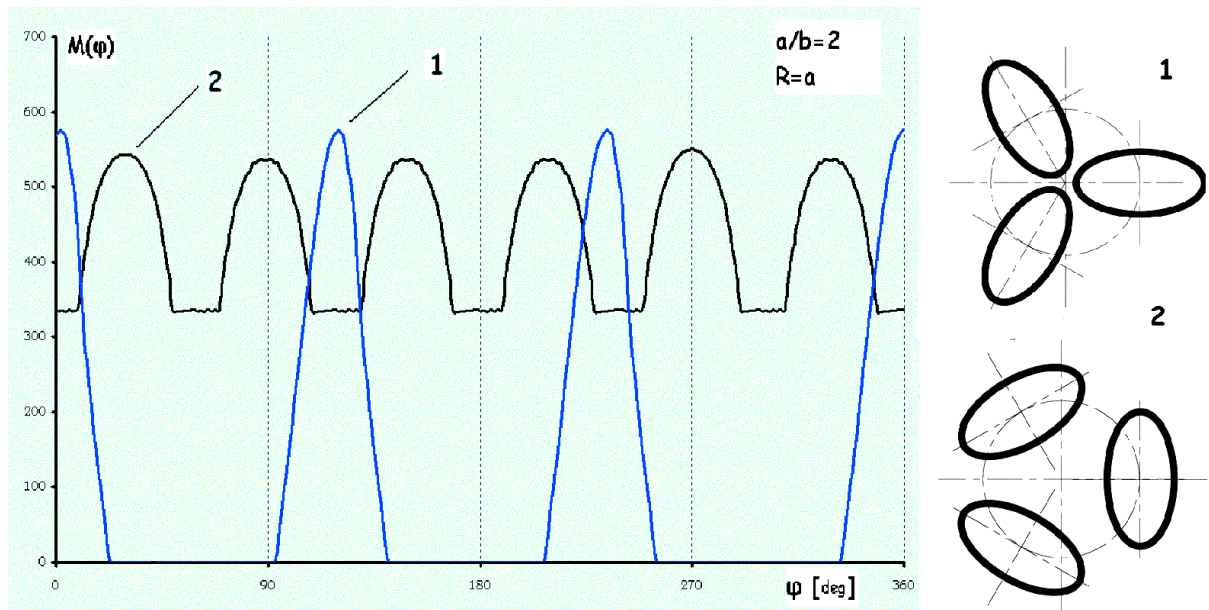
It is not necessary to do the integral on the whole boundary of the ellipse. Using the symmetrical properties, it is sufficient to execute the integration between the intersection points (13).

After transformation between the coordinate-systems the function of the torque is given:

$$\overline{M} = 2 \cdot I \cdot B \int_{\zeta_2}^{\zeta_1} \left( \sin\alpha \sqrt{\frac{a^2 - \zeta^2}{a^2 - \zeta^2 + a^2 \cdot b^2 \cdot \zeta^2}} + \cos\alpha \frac{\zeta \cdot a \cdot b}{\sqrt{a^2 - \zeta^2 + a^2 \cdot b^2 \cdot \zeta^2}} \right) \cdot \left( R \cdot \cos\alpha + \zeta \cdot \cos\alpha - \sin\alpha \cdot \sqrt{b^2 \cdot \left(1 - \frac{\zeta^2}{a^2}\right)} \right) +$$

$$+ \left( \cos\alpha \sqrt{\frac{a^2 - \zeta^2}{a^2 - \zeta^2 + a^2 \cdot b^2 \cdot \zeta^2}} + \sin\alpha \frac{\zeta \cdot a \cdot b}{\sqrt{a^2 - \zeta^2 + a^2 \cdot b^2 \cdot \zeta^2}} \right) \cdot \left( R \cdot \sin\alpha + \zeta \cdot \sin\alpha + \cos\alpha \cdot \sqrt{b^2 \cdot \left(1 - \frac{\zeta^2}{a^2}\right)} \right) d\zeta \quad (15)$$

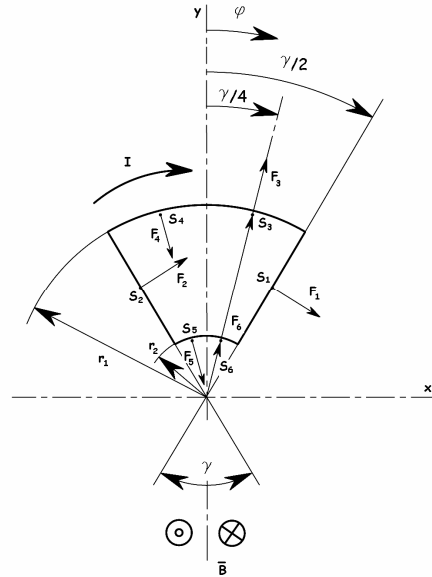
It is probably impossible to solve this equation by analytical method, so we solved it by a numerical integration process. The magnitude of the torque depends on the radius of the rotation (R), and not depends on the angular of the placement of the coils (**Fig. 4**). The best placement is, where the big axes of the coils are parallel to the circumferential velocity vector. This placement effects the smallest torque fluctuation.



**Fig. 4:** Torque function in case of different coil arrangement

## RING-SEGMENT SHAPED COIL

Ring-segment shape has two long, straight sections which is a very useful property because the segments can be placed side by side without a gap. So this shape allows the best economy in space among the various coil shapes.



**Fig. 5:** Vector sum of the magnetic force on the different sections of the coil

The resultant magnetic force vectors on the straight parts of the coil is:

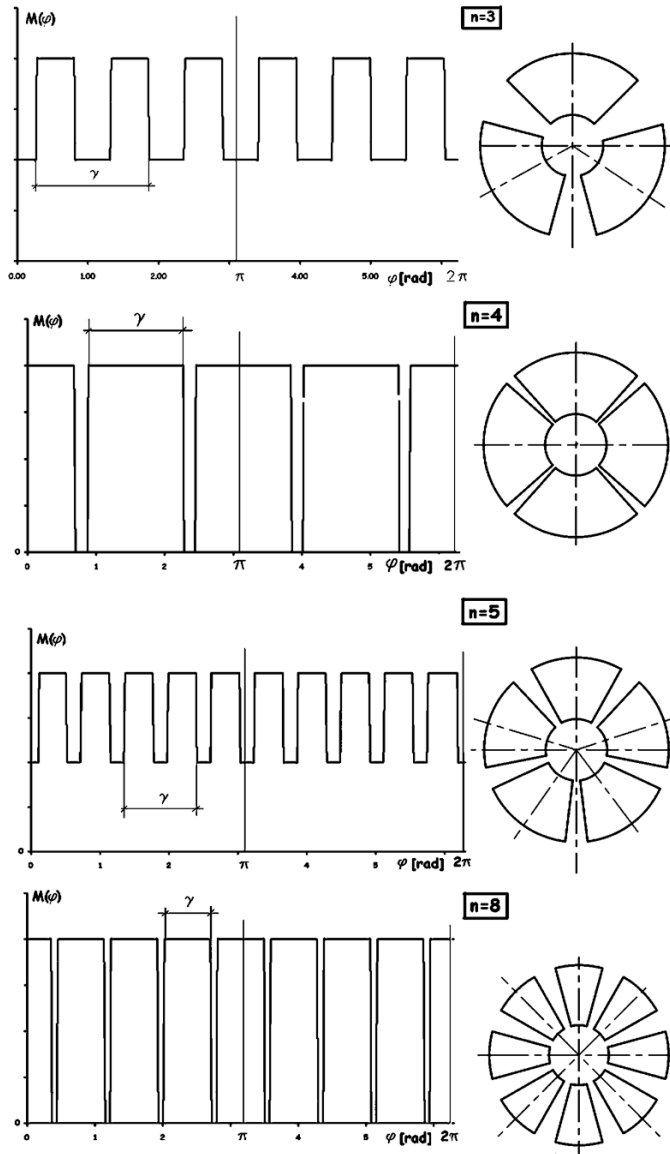
$$F_1 = F_2 = (r_2 - r_1) \cdot I \cdot B \quad (16)$$

We can discard the resultant magnetic force vectors on the curved parts of the coil, because these vectors are small in the second order beside the  $F_1$  and  $F_2$ .

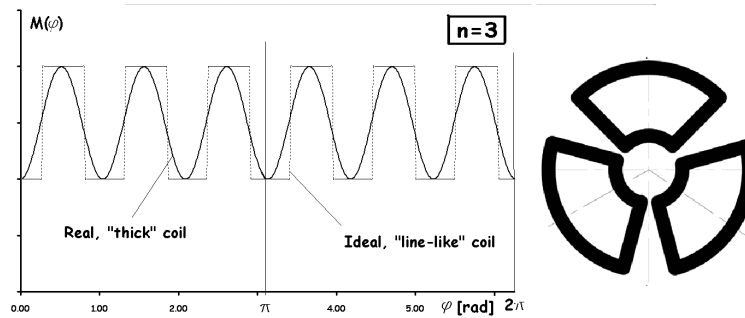
The generated torque is given by:

$$M_1 = M_2 = 2 \cdot \left( \frac{(r_2 - r_1)}{2} + r_1 \right) \cdot (r_2 - r_1) \cdot I \cdot B \quad (17)$$

Accordant to the **Fig. 6.**, using even number of coils is not advantageous because the amplitude of the torque fluctuation is twice greater than using odd number of coils. In actual slim motors the function of torque is not as square-shaped as the (17) results (**Fig. 6.**). Because of the thickness of the coils, and the force that generated on the curved parts, the function of torque is rather sinusoidal (**Fig. 7.**).



**Fig. 6:** Torque function in case of various number of coils



**Fig 7:** Estimated function of torque in actual motors



## CONCLUSION

The smallest amplitude of torque fluctuation can be reached by using at least 5 ring-segment shaped coils. This is the most space efficient construction for flat micromotors. Using ellipse shape coils, the properties of the torque function can be correct by choosing the convenient geometry.

## FURTHER RESEARCH

Torque generation of the actual "thick" coils can be described more accurately by finite element models. The model allows to consider the distribution of the real magnetic induction vectors between the magnet and the core.

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