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OPTIMAL CONTROL OF DISTRIBUTED PARAMETER SYSTEMS ON THE EXAMPLE OF A GLASS FEEDER

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ABSTRACT

In this work, different optimization based control approaches for distributed parameter systems are investigated on the example of a glass feeding process. After deriving the model equations for the glass feeder, an overview of optimization based control is given. Due to their practical advantages, the focus here is on direct methods, namely direct collocation and multi-stage control parametrization. Several simulation results are presented to illustrate the effectiveness and essential properties of optimization based control. As an alternative control approach, the flatness based Transportansatz is reviewed. Supplementary simulation experiments using this method for controller design are given in addition. Concluding, a comparison between different control approaches considered in this work is drawn.

Index Terms— distributed parameter systems, optimal control, numerical optimization, direct methods

1. INTRODUCTION

In many real-world applications, model-based controller design leads to systems with distributed parameters. Such systems are usually modeled using partial differential equations (PDEs), for which it is often impossible to find an analytical solution. The infinite dimensionality of the problem makes controller design a highly challenging task. Optimization based control approaches have proven as reliable methods to achieve the desired control performance for such systems. In recent years, differential-algebraic control approaches, particulary flatness based methods, have emerged as a promising new approach to control distributed parameter systems. In this work, different optimization based control methods are investigated on the example of an industrial glass feeding process. Comparing the performance of such differential-algebraic methods with optimization based control approaches is an interesting question, which will be investigated using representative simulation experiments.

The rest of this work is organized as follows. In section 2, the mathematical model for the example process is derived and the control problem is defined. In section 3, different methods for optimal control of PDEs are characterized. Section 4 gives a brief review of a recently developed flatness based control approach called Transportansatz. In section 5, simulation results for different optimal control problems on the described example process are provided and compared to the results obtained with the differential-algebraic control approach. The results are concluded in the last section, where also a brief outlook to future work is given.

2. CONTROL PROBLEM

In this work, optimal control of distributed parameter systems will be discussed on the example of an industrial glass feeding process for container glass production. The glass feeder under consideration is a channel that connects the smelting furnace, with the forming devices. The hot molten glass enters the feeder with inlet temperature and flows with a certain velocity through the channel. At the end of the feeder, it leaves the channel at outlet temperature and is formed to the final product. The main purpose of the feeder is the transport of the molten glass to the former and the precise achievement of a desired outlet temperature, which depends on the particular type of container glass that is currently produced. The temperature of the glass can be influenced using gas burners that are located along the feeder.

The glass feeder is divided into several segments or zones, which have different geometric properties and energetic behavior. Thermocouples at the beginning and end of each zone allow to measure the inlet and outlet temperature for each segment. Gas burners belonging to the same zone are jointly controlled. Therefore, for each zone a different temperature setpoint can be defined, and each zone can be controlled individually. Temperature control was originally done by using

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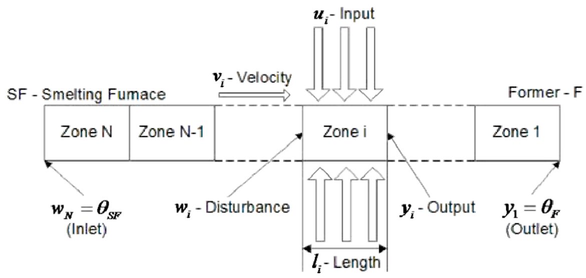


Fig. 1: Glass feeder divided in zones.

simple PI-controllers, and the main reason for the segmentation of the feeder in multiple zones was to avoid instabilities of the original controllers. The segmentation of the feeder is illustrated in fig. 1. As shown, zones are numbered in decreasing order in flow direction. The molten glass flows with a certain velocity v from the inlet of the N^{th} to the outlet of the first zone. Neighboring zones are coupled at the spatial boundary in flow direction. That means that the outlet temperature y_i of the i^{th} zone is equal to the inlet temparature w_{i-1} of the following zone. For the N^{th} zone, w_N is given by the temperatur of the glass melt leaving the smelting furnace. The inlet temperatur is considered as a disturbance. To achieve the desired outlet temperature, the distributed control u_i, located in form of gas burners along the feeder, is used.

Based on this description of the feeder, a mathematical model for a single zone can be formulated. For the sake of simplicity, the index i is in the following omitted. Because the length of a zone is large compared to the width of the feeder channel, it is sufficient to consider a one dimensional model. The dynamics of the spatially distributed temperatur $\theta(z,t)$ is described by the following transport process:

$$\frac{\partial}{\partial t}\theta(z,t)+v\frac{\partial}{\partial z}\theta(z,t)+k_1\theta(z,t) = k_2\beta(z)u(t)\,. \quad (1)$$

The parameter v denotes the velocity of the glass melt in flow direction, which is assumed to be constant. The spatial characteristic of the input $u(t)$ is described by $\beta(z)$. Note that this is a special case of a distributed input, because it is possible to describe the input with the concentrated control variable $u(t)$. The unknown parameters k_1 and k_2 have been identified in a previous work by Henkel et al. [1]. The glass feeder is not modeled using the classical heat equation because the flow velocity v is significantly higher than the temperature diffusity. Therefore, a transport process is enough to model the glass feeder with sufficient accuracy.
Consequently, only one boundary condition is necessary, namely

$$\theta(0,t) = w(t)\;, \quad (2)$$

which corresponds to the coupling of neighboring zones via their spatial boundary. The initial condition is given by

$$\theta(z,0) = \theta_0(z)\;, \quad (3)$$

while the output of the zone is defined by

$$y(t) = \theta(L,t)\;, \quad (4)$$

where L is the length of the feeder zone.

The system of equations (1)-(3) and (4) will be used as example control problem throughout the rest of this work. Regarding controller design, two main objectives are usually distinguished, namely stabilization around an operating point, and dynamic tracking of a reference trajectory. In this work, the focus lies on feed forward controller design for trajectory tracking, which is important for example when the operating temperature of the glass feeder is shifted due to changes in the production line.

3. OPTIMIZATION BASED CONTROL

Optimization based approaches have been widely used to control distributed parameter systems, see e.g. [2] or [3]. The general idea is to transform the control problem to a suitable optimization problem. Often, optimization based control approaches do not deliver a general control law, but a sequence of optimal, possible spatially distributed input values over a finite horizon t_f. The optimization problem must be formulated such, that this resulting input will achieve the desired control performance.

The result of any optimization depends mainly on the chosen cost functional. In the case of trajectory tracking, a straight forward way to formulate the cost functional is given by

$$J(\theta(z,t),u(t),t) = \int_0^{t_f} \underbrace{(y(t)-y_{\text{ref}}(t))^2}_{f_0}\,dt\;. \quad (5)$$

In other words, the mean squared deviation of the outlet temperature from the desired reference trajectory y_{ref} will be optimized. The optimal control problem is then given by

$$\min_{u(t)} J(\theta(z,t),u(t)) \quad (6)$$

subject to the system dynamics (1) and (4), boundary (2) and initial condition (3), as well as additional state and control constraints given by

$$g(\theta(z,t),u(t)) \leq 0\;. \quad (7)$$

The optimal control problem (6) is an infinite-dimensional problem in $u(t)$. In most cases, an analytical solution of the optimization problem cannot be found. Therefore, numerical methods play an important role in computing the optimal control. Numerical optimization methods may be divided into two main groups, namely indirect and direct methods.

3.1. Indirect methods

Indirect methods are based on variational calculus and make immediate use of the optimality conditions. In case of optimal control for systems governed by ordinary differential equations (ODEs), starting with the so called Hamiltonian the adjoint state p is derived, which is in case of distributed parameter systems also a distributed state. The adjoint state equation forms together with the original system dynamics the augmented (or canonic) system. Based on Pontryagin's maximum principle, first order optimality conditions can be derived. Together with some additional conditions and a boundary condition for the adjoint state, this forms a two-point boundary value problem, from whose solution the optimal control can be determined (see e.g. [4]). In case of distributed parameter systems, the problem becomes more complicated. One possibility is to spatially discretize the system and then apply common solution techinques to integrate the resulting system of ordinary differential equations (method of lines), see e.g. [5]. Another approach is to derive the optimality conditions analogically to the case of ODE systems. However, for systems goverened by PDEs this is a difficult problem and requires the user to have a profound knowledge on variational calculus to be able to derive the adjoint state equation and optimality conditions (see e.g. [2] for a detailed introduction).

A frequently used method to solve the boundary value problem is the so called multiple shooting. Originally developed for two-point boundary value problems in ordinary differential equations, shooting methods attempt to find the solution by iteratively improving the initial value until the terminal value is matched. The terminal value is computed numerically, using e.g. finite-differences (FDM) or finite-element methods (FEM). Multiple shooting divides the solution process in the time domain into several shooting approaches that are coupled via additional matching conditions. A more detailed explanation of multiple shooting in case of distributed parameter systems can be found e.g. in [6].

Multiple shooting has the advantage, that all kinds of state and input constraints are allowed, and that usually highly accurate solutions can be obtained. However, as with all indirect methods, there exist several difficulties that hinder their usuage in many applications. One of the main disadvantages is the need to derive the adjoint state equation and optimality conditions to set up the boundary value problem. When inequality constraints must be considered, the problem complexity grows further. In this case, the solution will switch at points where inequality constraints become active/inactive. Therefore, a priori knowledge about the solution structure, i.e. at least the number of switching points, is necessary. Another main disadvantage is the need for initial estimates for the state and the adjoint state. Moreover, the system might be very sensitive to small changes in the initial values, and numerical solutions may be ill-conditioned even for reasonable initial estimates. Due to these difficulties in application, the remainder of this work will focus on direct methods that are explained in the next subsection.

3.2. Direct methods

Direct methods aim to find the solution of the originally infinite dimensional optimization problem by approximating it with a finite dimensional problem [7]. For that, the continuous optimal control $u(t)$ could for instance be approximated with a finite sequence u_k. In case of a distributed control input, it is moreover necessary, to spatially approximize the control, for instance via discretization. The cost function can then be directly optimized, using the approximation parameters as optimization variables. As the main advantage of direct methods, the user is not forced to derive the adjoint state and deal with the optimality conditions from variational calculus. Moreover, any kind of state and control constraints can be relatively easy included in the optimization process. However, the solution obtained is usually less accurate compared to indirect methods. In addition, the PDE and the state constraints are usually only satisfied at the approximation points. This makes indirect methods more adequate for critical applications such as e.g. trajectory planning in aeronautics.

In most applications, however, the accuracy provided by direct methods is more than sufficient to achieve the desired control performance. In the following, two different direct approaches for optimal control of the glass feeder are presented. The first is called direct collocation, where both the control and the state are approximated at a finite number of points in time, the so called collocation points. Since the state is a spatially distributed variable, it must be spatially discretized as well, using appropriate discrete approximations of the spatial derivatives. Consequently, the direct collocation method implies a full discretization of the state. The resulting nonlinear optimization problem has the approximated control and state as optimization parameters, and the discretized PDE and the additional conditions (7) as constraints. Depending on the chosen discretization grid size, the optimization problem becomes quickly very large. However, under certain conditions it is possible to exploit the particular structure which results from the discretization process, to find the solution in a memory efficient way.

Another direct method is the so called control parametrization. As the name implies, here the control is approximated, e.g. as piecewise constant, and the approximation parameters are taken as optimization variables. To evaluate the cost functional, the system equations are numerically integrated. In case of distributed parameter systems, integration could be done either using PDE solvers like FDM or FEM, or again using the method of lines, i.e. using ODE solvers after spatial discretization. Control constraints can be directly taken into account during the optimization. General state and control constraints (7) can also be considered, however, in general they will be satisfied only at a finite number of approximation points. The result is a large unstructured non-linear optimization problem, which can be numerically solved using e.g. sequential quadratic programing (SQP). To improve the numerical condition of the problem, the multi-stage control parametrization approach divides the time domain into multiple so called stages, which are coupled via additional matching conditions. By that, the problem turns into a structured optimization problem, which can be utilized by an adequat solver. Moreover, the sensitivity to the approximation parameters decreases, and thus the numerical solution can be obtained more efficiently.

4. ALTERNATIVE CONTROL APPROACH

Among alternative control approaches for distributed parameter systems, differential-algebraic methods have drawn a lot of attention during recent years. Of particular interest are flatness based approaches, which provide a more profound insight into the structural properties of a dynamical system, see e.g. [8], [9], or [10]. Characteristic for a flat system is that its dynamical behavior can be fully described by using only the so called flat output and its derivatives. This implies a kind of system inversion, which allows to describe the states and input of the flat system only depending on the flat output. Consequently, the control for almost arbitrary reference trajectories can be easily computed for such systems.

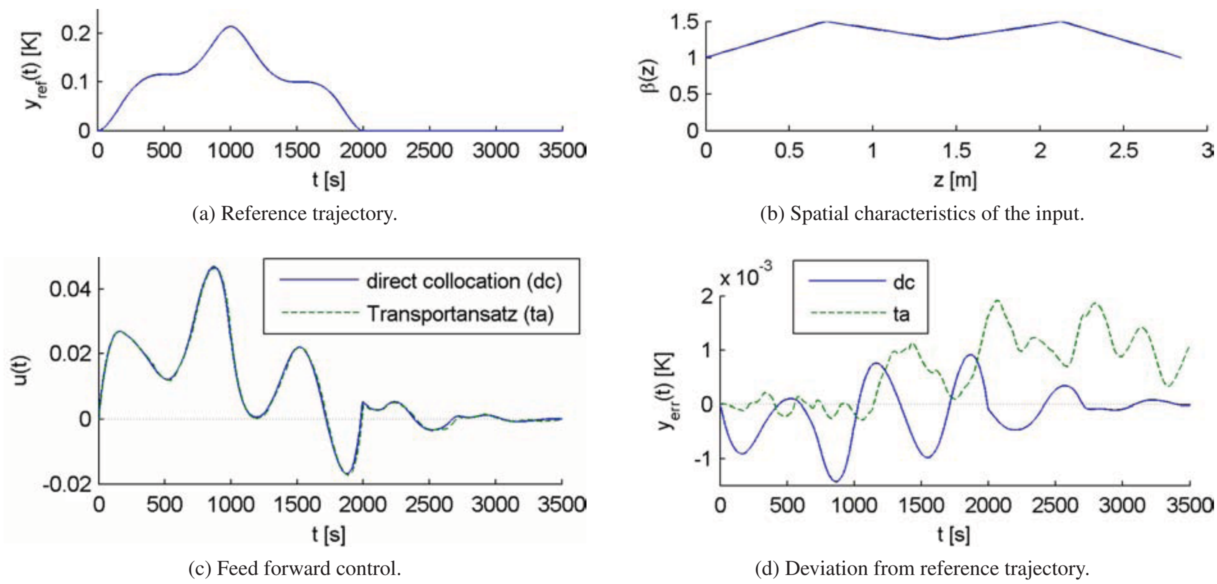
Recently, a differential-algebraic control approach for transportation systems with spatially distributed control, called Transportansatz, has been presented by Malchow et al. [11]. Based on the exact solution of (1)–(4), the authors develop a recursive formula to compute the feed-forward control that will produce the desired output trajectory. The resulting control law contains an infinite series, whose subsequent series elements compensate for the control error caused by preceding elements. In practice, the infinite series must be approximated by a finite one. For that, the infinite series is truncated after the m^{th} element. As a side effect of this method, it is possible to give an a priori estimation of the resulting control error. Representative for differential-algebraic methods, the Transportansatz will be used in the following section for a comparison to optimization based control approaches.

5. SIMULATION RESULTS

The simulation model for a single zone of a glass feeder has been implemented in MATLAB/Simulink based on (1), extended by an additional diffusive term $-\alpha^{\partial^2\theta}/_{\partial z^2}$, using finite differences. The system parameters for the zone are $\alpha = 7.3 \cdot 10^{-7}\,\mathrm{m^2s^{-1}}$, $v = 4.0 \cdot 10^{-3}\,\mathrm{ms^{-1}}$, $k_1 = 2.2 \cdot 10^{-3}\,\mathrm{s^{-1}}$, $k_2 = 0.014\,\mathrm{Ks^{-1}}$ and $L = 2.844\,\mathrm{m}$, based on the results in [1]. The FDM model has 640 spatially discretization points, corresponding to a spatial grid size of approximately 4.44 mm. The time step size is 1 s. Controller synthesis for the direct collocation approach and the Transportansatz based approach is done using MATLAB. For the Multi-stage control parametrization, the C++ tool HQP/Omuses has been used, which provides efficient solution methods exploiting the structure of the optimization problem. In the following, simulation results are presented that show the effectivity of optimization based feed forward controller synthesis and illustrate the differences to differential-algebraic methods.

In the first simulation, a feed forward control for the polynomial reference trajectory shown in fig. 2(a) is computed using the direct collocation approach and the Transportansatz. The location dependent parameter $\beta(z)$ is distributed as shown in fig. 2(b). No control or state constraints are considered, and the disturbance at the inlet is set to be $w(t) \equiv 0$. For the direct collocation approach, the PDE is discretized at 876 spatial and 143 temporal discretization points. In the absence of any constraints, the problem degenerates to a system of 126 143 linear equations, corresponding to the 125 268 collocation conditions and 875 time steps of the discretized boundary condition. In this case, the feed forward control can be easily computed, see blue line in fig. 2(c). The green dashed line in the same figure shows the feed forward control that was computed using the Transportansatz. For the implementation, the infinite series was approximated using six series elements. The number of series elements was chosen such that the error using either method is similar, see fig. 2(d). However, in this case, computing the control using the direct collocation is significantly faster (approx. 40 times) than using the Transportansatz, which takes about ten seconds on an Intel Core 2 Duo @ 2.5 GHz.

In the second simulation, a change of the operating point by 4 K over a transition time of 750 s is considered, as shown in fig. 3(a) (blue line). Again, no disturbance at the inlet of the zone and no state constraints are considered. Moreover, the spatial characteristic of the input is assumed to be $\beta(z) = 1$ for $z \in [0, L]$. However, in this simulation the control is constrained



(a) Reference trajectory.

(b) Spatial characteristics of the input.

(c) Feed forward control.

(d) Deviation from reference trajectory.

Fig. 2: Unconstrained trajectory tracking.

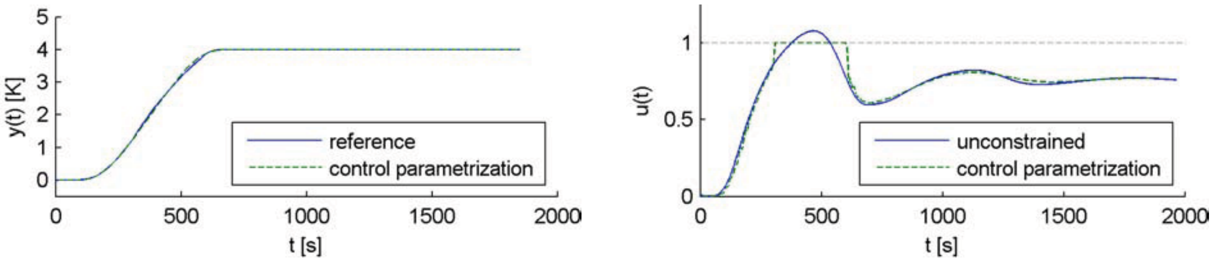
to $|u(t)| \leq 1$. As shown in fig. 3(b) (blue line), the control constraint (here computed using the direct collocation approach) will be violated by the resulting feed forward control, if it is not considered during computation. Taking the input constraint into account, the direct collocation approach results in a least squares problem with linear inequality constraints. Matlab is not very well suited for that problem, because it doesn't provide a method to exploit the sparsity of the inequality constraints, and thus quickly runs into memory problems. Therefore, the optimal control problem has been solved using a multi-stage control parametrization approach using HQP/Omuses. The resulting feed forward control shown in fig. 3(b) (green dashed line) satisfies the given control constraint, resulting only in a slight deviation from the reference trajectory (compare fig. 3(a), green dashed line). Using the Transportansatz to compute the control has the disadvantage, that the control constraint cannot be taken into account. This means, that the reference trajectory must be explicitly adjusted. For the very simple trajectory chosen in this case, only one parameter – namely the transition time – can be adjusted, which must be set to at least 900 s to satisfy the constraints.

The last simulation is meant to illustrate the problem of state constraints. Again, no disturbance at the inlet and a uniform spatial characteristic of the control are assumed. The reference temperature at the outlet of the zone is supposed to increase by 4 K and then decrease by 4 K again after 200 s, see fig. 4(a). For this simulation experiment, an arbitrary state constraint was chosen: $|{\partial\theta}/{\partial t}| \leq 0.01$. Such a constraint on the temperature change might be motivated by the production process. With a transition time of 800 s for both the increase and decrease of the outlet temperature, the state constraint is satisfied at the outlet $z = L$ as shown by the blue curve in fig. 4(b). However, looking at the evolution of the temperature at $z = 1.65\,\mathrm{m}$ as indicated by the green dashed curve, the constraint may be violated at other locations along the feeder. In consequence, the full distributed state must be considered, when computing the control if state constraints exist.

6. CONCLUSIONS

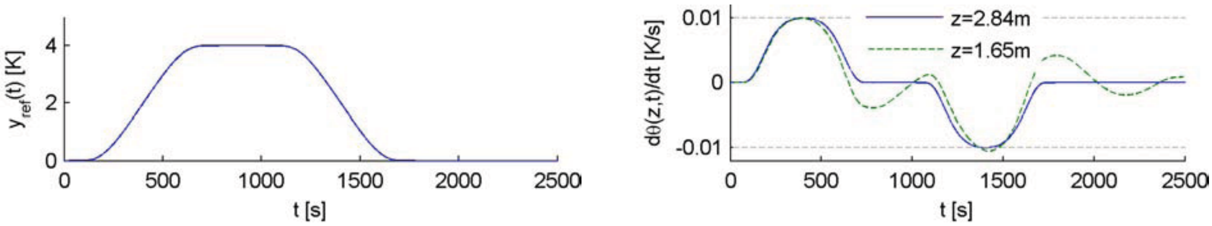
In this work, optimal control of systems governed by PDEs was investigated on the example of a glass feeder process. The main focus was on direct methods for the numerical solution of the optimization problem, because of the easy application compared to indirect methods. One of the main advantages is the fact that control constraints can be easily accounted for. The direct collocation method turns out to be very well suited for the unconstrained case. If constraints exist, memory requirements quickly pose a limit on the precision of the solution and require for optimized solution methods. The multi-stage control parametrization offered an overall good performance. Control constraints as well as state constraints can be relatively easy implemented. The flatness based Transportansatz has the advantage, that it directly provides a way to compute the control based on a given reference trajectory. Moreover, it provides means to a priori calculate the control error. However, since control constraints cannot be considered at design time, an iterative approach must be chosen, during which the reference trajectory is adjusted to satisfy the constraints. Therefore, it is necessary to design feasible reference trajectories with a sufficient degree of freedom.

Concerning arbitrary state constraints, it is still not



(a) Outlet temperature. (b) Feed forward control.

Fig. 3: Set point change under control constraints.



(a) Reference trajectory. (b) Time derivative of the outlet temperature.

Fig. 4: Set point change under state constraints.

clear how to consider those when using the differential-algebraic controller design approach. While this can be relatively easy done for optimization based controller design, more research is necessary to investigate how they can be considered using the Transportansatz. It would be of further interest to investigate the possibility to combine the flatness based approach with the optimization based control design approach, to exploit any synergetic effects that might exist. The presented methods for controller design should be applied to different dynamical systems in future works. Based on that, the goal is to derive a benchmark that allows to systematically compare different methods for feedforward controller design.

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