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INVESTIGATION OF MOTOR, GENERATOR, AND BRAKING OPERATING MODES OF SPHERICAL MULTI COORDINATE INDUCTION MACHINES

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ABSTRACT

In this paper a spherical multi coordinate induction machine for transport purposes is considered. On the basis of a non-traditional mathematical model, which is based on Integral Equation Method, motor, generator, and braking operating modes are analyzed.

Index Terms **-** Multi Coordinate Spherical Induction Drive, Integral Equation Method

1. INTRODUCTION

The most existing multi coordinate drives were developed for high precision and high dynamics applications such as semiconductor manufacturing systems, machining tools, laser-cutting systems and others. At the same time there are number of applications such as special transport and logistic systems, where reliable, robust, and low-cost planar or spherical multi coordinate drives are needed without high requirements to precision and dynamics. For such requirements, the induction motor principle seems to be well suitable, which has following structural advantages:

- Relatively simple construction,
- Homogeneous and unstructured secondary part,
- Arbitrary orientation of propulsion.

In this paper a spherical multi coordinate induction machine is considered. The aim is to develop a mathematical model which is well suitable for model based design of such machines, by taking into account of all relevant operating modes: motor, generator, and braking operating modes. The modeling of spherical induction machines by means of traditional methods which are based on the solving of a boundary problem for the Laplace equation in terms of magnetic scalar or vector potential was already considered by several authors [1]. But the resulting equations for calculation of integral function parameters are very difficult because of appearance of special functions. Therefore their suitability for model based design is limited.

In the present paper a non-traditional modeling approach is proposed which is based on the Integral Equation Method. Using this method the boundary value problem can be reduced to the integral equation for a flow function, which can be chosen, so that it is non zero only in the conductive layer. Because in practice the conductive layer of the rotor is very thin in comparison with other dimensions, the conductive layer can be assumed as infinite thin. Therefore an originally spatial problem can be transformed into a surface problem. Furthermore, the Integral Equation Method is more economical in numerical realization.

2. PROBLEM STATEMENT

Consider a spherical induction machine. Its configuration is shown in figure 1.

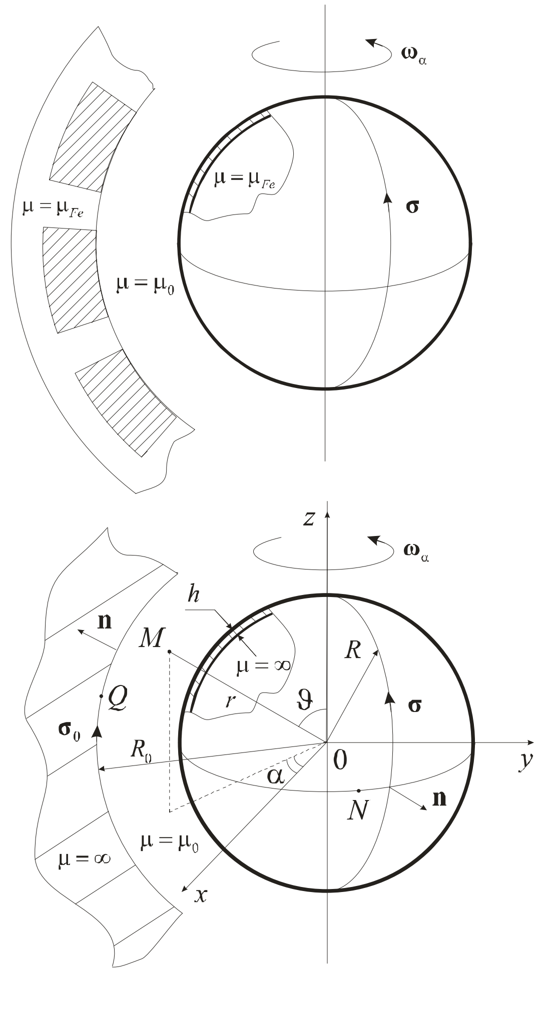

Figure 1 a) Structure of the spherical induction machine, b) graphical representation of the mathematical model assumptions and relevant geometric parameters

The following assumptions are made:

- The machine has a smooth stator (primary) of interior radius R_0 and a smooth rotor (secondary) of radius R.
- All materials of the system have isotropic and homogeneous properties.
- The stator has a macro current with linear density $\boldsymbol{\sigma}_0$ on its surface.
- The influence of stator teeth is taken into account by Carter's coefficient.
- The rotor consists of iron ball and thin spherical layer with conductivity γ and thickness h. In practice the conductive layer of the rotor is very thin in comparison with other dimensions and therefore it can be considered as a surface of radius R and equivalent linear conductivity γ^*.
- The angular velocity of the rotor is ω_α.
- The primary current sheet has m pairs of magnetic poles. m is an integer number.
- The relative permeability of iron in the stator and the rotor is infinite.
- The relative permeability of other machine parts is 1.
- Relativity effects are negligible.

Two-dimensional fields of primary ($\boldsymbol{\sigma}_0$) and secondary ($\boldsymbol{\sigma}$) current densities are solenoidal. Therefore they can be represented by scalar flow functions τ_0 and τ respectively.

$$\boldsymbol{\sigma} = \left[\nabla_S \tau, \mathbf{n}\right], \quad \boldsymbol{\sigma}_0 = \left[\nabla_S \tau_0, \mathbf{n}\right] \tag{1}$$

with calibration conditions

$$\oint\!\!\!\oint_{r=R} \tau dS = 0, \quad \oint\!\!\!\oint_{r=R_0} \tau_0 dS = 0. \tag{2}$$

For the present problem the spherical coordinate system (see Figure 1) is taken. If the primary current is sinusoidal, then for the corresponding flow function follows

$$\tau_0(\theta,\alpha,t) = \tau_{0\,\max}(\theta)\,\cos(\omega t - m\alpha) =$$
$$= \sum_{n=m}^{\infty} A_{0n} P_n^m\left(\cos(\theta)\right)\frac{1}{2}\left(e^{j(\omega t - m\alpha)} + e^{-j(\omega t - m\alpha)}\right).$$

The primary current density generates a traveling magnetic field. Further the analysis is made for any harmonic component of the primary current

$$\tau_{0n}^{\pm}(\theta,\alpha,t) = A_{0n} P_n^m(\cos\theta) e^{\pm j(\omega t - m\alpha)}.$$

General the resulting primary current are obtained by superposition of individual harmonics. Because the spherical functions $P_n^m(\cos\theta)e^{\pm jm\alpha}$ are orthogonal, the integral functional parameters can also be obtained by superposition. Denote selected harmonic component by

$$\tau_0 = \tau_{0n}^{\pm}. \tag{3}$$

The same assumptions are made for secondary current density and its potentials.

3. EQUATIONS FOR THE SECONDARY SOURCES

In the present case there are no sources of the magnetic field in the area $R < r < R_0$. Therefore the intensity $\mathbf{H}$ is a potential field and can be represented by $\mathbf{H} = -\operatorname{grad}\varphi$, $r \neq R$, $r \neq R_0$ with calibration

$$\oint\!\!\!\oint_{r \neq R_0, R} \varphi dS = 0.$$

The boundary problem for φ has the following form

$$\Delta\varphi = 0, \;\; R < r < R_0,$$
$$\varphi = 0, \;\; R > r > R_0,$$
$$\varphi^+ - \varphi^- = \tau, \;\; r = R,$$
$$\varphi^+ - \varphi^- = \tau_0, \;\; r = R_0$$

Symbols "$+$" and "$-$" in superscript denote the limiting values along positive and negative directions of the normal respectively.

The solution of this problem can be represented as following[1]

$$\begin{aligned}\varphi(M) = {} & \frac{1}{4\pi}\oint\!\!\!\oint_{r=R} \tau(N)\frac{\partial}{\partial n_N}\frac{1}{r_{NM}} dS_N + \\ & + \frac{1}{4\pi}\oint\!\!\!\oint_{r=R_0} \tau_0(Q)\frac{\partial}{\partial n_Q}\frac{1}{r_{QM}} dS_Q + \\ & + \frac{1}{4\pi}\oint\!\!\!\oint_{r=R} \frac{\rho(N)}{r_{NM}} dS_N + \frac{1}{4\pi}\oint\!\!\!\oint_{r=R_0} \frac{\rho_0(Q)}{r_{QM}} dS_Q.\end{aligned} \tag{4}$$

τ_0 is given, $\rho = -\dfrac{\partial\varphi^+}{\partial n}$, $r = R$; $\rho_0 = \dfrac{\partial\varphi^-}{\partial n}$, $r = R_0$, for τ is valid the equation

$$\Delta_S \tau = -\gamma^* \mu_0 \left(\omega_\alpha \frac{\partial}{\partial\alpha} + \frac{\partial}{\partial t}\right)\frac{\partial\varphi^+}{\partial n}, \; r = R. \tag{5}$$

Equation (5) follows from the equation $\operatorname{rot}_n \boldsymbol{\delta} = \operatorname{rot}_n \gamma\left(\mathbf{E} + \left[\mathbf{v}, \mathbf{B}\right]\right)$ after integration over the thickness of the conductive layer by taking into account of following equations: (1), $\boldsymbol{\sigma} = \int_{-h}^{0} \boldsymbol{\delta} dn$, $\operatorname{rot}\mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$, and $\mathbf{B} = \mu_0 \mathbf{H}$. n is the coordinate along the normal to the sphere. Zero point of this coordinate is situated on the sphere $r = R$.

The linear conductivity γ^* from (5) is represented [3] as following[2] $\gamma^* = \dfrac{\gamma}{p}\tanh ph$,

---

[1] Variable t in arguments is excluded for short.

[2] According to [2] electromagnetic wave has primary direction along the thickness of the layer whenever this layer is thin. Massive part of the rotor is removed by image method [4].

$p=\sqrt{\pm j\left(\omega_\alpha m-\omega\right)}$ in the case if skin effect is taken into account. Otherwise $\gamma^* = \gamma h$.

By integrating over the thickness of the conductive layer from (5) follows

$$\Delta_S \tau = \gamma\mu_0\left(\omega_\alpha \frac{\partial}{\partial\alpha}+\frac{\partial}{\partial t}\right)\int_{-h}^{0} H_n dn .$$

The integral on the right side of the equation can be represented by $H_n\left(-h,\theta,\alpha\right)$. The curvature of the layer and derivatives along the layer in the Laplace operator are negligible. Hence the following approximate equation can be considered.

$$\frac{\partial^2 \mathbf{H}}{\partial n^2} = \pm j\gamma\mu_0\left(\omega_\alpha m-\omega\right)\mathbf{H},\ 0<n<-h \tag{6}$$

for any harmonic component of the magnetic field instead of the equation $\Delta\mathbf{H} = \gamma\mu_0\left(\omega_\alpha \frac{\partial}{\partial\alpha}+\frac{\partial}{\partial t}\right)\mathbf{H}$.

Then the method of Images is used (Figure 2). The normal component for the solution of equation (6) with even symmetry (fig.2) has the following form $H_n\left(n,\theta,\alpha\right) = C\left(\theta,\alpha\right)\cosh(pn)$.

$$C\left(\theta,\alpha\right) = \frac{p}{\mathrm{sh}(ph)}\int_{-h}^{0} H_n dn$$

or

$$C\left(\theta,\alpha\right) = \frac{1}{\mathrm{ch}(ph)} H_n\left(-h,\theta,\alpha\right).$$

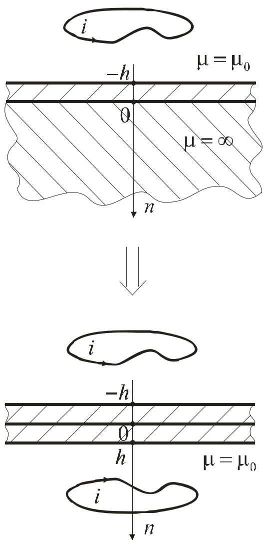

Figure 2

Thus

$$\gamma\int_{-h}^{0} H_n dn = \frac{\gamma}{p}\tanh\left(ph\right)H_n\left(-h,\theta,\alpha\right) = -\gamma^*\frac{\partial\varphi^-}{\partial n}\left(\theta,\alpha\right)$$

.

Note that the used assumptions are valid if $h \ll R$ and $m^2 \ll \gamma\mu_0\left|\omega_\alpha m-\omega\right|R^2$. In this case approximate solution for the eddy currents density $\boldsymbol{\delta}$ has a similar form. Namely

$$\boldsymbol{\delta}\left(n,\theta,\alpha\right) = \frac{\boldsymbol{\sigma}\left(\theta,\alpha\right)p}{\sinh\left(ph\right)}\cosh\left(pn\right). \tag{7}$$

In this way the approximate distribution of the current in all conductive volume using the density $\boldsymbol{\sigma}$ is determined.

Consider last two integrals from (4). They are potentials of single layer. However they can be represented as potentials of double layer with densities τ^* and τ_0^*. Using the basic integral formula of the harmonic function theory [5]

$$\frac{1}{4\pi}\oiint_S \frac{\partial\psi}{\partial n}\left(N\right)\frac{1}{r_{NM}}dS_N - \frac{1}{4\pi}\oiint_S \psi\left(N\right)\frac{\partial}{\partial n_N}\frac{1}{r_{NM}}dS_N =$$
$$= \begin{cases} \psi\left(M\right), & M \in V; \\ 0, & M \notin V \cup S, \end{cases}$$

where ψ is a harmonic function in a finite volume V with boundary S; $\mathbf{n}$ is the exterior normal to the surface S. If V is infinite the equation is valid by $\psi\left(M\right)\xrightarrow[M\to\infty]{} 0$.

Using (3) the boundary conditions of our problem can be represented by

$$\tau^* = -\frac{R}{n}\frac{\partial\varphi^+}{\partial n},\ r=R,\ \tau_0^* = \frac{R_0}{n+1}\frac{\partial\varphi^-}{\partial n},\ r=R_0. \tag{8}$$

The densities τ^* and τ_0^* have physical explanation. They are flow functions of surface micro currents, which are induced on the spheres $r=R$ and $r=R_0$.

Using (8) it follows from (4) that

$$\varphi\left(M\right) = \tilde{\varphi}\left(M\right)+\tilde{\varphi}^0\left(M\right) = \frac{1}{4\pi}\oiint_{r=R}\tilde{\tau}\left(N\right)\frac{\partial}{\partial n_N}\frac{1}{r_{NM}}dS_N +$$
$$+\frac{1}{4\pi}\oiint_{r=R_0}\tilde{\tau}_0\left(Q\right)\frac{\partial}{\partial n_Q}\frac{1}{r_{QM}}dS_Q,\ R<r_M<R_0, \tag{9}$$

where $\tilde{\tau} = \tau+\tau^*$, $\tilde{\tau}_0 = \tau_0+\tau_0^*$.

The following equations are needed for the later calculations:

$$\begin{aligned}
&\frac{\partial}{\partial n}\frac{1}{4\pi}\oiint_{r=R}\xi\left(N\right)\frac{\partial}{\partial n_N}\frac{1}{r_{NM}}dS_N = \Delta_S \mathrm{K}\xi\left(M\right),\ r_M \to R;\\
&\frac{\partial}{\partial n}\frac{1}{4\pi}\oiint_{r=R_0}\xi\left(Q\right)\frac{\partial}{\partial n_Q}\frac{1}{r_{QM}}dS_Q = \frac{R}{R_0}\Delta_S \mathrm{K}_{01}\xi\left(M\right),\\
&\qquad r_M = R;\\
&\frac{\partial}{\partial n}\frac{1}{4\pi}\oiint_{r=R}\xi\left(N\right)\frac{\partial}{\partial n_N}\frac{1}{r_{NM}}dS_N = \frac{R_0}{R}\Delta_S \mathrm{K}_{10}\xi\left(M\right),\\
&\qquad r_M = R_0;\\
&\frac{\partial}{\partial n}\frac{1}{4\pi}\oiint_{r=R_0}\xi\left(Q\right)\frac{\partial}{\partial n_Q}\frac{1}{r_{QM}}dS_Q = \Delta_S \mathrm{K}_0\xi\left(M\right),\\
&\qquad r_M \to R_0,
\end{aligned} \tag{10}$$

where

$$\mathrm{K}\xi(M)=\frac{1}{4\pi}\oiint_{r=R}\frac{\xi(N)}{r_{NM}}dS_N,\ r_M=R;$$

$$\mathrm{K}_0\xi(M)=\frac{1}{4\pi}\oiint_{r=R_0}\frac{\xi(N)}{r_{NM}}dS_N,\ r_M=R_0;$$

$$\mathrm{K}_{01}\xi(M)=\frac{1}{4\pi}\oiint_{r=R_0}\frac{\xi(N)}{r_{NM}}dS_N,\ r_M=R;$$

$$\mathrm{K}_{10}\xi(M)=\frac{1}{4\pi}\oiint_{r=R}\frac{\xi(N)}{r_{NM}}dS_N,\ r_M=R_0.$$

Integral on the right side is defined as limit for points M along exterior normal to the sphere $r=R$ or $r=R_0$ accordingly. $\Delta\frac{1}{r_{QM}}=0$ for $r_M\neq R,R_0$ is taken into account. These identities are held for any continuously differentiable function ξ on spheres with common center (see Appendix).

On substitutions (10) and (9) in equations (5) and (6)

$$\tilde{\tau}=-\gamma^*\mu_0\left(\omega_\alpha\frac{\partial}{\partial\alpha}+\frac{\partial}{\partial t}\right)\mathrm{K}\tilde{\tau}-$$

$$-\gamma^*\mu_0\frac{R}{R_0}\left(\omega_\alpha\frac{\partial}{\partial\alpha}+\frac{\partial}{\partial t}\right)\mathrm{K}_{01}\tilde{\tau}_0+\tau^*,\ r=R;$$

$$\tau^*=-\frac{R}{n}\Delta_S\mathrm{K}\tilde{\tau}-\frac{R^2}{nR_0}\Delta_S\mathrm{K}_{01}\tilde{\tau}_0,\ r=R;$$

$$\tau_0^*=\frac{R_0^2}{R(n+1)}\Delta_S\mathrm{K}_{10}\tilde{\tau}+\frac{R_0}{n+1}\Delta_S\mathrm{K}_0\tilde{\tau}_0,\ r=R_0;$$

Note that first equation follows from (5) after removing Laplace operator in both sides. However using (2) the constant of integration is equal to zero.

It is important, that the spherical functions $Y_n^m(\theta,\alpha)=P_n^m(\cos\theta)e^{\pm jm\alpha}$ are eigenfunctions of both Laplace operator Δ_S on the spheres $r=R,\ r=R_0$ and operators $\mathrm{K},\mathrm{K}_0,\mathrm{K}_{01},\mathrm{K}_{10}$. Its proof is trivial for Laplace operator. For integral operators this is proved using by spherical harmonics expansion [5] for their kernels. The obtained equations are

$$\Delta_S Y_n^m=-\frac{n(n+1)}{R^2}Y_n^m,\ r=R;$$

$$\Delta_S Y_n^m=-\frac{n(n+1)}{R_0^2}Y_n^m,\ r=R_0;$$

$$\mathrm{K}Y_n^m=\frac{R}{2n+1}Y_n^m,\ \mathrm{K}_0Y_n^m=\frac{R_0}{2n+1}Y_n^m,$$

$$\mathrm{K}_{01}Y_n^m=\left(\frac{R}{R_0}\right)^{n-1}\mathrm{K}Y_n^m,\ \mathrm{K}_{10}Y_n^m=\left(\frac{R}{R_0}\right)^{n+1}\mathrm{K}Y_n^m;$$

$m\in\overline{1,n}$.

Using these formulas one can get

$$\tilde{\tau}=-\gamma^*\mu_0\frac{R}{2n+1}\left(\omega_\alpha\frac{\partial}{\partial\alpha}+\frac{\partial}{\partial t}\right)\left[\tilde{\tau}+\left(\frac{R}{R_0}\right)^n\tilde{\tau}_0\right]+\tau^*,$$

$r=R;$

$$\tau^*=\frac{n+1}{2n+1}\left[\tilde{\tau}+\left(\frac{R}{R_0}\right)^n\tilde{\tau}_0\right],\ r=R;$$

$$\tau_0^*=-\frac{n}{2n+1}\left[\left(\frac{R}{R_0}\right)^{n+1}\tilde{\tau}+\tilde{\tau}_0\right],\ r=R_0.$$

Then

$$\tilde{\tau}_0=\frac{2n+1}{3n+1}\cdot\left(\tau_0-\frac{n}{2n+1}\left(\frac{R}{R_0}\right)^{n+1}\tilde{\tau}\right),\ r=R_0; \qquad (11)$$

$$\tilde{\tau}=\frac{n+1}{2n+1}\tilde{\tau}-\gamma^*\mu_0\frac{R}{2n+1}\left(\omega_\alpha\frac{\partial}{\partial\alpha}+\frac{\partial}{\partial t}\right)\times$$

$$\times\left[\tilde{\tau}+\left(\frac{R}{R_0}\right)^n\tilde{\tau}_0\right]+\frac{n+1}{2n+1}\left(\frac{R}{R_0}\right)^n\tilde{\tau}_0,\ r=R. \qquad (12)$$

According to introduced notation

$$\tilde{\tau}_0^\pm(\theta,\alpha,t)=\tilde{A}_{0n}^\pm P_n^m(\cos(\theta))e^{\pm j(\omega t-m\alpha)},$$

$$\tilde{\tau}^\pm(\theta,\alpha,t)=\tilde{A}_n^\pm P_n^m(\cos(\theta))e^{\pm j(\omega t-m\alpha)}.$$

In terms of coefficients $\tilde{A}_{0n}^\pm$, $\tilde{A}_n^\pm$ equations (11) and (12) are presented as follows

$$\tilde{A}_{0n}^\pm(t)=\frac{2n+1}{3n+1}\left[A_{0n}-\frac{n}{2n+1}\left(\frac{R}{R_0}\right)^{n+1}\tilde{A}_n^\pm(t)\right],$$

$$\tilde{A}_n^\pm(t)\left\{\frac{n}{2n+1}+\frac{n+1}{2n+1}\frac{n}{3n+1}\left(\frac{R}{R_0}\right)^{2n+1}\pm\right.$$

$$\left.\pm j\mu_0\gamma^*\frac{R}{2n+1}(\omega-m\omega_\alpha(t))\left(1-\frac{n}{3n+1}\left(\frac{R}{R_0}\right)^{2n+1}\right)\right\}+$$

$$+\mu_0\gamma^*\frac{R}{2n+1}\left(1-\frac{n}{3n+1}\left(\frac{R}{R_0}\right)^{2n+1}\right)\frac{d\tilde{A}_n^\pm(t)}{dt}=A_{0n}\times$$

$$\times\left\{\frac{n+1}{3n+1}\left(\frac{R}{R_0}\right)^n\mp j\mu_0\gamma^*\frac{R}{3n+1}(\omega-m\omega_\alpha(t))\left(\frac{R}{R_0}\right)^n\right\}.$$

Last equation is a first-order differential equation with variable coefficients for motor and brake operation modes. The solution of this equation $\tilde{A}_n^\pm$ are complex conjugate quantities.

$$\frac{d}{dt}\tilde{A}_n^\pm(t)+a_n^\pm(t)\tilde{A}_n^\pm(t)=b_n^\pm(t), \qquad (13)$$

$$a_n^\pm(t)=\frac{1}{c}\left\{\frac{n}{2n+1}+\frac{n+1}{2n+1}\frac{n}{3n+1}\left(\frac{R}{R_0}\right)^{2n+1}\pm\right.$$

$$\left.\pm j\mu_0\gamma^*\frac{R}{2n+1}(\omega-m\omega_\alpha(t))\left(1-\frac{n}{3n+1}\left(\frac{R}{R_0}\right)^{2n+1}\right)\right\},$$

$$b_n^\pm(t)=\frac{A_{0n}}{c}\left\{\frac{n+1}{3n+1}\left(\frac{R}{R_0}\right)^n\mp\right.$$

$$\pm j\mu_0\gamma^* \frac{R}{3n+1}\left(\omega - m\omega_\alpha(t)\right)\left(\frac{R}{R_0}\right)^n \Bigg\},$$

$$c = \mu_0\gamma^* \frac{R}{2n+1}\left(1 - \frac{n}{3n+1}\left(\frac{R}{R_0}\right)^{2n+1}\right).$$

The Solutions of equation (13) has the form

$$\tilde{A}_n^\pm(t) = \left\{\tilde{A}_n^\pm(0) + \int_0^t b_n^\pm(\xi)e^{-g(\xi)}d\xi\right\}e^{g(t)},$$

$$g(t) = -\int_0^t a_n^\pm(\xi)\,d\xi .$$

In steady state mode ($\omega_\alpha = const,\ \tilde{A}_n^\pm = const$) equation (13) is a simple algebraic equation with solutions $\tilde{A}_n^\pm = \dfrac{b_n^\pm}{a_n^\pm}$.

4. CALCULATING OF INTEGRAL FUNCTIONAL PARAMETERS

Consider sum of two spherical harmonics for primary and secondary sources both. These harmonics generate a travelling magnetic field. They are trigonometric harmonics at $\theta = const$. The flow functions for macro currents of the stator and the rotor are expressed as

$$\tau_{0n} = A_{0n}P_n^m\left(\cos(\theta)\right)\frac{1}{2}\left(e^{j(\omega t - m\alpha)} + e^{-j(\omega t - m\alpha)}\right)$$

$$\tau_n = P_n^m\left(\cos(\theta)\right)\frac{1}{2}\left(A_n^+ e^{j(\omega t - m\alpha)} + A_n^- e^{-j(\omega t - m\alpha)}\right)$$

Taking into account micro currents flow functions for summary currents are

$$\tilde{\tau}_{0n} = P_n^m\left(\cos(\theta)\right)\frac{1}{2}\left(\tilde{A}_{0n}^+ e^{j(\omega t - m\alpha)} + \tilde{A}_{0n}^- e^{-j(\omega t - m\alpha)}\right)$$

$$\tilde{\tau}_n = P_n^m\left(\cos(\theta)\right)\frac{1}{2}\left(\tilde{A}_n^+ e^{j(\omega t - m\alpha)} + \tilde{A}_n^- e^{-j(\omega t - m\alpha)}\right)$$

Coefficients are associated by

$$A_n^\pm = \frac{n}{2n+1}\left(1 + \frac{n+1}{3n+1}\left(\frac{R}{R_0}\right)^{2n+1}\right)\tilde{A}_n^\pm - \frac{n+1}{3n+1}\left(\frac{R}{R_0}\right)^n A_{0n}$$

Using virtual displacement method the instantaneous value of the torque can be calculated. In fact we differentiate with respect to α the mutual energy that is represented in the form

$$W = \mu_0 \iiint_{V_\infty} \nabla\tilde{\varphi}\nabla\tilde{\varphi}^0 dV =$$

$$= \mu_0 \iint_{r=R}\left(\tilde{\varphi}^+ - \tilde{\varphi}^-\right)\frac{\partial\tilde{\varphi}^0}{\partial n}dS = \mu_0 \iint_{r=R}\tilde{\tau}\frac{\partial\tilde{\varphi}^0}{\partial n}dS,$$

where first Green formula is used.

The obtained expression is

$$T_n(t) = \mu_0 \oint\!\!\!\oint_{r=R}\frac{\partial\tilde{\tau}_n}{\partial\alpha}(t)\frac{\partial\tilde{\varphi}^0}{\partial n}(t)\,dS =$$

$$= \frac{2\pi R\mu_0 m}{2n+1}\,\frac{n(n+1)}{3n+1}\,\frac{(n+m)!}{(n-m)!}\left(\frac{R}{R_0}\right)^n A_{0n}\ \mathrm{Im}\left(\tilde{A}_n^+(t)\right).$$

Curves of the torque dependence on the slip for various radius of the rotor are shown in Figure 3. It is assumed that $\gamma = 5.8\cdot10^7\cdot1/\Omega\cdot$m, $h = 0.01$ m, $R_0 = 1$ m, $n = m = 3$. The skin effect is neglected.

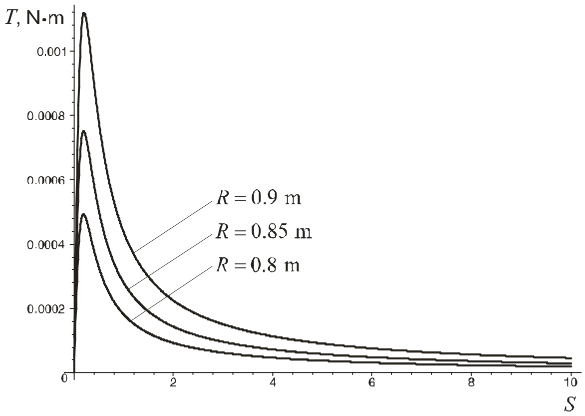

Figure 3

The instantaneous power losses in the conductive layer is defined by the following formula

$$P_n(t) = \frac{1}{\gamma}\iiint_\Omega\left|\boldsymbol{\delta}(t)\right|^2 d\Omega = \frac{1}{\gamma}\oint\!\!\!\oint_{r=R} dS\int_{-h}^{0}\left|\boldsymbol{\delta}(t)\right|^2 dn =$$

$$= \frac{k}{\gamma h}\oint\!\!\!\oint_{r=R}\left|\boldsymbol{\sigma}(t)\right|^2 dS = \frac{k}{\gamma h}\oint\!\!\!\oint_{r=R}\left|\nabla_S\tau_n(t)\right|^2 dS =$$

$$= \frac{2\pi k}{\gamma h}\,\frac{n(n+1)}{2n+1}\,\frac{(n+m)!}{(n-m)!}\times$$

$$\times\left(\left|\mathrm{Re}\left(\tilde{A}_n^+(t)\right)\frac{n}{2n+1}\left(1 + \frac{n+1}{3n+1}\left(\frac{R}{R_0}\right)^{2n+1}\right) - \right.\right.$$

$$\left. - \frac{n+1}{3n+1}\left(\frac{R}{R_0}\right)^n A_{0n}\right|^2 + \left|\mathrm{Im}\left(\tilde{A}_n^+(t)\right)\times\right.$$

$$\left.\left.\times\frac{n}{2n+1}\left(1 + \frac{n+1}{3n+1}\left(\frac{R}{R_0}\right)^{2n+1}\right)\right|^2\right),$$

where

$$k = \frac{h}{\chi}\frac{\mathrm{sh}\left(\dfrac{h}{\chi}\right) \pm \sin\left(\dfrac{h}{\chi}\right)}{\mathrm{ch}\left(\dfrac{h}{\chi}\right) - \cos\left(\dfrac{h}{\chi}\right)},\quad \chi = \sqrt{\frac{2}{\left|\omega_\alpha m - \omega\right|\mu_0\gamma}}\ .$$

The coefficient k takes into account variation of the current density distribution along the thickness of the layer.

The power losses versus slip characteristics for steady state mode are shown in Figure 4.

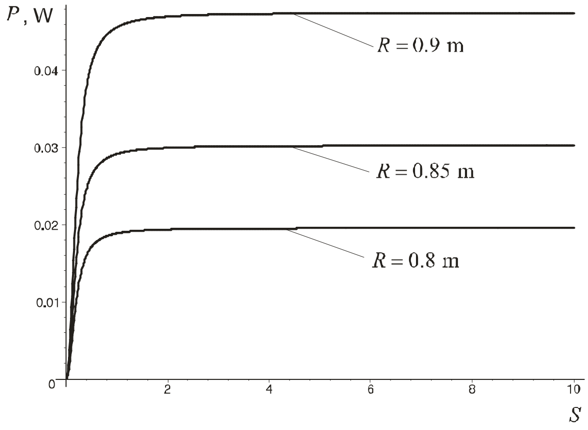

Figure 4

The operation mode of uniform acceleration is considered. The primary current has constant amplitude. The secondary current is equal to the current in steady state mode at the initial time.

The torque as function of time for both steady state and uniform acceleration modes are shown in Figure 5 and 6. The above described assumptions are used. $R = 0.9$ m, $\omega_\alpha(t) = 10t$ and $\omega_\alpha(t) = 50 - 10t$ respectively. $t \in [0..5]$ for both cases.

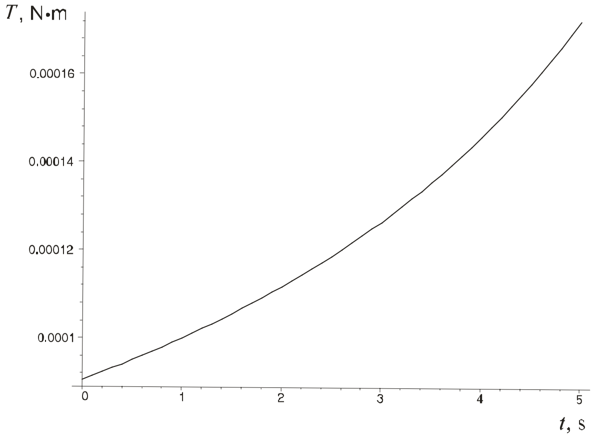

Figure 5

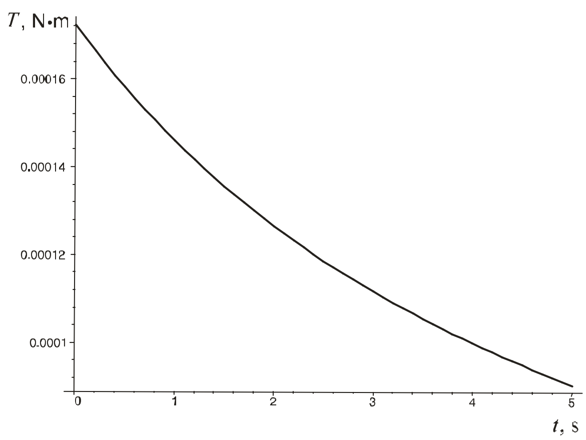

Figure 6

Differences between curves are insignificant in each considered operation modes for both cases. Thus the contribution of acceleration is negligible.

5. CONCLUSION

On the non-traditional way by means of Integral Equation Method a mathematical model for spherical multi coordinate induction machine has been created. The applied method allows reducing the original spatial boundary problem to a surface integral equation.

The simple algebraic formulas for calculation of torque and power losses without special functions are obtained. The obtained formulas allow calculating of these parameters for all relevant operating modes: motor, generator, and braking operating modes.

In present paper the dependence of torque and power losses on slip only for steady state mode is considered. The time-dependence of torque is calculated assuming a uniform acceleration. The case considered is, when the rotation axis of the rotor and the axis of traveling magnetic field are coincident. Later the arbitrary orientation of these axes will be considered.

6. APPENDIX

Consider the first identity from (10)

$$\frac{\partial}{\partial n_M}\frac{1}{4\pi}\oint\!\!\!\oint_{r=R_0}\xi(Q)\frac{\partial}{\partial n_N}\frac{1}{r_{NM}}dS_N =$$

$$-\mathbf{n}_M \,\mathrm{grad}_M \frac{1}{4\pi}\oint\!\!\!\oint_{r=R_0}\xi(N)\mathbf{n}_N \,\mathrm{grad}_M \frac{1}{r_{NM}}dS_N =$$

$$= -\mathbf{n}_M \,\mathrm{grad}_M \,\mathrm{div}_M \frac{1}{4\pi}\oint\!\!\!\oint_{r=R_0}\xi(N)\frac{\mathbf{n}_N}{r_{NM}}dS_Q = \qquad (14)$$

$$= -\,\mathbf{n}_M \,\mathrm{rot}\,\mathrm{rot}\frac{1}{4\pi}\oint\!\!\!\oint_{r=R_0}\xi(N)\frac{\mathbf{n}_N}{r_{NM}}dS_N =$$

$$= \mathrm{rot}_n \frac{1}{4\pi}\oint\!\!\!\oint_{r=R_0}\xi(N)\left[\mathrm{grad}_N \frac{1}{r_{NM}}, \mathbf{n}_N\right]dS_N,$$

here $\mathrm{grad}_N \frac{1}{r_{NM}} = -\mathrm{grad}_M \frac{1}{r_{NM}}$ and identities for vector analysis [6] are used.

Note that $\mathrm{grad}_N \frac{1}{r_{NM}} = \frac{\mathbf{r}_{NM}}{r_{NM}^3}$. From Figure 7

$$\left[\mathrm{grad}\frac{1}{r_{NM}}, \mathbf{n}_N\right] = \left[\frac{\mathbf{r}_{NM}}{r_{NM}^3}, \mathbf{n}_N\right] = \left[\frac{\mathbf{r}_{NM}}{r_{NM}^3}, \mathbf{n}_M\right] +$$

$$+\left[\frac{\mathbf{r}_{NM}}{r_{NM}^3}, \mathbf{n}_N - \mathbf{n}_M\right] = \left[\frac{\mathbf{r}_{NM}}{r_{NM}^3}, \mathbf{n}_M\right] = \left[\mathrm{grad}\frac{1}{r_{NM}}, \mathbf{n}_M\right].$$

Then

$$\mathbf{n}_M \operatorname{rot}\frac{1}{4\pi}\oiint\limits_{r=R}\xi(N)\left[\operatorname{grad}_N\frac{1}{r_{NM}},\mathbf{n}_M\right]dS_N=$$

$$=-\frac{1}{4\pi}\oiint\limits_{r=R}\xi(N)\operatorname{div}_{S(M)}\left[\mathbf{n}_M,\left[\operatorname{grad}_N\frac{1}{r_{NM}},\mathbf{n}_M\right]\right]dS_N=$$

$$=-\frac{1}{4\pi}\oiint\limits_{r=R}\xi(N)\Bigg(\operatorname{div}_{S(M)}\operatorname{grad}_N\frac{1}{r_{NM}}-$$

$$-\operatorname{div}_{S(M)}\left(\mathbf{n}_M\left(\operatorname{grad}_N\frac{1}{r_{NM}},\mathbf{n}_M\right)\right)\Bigg)dS_N=$$

$$=\frac{1}{4\pi}\Delta_S\oiint\limits_{r=R}\frac{\xi(N)}{r_{NM}}dS_N,$$

where $\mathbf{n}\operatorname{rot}\mathbf{a}=-\operatorname{div}_S[\mathbf{n},\mathbf{a}]$ is used.

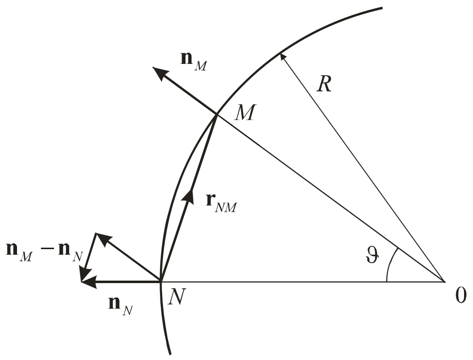

Figure 7

Formula (14) is carried out for the second identity from (10) if R is replaced by R0. At that $\operatorname{grad}_Q\frac{1}{r_{QM}}=\frac{\mathbf{r}_{QM}}{r_{QM}^3}$. From fig. 8 we see $\mathbf{r}_{QM}=\mathbf{r}_{NM}-a\mathbf{n}_Q$, $a=R_0-R$.

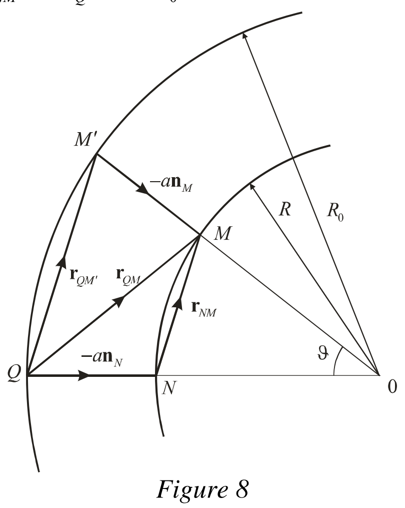

Figure 8

Then

$$\left[\operatorname{grad}\frac{1}{r_{QM}},\mathbf{n}_Q\right]=\left[\frac{\mathbf{r}_{QM}}{r_{QM}^3},\mathbf{n}_Q\right]=\left[\frac{\mathbf{r}_{NM}-a\mathbf{n}_Q}{r_{QM}^3},\mathbf{n}_Q\right]=$$

$$=\left[\frac{\mathbf{r}_{NM}}{r_{QM}^3},\mathbf{n}_Q\right]=\left[\frac{\mathbf{r}_{NM}}{r_{QM}^3},\mathbf{n}_M\right]+$$

$$+\left[\frac{\mathbf{r}_{NM}}{r_{QM}^3},\mathbf{n}_Q-\mathbf{n}_M\right]=\left[\frac{\mathbf{r}_{NM}}{r_{QM}^3},\mathbf{n}_M\right]$$

Thus we obtain

$$-\operatorname{rot}_n\frac{1}{4\pi}\oiint\limits_{r=R_0}\xi(Q)\left[\operatorname{grad}_Q\frac{1}{r_{QM}},\mathbf{n}_Q\right]dS_Q=$$

$$=\operatorname{rot}_n\frac{1}{4\pi}\oiint\limits_{r=R_0}\xi(Q)\left[\frac{\mathbf{r}_{NM}}{r_{QM}^3},\mathbf{n}_M\right]dS_Q=$$

$$=-\frac{1}{4\pi}\oiint\limits_{r=R_0}\xi(Q)\operatorname{div}_{S(M)}\left[\mathbf{n}_M,\left[\frac{\mathbf{r}_{NM}}{r_{QM}^3},\mathbf{n}_M\right]\right]dS_Q=$$

$$=-\frac{1}{4\pi}\oiint\limits_{r=R_0}\xi(Q)\Bigg(\operatorname{div}_{S(M)}\frac{\mathbf{r}_{NM}}{r_{QM}^3}-$$

$$-\operatorname{div}_{S(M)}\left(\mathbf{n}_M\frac{(\mathbf{r}_{NM},\mathbf{n}_M)}{r_{QM}^3}\right)\Bigg)dS_Q=$$

$$=-\frac{1}{4\pi}\frac{R}{R_0}\oiint\limits_{r=R_0}\xi(Q)\operatorname{div}_{S(M)}\frac{\mathbf{r}_{QM}}{r_{QM}^3}dS_Q=$$

$$=-\frac{1}{4\pi}\frac{R}{R_0}\oiint\limits_{r=R_0}\xi(Q)\operatorname{div}_{S(M)}\operatorname{grad}_Q\frac{1}{r_{QM}}dS_Q=$$

$$=\frac{1}{4\pi}\frac{R}{R_0}\Delta_S\oiint\limits_{r=R_0}\xi(Q)\frac{1}{r_{QM}}dS_Q,$$

where representation $\mathbf{r}_{NM}=\frac{R}{R_0}\mathbf{r}_{NM'}=\frac{R}{R_0}\left(\mathbf{r}_{QM}+a\mathbf{n}_M\right)$ is used.

Prove the third identity using Figure 9. $\operatorname{grad}_N\frac{1}{r_{NM}}=\frac{\mathbf{r}_{NM}}{r_{NM}^3}$, $\mathbf{r}_{NM}=\mathbf{r}_{QM}+a\mathbf{n}_N$, $a=R_0-R$.

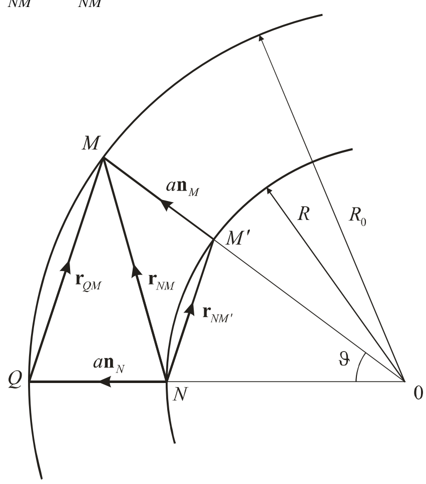

Figure 9

The previous proof is held for it with following changes: $r=R_0$, Q is replaced by N and $\mathbf{r}_{QM}=\frac{R_0}{R}\mathbf{r}_{NM'}=\frac{R_0}{R}\left(\mathbf{r}_{NM}-a\mathbf{n}_M\right)$.

The proof of the fourth identity looks like first one but R is replaced by R_0.

7. NOMENCLATURE

R_0	Interior radius of the stator.
R	Mean radius of the conductive layer.
γ	Conductivity of material.
γ^*	Equivalent complex conductivity.
h	The thickness of the conductive layer.
μ_0	Magnetic constant, $=4\pi\cdot 10^{-7}$ H/m .
ω	Radian frequency.
ω_α	Angular velocity.
$\mathbf{v}$	Linear velocity.
$\mathbf{B}$	Induction of magnetic field.
$\mathbf{H}$	Intensity of magnetic field.
$\mathbf{E}$	Intensity of electric field.
φ	Scalar magnetic potential.
$\boldsymbol{\delta}$	Current density of the rotor.
$\boldsymbol{\sigma}_0$	Linear current density of the stator.
$\boldsymbol{\sigma}$	Linear current density of the rotor.
τ_0	Flow function of stator's current density.
τ	Flow function of rotor's current density.
r, θ, α	Standard spherical coordinates.
r_{NM}	Distance between points N and M .
m	Number of magnetic poles, $=1, 2 \dots$.
n	Degree of associated Legendre polynomial.
P_n^m	Associated Legendre polynomial.
$\mathbf{n}$	Unit exterior normal vector to the spheres.
$\frac{\partial\,\cdot}{\partial n}$	Normal derivative.
$\int_{-h}^{0} \cdot\, dn$	Integral over thickness of layer.
∇_S	Two-dimensional surface gradient.
Δ_S	Surface Laplace operator.
ω_Φ	Velocity of travelling magnetic field, $=\frac{\omega}{m}$.
S	slip, $=\frac{\omega_\Phi - \omega_\alpha}{\omega_\Phi}$.

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