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FINITE ELEMENT ANALYSIS OF DISPROPORTIONATE REGIONS

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ABSTRACT

The paper concerns the detailed electromagnetic field calculation in geometrically disproportionate small regions. For example they could be presented by cracks, small air gaps or inhomogeneous inclusions. Unfortunately each of them is possible to be a part of investigated device. In the electrotechnic this is very often encountered problem. The task is interesting in computational aspect mostly as the field values, forces or pressures at this problem region are of important meaning for normal functioning of the device. The similar solution by finite element method would lead either to disproportional dimensions of elements or to huge number of elements. Then the difficulties which have to be overcome are vastly increasing errors or limitations in the computers memory. This impedes the solution of the forward, as well as the coupled and inverse problems in the device. The correct solution is proposed by iteration procedure.

***Index Terms* -** Incommensurable regions, 3D FEM analysis

1. INTRODUCTION

In electromagnetic field analysis one of the fundamental computational problems is examination of incommensurable regions. These are cases when detailed field calculation is needed to be applied in a geometrically small region (cracks in the material, small air gaps, inhomogeneous inclusions and etc.) which is a part of large one. In investigation of electrotechnical devices this problem encounters very often many obstacles. It is interesting in computational aspect mostly as the field values, forces or pressures at the problem regions are of important meaning for normal functioning of the device. Unfortunately the similar task solution by finite element method would lead either to disproportional dimensions of elements or to enormous number of elements. The difficulties under these circumstances are well known. The researcher has to overcome vastly increasing errors or limitations in the computers memory.

The problem is proposed to be solved by iteration procedure. The presented approach is illustrated by an example on the benchmark non-linear DC electromagnet. The computed forces results in the small air gap of the device are compared with the experimental ones. The paper deals with formulation of the mathematical model and the methodology of its solution.

2. THE PROPOSED APPROACH

Modeling and correct solution of the task requires implementation of iteration procedure. It is formulated on the basis of the theorem of the unique electromagnetic field solution. The forward step involves electromagnetic field analysis without taking into account the problem region in detail. After that it is surrounded by own buffer zone near by including problem region. Because of the fact that it is relatively too small its influence on the field distribution outside the buffer zone is considered to be negligible. This is the reason why the next (second) step is carried out by applying the magnetic vector potential field values, obtained on the buffer zone boundaries (outside boundaries) as a results from the previous analysis. These values are used as boundary conditions in the next second step procedure. Now the field is solved inner in the previous buffer zone including the problem region. The fine meshing in the problem region with great number of elements of “good quality” is possible. Because of this observed field values, forces and pressures are more exactly calculated. In the end of this step new (second) buffer zone is formatted closely including the problem region but outside it. Determining the field values on the second buffer zone boundaries (inner boundaries) the back (third) step of the procedure consists in field calculations only outside the inner boundaries. In this step influence of the small region field over the outside field distribution is taking into account. The iterative procedure terminates when desired accuracy with respect to the boundary values on the outside and the inner boundaries is reached.

3. INVESTIGATED OBJECT

As there is no analytical solution for the 3D static force problem the verification is carried out for a suitable standard model comparing the computed with experimental results. The main features of the model are that the flux is distributed non-uniformly and three-dimensionally, the magnetic saturation is taken into account because of the nonlinearity of the yoke and the center pole steel, the flux density is changed suddenly near the integration path for the Maxwell stress tensor method, the energy is not linearly changed with the displacement of the movable body. The verification is carried out on a suitable standard model-electromagnet, comparing the computed with experimental results. The benchmark non-linear DC electromagnet (TEAM Workshop Problem □ 20) is regarded. Electromagnetic field is analyzed by 3D FEM computation applying Comsol Multiphysics software package, version 3.5. It should be noted that the calculation of the force between the center pole and the yoke of the electromagnet is quite difficult problem [1] due to the presence of two small air gaps where the forces are in opposite directions. The 3D model is shown in Fig.1. The center pole and yoke are made of nonlinear steel. The coil has 381 turns. The exciting direct current is changed in large diapason and taking high values causes saturation of the device. The ampere-turns (DC) are chosen to be 1000, 3000, 4500 and 5000 in order to be considered influence of the saturation effect. The z-component Fz of the center pole is calculated under the pole, at point with coordinates in millimeters: P(0.0, 0.0, 25.75).

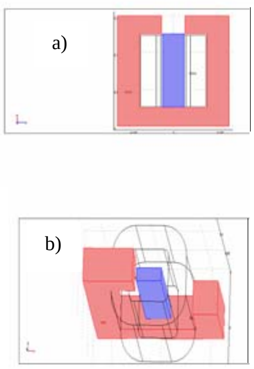

Figure 1 DC electromagnet: a) vertical view; b) 3D view suppressing upper right side of the yoke

In this model the computational exactness is strongly influenced by the accuracy of the field values, especially when the material is highly saturated. Because of this it is very useful the proposed computational approach to be tested. Its verification is possible comparing the forces results at point P in all diapason of the exciting current changes, including the presents of nonlinearity in benchmark electromagnet. The compared forces values are the ones obtained in the process of computation as follows: in the forward (first) step, in the second step by the use of fine meshing in the air gap region with great number of elements of "good quality" and as last with measured experimental results. In the forward solution one fourth of the domain is meshed by 14490 tetrahedral elements. In the precise calculations of the air gap electromagnetic field and force values in the second step their number increase approximately two times.

4. MATHEMATICAL MODEL

The electromagnetic problem is solved as stationary and the field model is with respect to the magnetic vector potential A based on the equation

$$rot\left(\frac{1}{\mu} rot\vec{A}\right) = \vec{J}, \qquad (1)$$

where μ is magnetic permeability and J is the exciting current density.

Our interest of the local forces distribution by Maxwell stress tensor is satisfied computing only its components in given point. One recent attempt is the method of the nodal force calculation [3]. In accordance of this method the work ΔA due to the force action for the shifting Δu is presented by the expression

$$\Delta A = -\iiint \tau_{ik} \frac{\partial(\Delta u_i)}{\partial k} d\,\mathrm{v}, \qquad (i,k = x,y,z)$$

Here by τ_{ik} the relevant component of Maxwell stress tensor is noted and dv is elementary volume of the element. In 3-D investigations the Maxwell stress tensor T includes the following elements

$$T = \frac{1}{\mu_0}\begin{bmatrix} B_x^2 - \frac{1}{2}B^2 & B_x B_t & B_x B_z \\ B_y B_x & B_y^2 - \frac{1}{2}B^2 & B_y B_z \\ B_z B_x & B_z B_y & B_y^2 - \frac{1}{2}B^2 \end{bmatrix}. \qquad (2)$$

In equation (2) the quantities B_x, B_y and B_z are magnetic flux density components and B its module in the element. For shifting of the i-th element is obtained

$$\Delta u_i = \sum_n N_i \Delta u_{ni} \qquad (3)$$

where n is the number of nodes and N_i is the shape function of this tetrahedral element

$$N_i = a_i + b_i x + c_i y + d_i z \qquad (i = 1,...,4).$$

Replacing (3) in (1) the following term for the work is found

$$\Delta A = \sum_n \left(-\iiint \tau_{ik} \frac{\partial(N_i)}{\partial k} d\,\mathrm{v} \right) \Delta u_i$$

But this work is possible to be determined also as action of the components of all nodal forces

$$\Delta A = \sum_{n} f_{ni} \Delta u_{ni}\,, \qquad (i = x, y, z)$$

considering f_{ni} as i-th components of the force at node number n. Comparing the last two equations the next expression for nodal force is obtained

$$f_{ni} = -\iiint \tau_{ik} \frac{\partial N_n}{\partial k} \mathrm{dv}$$

On the basis of this term the z component of the force at node number k is calculated by the following equation

$$f_{kz} = -(\tau_{zx} b_k + \tau_{zy} c_k + \tau_{zz} d_k).V_e\,.$$

Here the quantities b_k, c_k and d_k are coefficients of the shape function and V_e is the volume of the observing tetrahedral element. Consecuantly the local force exerted at the node is calculated integrating the function for the partial local forces f_{kz} of all tetrahedral elements to which this node belongs.

5. COMPARISON OF THE NUMERICAL AND EXPERIMENTAL RESULTS

Software is verified for 3-D static force calculation on a benchmark non-linear DC electromagnet. The mentioned above methods are the most commonly used for force calculations. They are based on either Maxwell stress tensor (MST) or the virtual work principle (VWP). It is known that MST is derived by starting from the Lorenz force expression, whereas the virtual work principle is based on the mechanical concepts of forces being related to the change in stored energy [3]. Using MST usually is calculated the global force by integrating force densities [4]. This is realized over the surface in 3-D application. The local forces distribution by Maxwell stress tensor is obtained by the computing only its components in given point. The major benefit of MST is that it requires a single solution of the problem only. The virtual work principle computes the force on a body by a virtual displacement related to the change in the co-energy of the system. But the gradient of the co-energy function is usually not easily available and at least two field solutions are required. Because of these disadvantages recently the method of the nodal force calculation method (NFM) is preferred.

The z-component Fz at point P(0.0, 0.0, 25.75) under the center pole is calculated and the results are demonstrated in Fig.2. The compared results are obtained applying Maxwell stress tensor (fz_{MST}), the nodal force method (fz_{NFM}) and the nodal force method under the proposed iteration procedure (fz_{IPR}).

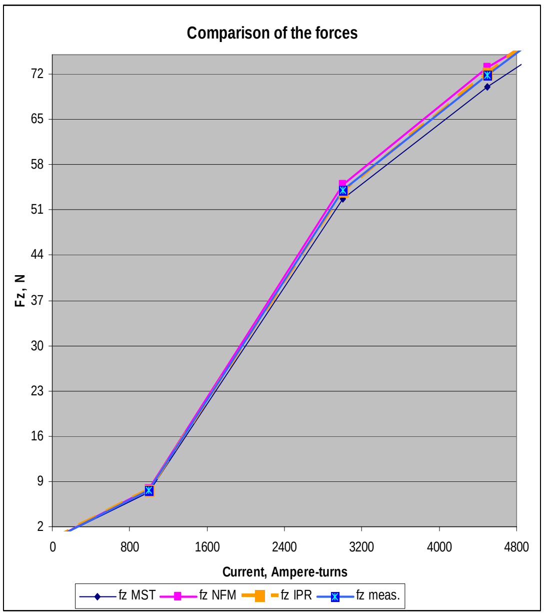

Figure 2 The z-component of the force at point P(0, 0, 25,75) under the central pole

The relative errors in all diapason of the ampere-turns change is shown in Fig.3. The first error ε1,% presents the differences between Fz values obtained by the nodal force calculation method and the experimental results. The second one ε2,% is taking into account the declinations between Fz values obtained by the nodal force calculation method combined with the proposed in the paper iteration procedure.

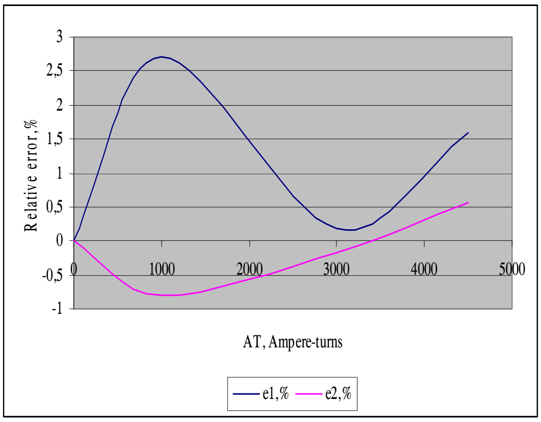

Figure 3 The relative error in F_z computations

The results show obvious efficacious of the proposed approach.

6. CONCLUSIONS

The paper concerns the cases when detailed electromagnetic field calculation is needed to be applied in geometrically small regions. The problem is interesting in computational aspect mostly as the field values, forces or pressures in these problem regions are of important meaning for normal working mode of the device. The numerical solution of similar task by finite element method leads either to disproportional dimensions of the elements or to enormous number of elements. Because of this the researcher has to overcome either vastly increasing errors or limitations in the computers memory.

In the paper the problem is formulated and is proposed to be solved correctly by iteration procedure. The suitable algorithm is described and applied successively to the real problem solution. The presented approach is illustrated by an example on the benchmark non-linear DC electromagnet. The computed forces results in the small air gap of the device are compared with the experimental ones. The comparison shows one good agreement and the computational reliability of the presented algorithm.

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7. REFERENCES

[1] N. Takahashi, T. Nakata and al., "Investigation of a model to verify software for 3-D static force calculation," IEEE Trans. Magn., vol.30, no.5, Sept. 1994, pp.3483-3487.

[2] K. Kim, H. Lee, Y. Chun, and J. lee, "Analysis of Electromagnetic Force Distribution on End Winding for Motor Reliance," IEEE Trans. Magn., vol.41, no.10, Oct. 2005, pp.4072-4074.

[3] C. Trowbridge and J. Sykulski, "Some key developments in computational electromagnetics and their attribution", IEEE Trans. Magn., vol.42, no.4, April. 2006, pp.503-508.

[4] I. Nishiguchit, A. Kameari and K. Haseyama," On the local force computation of deformable bodies in magnetic field", IEEE Trans. Magn., vol.35, no.3, May 1999, pp. 1650-1653.