51. IWK
Internationales Wissenschaftliches Kolloquium
International Scientific Colloquium

PROCEEDINGS

11-15 September 2006

FACULTY OF ELECTRICAL ENGINEERING AND INFORMATION SCIENCE

INFORMATION TECHNOLOGY AND ELECTRICAL ENGINEERING - DEVICES AND SYSTEMS, MATERIALS AND TECHNOLOGIES FOR THE FUTURE

TECHNISCHE UNIVERSITÄT
ILMENAU

Impressum

Herausgeber: Der Rektor der Technischen Universität Ilmenau
Univ.-Prof. Dr. rer. nat. habil. Peter Scharff

Redaktion: Referat Marketing und Studentische Angelegenheiten
Andrea Schneider

Fakultät für Elektrotechnik und Informationstechnik
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Redaktionsschluss: 07. Juli 2006

Technische Realisierung (CD-Rom-Ausgabe):
Institut für Medientechnik an der TU Ilmenau
Dipl.-Ing. Christian Weigel
Dipl.-Ing. Marco Albrecht
Dipl.-Ing. Helge Drumm

Technische Realisierung (Online-Ausgabe):
Universitätsbibliothek Ilmenau
ilmedia
Postfach 10 05 65
98684 Ilmenau

Verlag: isle
Verlag ISLE, Betriebsstätte des ISLE e.V.
Werner-von-Siemens-Str. 16
98693 Ilrnenau

ISBN (Druckausgabe): 3-938843-15-2
ISBN (CD-Rom-Ausgabe): 3-938843-16-0

Startseite / Index:
http://www.db-thueringen.de/servlets/DocumentServlet?id=12391

51st Internationales Wissenschaftliches Kolloquium
Technische Universität Ilmenau
September 11 – 15, 2006

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Inverse problems of electromagnetic Field Utilizing Sensitivity Analysis.

Introduction

Sensitivity analysis has proved to be a very robust method for solution of inverse problems in electromagnetics [1],[2]. It determines the dependence of global or local electromagnetic quantities on geometrical or physical parameters expressed in form of an objective function. The final aim of field calculation methods is generally the design of an electromagnetic device. Solving of inverse problem on the base of finite elements method (FEM) makes the optimal shape design possible [8],[9], as well as the identification of material cracks and flaws inside conducting materials with the help of eddy-current method [7]. This tasks can be defined similarly to recognition of space distribution of material parameters. The recognition processes in iterative manner based on the gradient information derived from sensitivity analysis. Sensitivity analysis belongs to the most important tools in optimization theory. For several objective functions the sensitivity may be directly calculated differentiating the objective function versus one of the material or geometric parameters. For the tasks based on local quantities the direct calculation is also possible, but it requires very large computational effort. A number of well described sensitivity evaluation methods exists in electric circuit theory. These methods could be adapted to electromagnetic field analysis programs, too. The first one bases on a version of Tellegen's theorem for electromagnetic field theory [7]. This method allows to calculate the sensitivity of chosen local quantities, as magnetic vector potential, impedance of a coil or induced voltage, versus all material parameters in a whole region at once. The second method bases on differentiation of stiffness matrix of the finite elements [8],[9]. The stiffness matrix contains complete information about geometric and material properties of the model. On the contrary to Tellegen's method, this method allows to calculate the sensitivity of all local quantities versus only one chosen parameter.

1. Inverse task utilizing sensitivity information

The task of conductivity recognition may be used for identification of crack shape. When using the eddy-current defectometer, the user should determine the search region, where the crack shape will be recognized. Then, the user should carry out the sufficient number of measurements of flux density around the crack. The number of measurements depends on discretization of the search region in finite elements. The discretization should be fine enough for modeling shape of the crack. To obtain the proper inverse job, the number of measurement points has to be greater or equal to the number of finite elements in search region. The dependence between conductivity □ inside of finite elements and the field distribution over conducting plate represented by the magnetic vector potential $\boldsymbol{R}$, is given by the following system of equations:

$$
\begin{bmatrix} \Delta R_1 \\ \Delta R_2 \\ \Delta R_3 \\ .... \\ \Delta R_j \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & .... & S_{1i} \\ S_{21} & S_{22} & S_{23} & .... & S_{2i} \\ S_{31} & S_{32} & S_{33} & .... & S_{3i} \\ .... & .... & .... & .... & .... \\ S_{j1} & S_{j2} & S_{j3} & .... & S_{ji} \end{bmatrix} \cdot \begin{bmatrix} \Delta \gamma_1 \\ \Delta \gamma_2 \\ \Delta \gamma_3 \\ .... \\ \Delta \gamma_i \end{bmatrix} \tag{1}
$$

where: j – number of measurement points, i – number of elements in the search region, $\boldsymbol{S}$ - sensitivity matrix. The field measurements can be carried out with constant feeding frequency ω. However, when identifying an inner crack, the multi-frequency method can be used. In both cases, the sensitivity values S_{ji} are evaluated in the frequency domain. If the eddy-currents are induced by the coil driven with non-harmonic current impulse, the time-domain evaluation is necessary.

The objective function in the conductivity recognition problems is a nonlinear function over the material conductivity. So the iterative procedure of mathematical programming using sensitivity information has to be adopted. For the examples shown below, the gradient method was chosen. After each iteration the results are compared with that of measurements and the new ΔR_j values for Eq.(1) are obtained. The described methods require access to the source code of a finite element package. However, the obtained algorithms are very time effective.

2. Evaluation of sensitivity in the frequency domain.

Neglecting second terms of perturbations of the variable parameters one can obtain the following sensitivity equation in the frequency domain [2]:

$$
\begin{aligned}
&\int_V (\boldsymbol{J}_0^+ \cdot \Delta \boldsymbol{E} - \boldsymbol{L}_0^+ \cdot \Delta \boldsymbol{H})\, \mathrm{dV} = \\
&\int_V (\boldsymbol{E} \cdot \boldsymbol{E}^+ \Delta\gamma + \mathrm{j}\omega\, \boldsymbol{E} \cdot \boldsymbol{E}^+ \Delta\varepsilon - \mathrm{j}\omega\, \boldsymbol{H} \cdot \boldsymbol{H}^+ \Delta\mu)\mathrm{dV} + \\
&\int_S (\boldsymbol{H}^+ \times \Delta \boldsymbol{E} + \boldsymbol{E}^+ \times \Delta \boldsymbol{H}) \cdot \boldsymbol{n}\, \mathrm{dS}\ .
\end{aligned} \tag{2}
$$

The proper boundary conditions of both systems cause vanishing of the surface integral in Eq.(2). The above equation determines how to construct the adjoint ($^+$) model. The excitation current $\boldsymbol{J}_0^+$ should be driven into this node, where the sensitivity value of $\boldsymbol{E}$ has to be obtained. Similarly, the magnetic current $\boldsymbol{L}_0^+$ makes possible the sensitivity calculation of $\boldsymbol{H}$. It means, that the original and adjoint systems differ only in excitations and boundary conditions. The geometrical properties and material parameters are the same. Further, the stiffness matrix of both systems is the same and requires only one factorization.
The other method consists in differentiation of the stiffness and mass matrices of the finite elements. The stiffness matrix $\boldsymbol{A}$ is computed in the standard finite element code:

$$
\boldsymbol{A}\boldsymbol{R} = \boldsymbol{b} \tag{3}
$$

Variation of electric conductivity γ of the material causes only changes of magnetic vector potential $\boldsymbol{R}$, that the excitation vector remains unchanged:

$$\frac{\partial \boldsymbol{A}}{\partial \gamma}\,\boldsymbol{R} + \boldsymbol{A}\frac{\partial \boldsymbol{R}}{\partial \gamma} = \boldsymbol{0}. \tag{4}$$

This formula allows to formulate the nodal sensitivity versus the conductivity γ:

$$\boldsymbol{S} = \frac{\partial}{\partial \gamma}\boldsymbol{R} = -\,\boldsymbol{A}^{-1}\frac{\partial \boldsymbol{A}}{\partial \gamma}\,\boldsymbol{R}. \tag{5}$$

In numerical implementation the sensitivity is calculated on the basis of the equation system (4). Unlike the Tellegen' method, in this method the sensitivity of all nodal potentials is obtained versus conductivity in one finite element. To calculate the sensitivity for other elements, the derivative of stiffness matrix should be determined anew. The terms of mass matrix are linear functions of electrical conductivity γ, so the matrix of derivatives contains only constants and zeroes.

Assuming the same number of finite elements and the nodes, for which the sensitivity is obtained, both methods seem to be equivalent in the necessary calculation time. As a result of both methods, the sensitivity matrix $\boldsymbol{S}$ is obtained. The matrix is necessary to solve Eq.(1).

3. Evaluation of sensitivity in time domain.

The dependence of magnetic field distribution on the variation of electric conductivity γ is represented by the sensitivity matrix $\boldsymbol{S}(t)$ for consecutive time steps j :

$$\begin{bmatrix} \Delta R(t_1) \\ \Delta R(t_2) \\ \Delta R(t_3) \\ .... \\ \Delta R(t_j) \end{bmatrix} = \begin{bmatrix} S_1(t_1) & S_2(t_1) & S_3(t_1) & .... & S_i(t_1) \\ S_1(t_2) & S_2(t_2) & S_3(t_2) & .... & S_i(t_2) \\ S_1(t_3) & S_2(t_3) & S_3(t_3) & .... & S_i(t_3) \\ .... & .... & .... & .... & .... \\ S_1(t_j) & S_2(t_j) & S_3(t_j) & .... & S_i(t_j) \end{bmatrix} \cdot \begin{bmatrix} \Delta\gamma_1 \\ \Delta\gamma_2 \\ \Delta\gamma_3 \\ .... \\ \Delta\gamma_i \end{bmatrix} \tag{6}$$

with : i - number of finite elements in the search region,
j - number of time steps of electromagnetic field evaluation.

The sensitivity matrix $\boldsymbol{S}(t)$ may be calculated by direct evaluation for small changes of the conductivity value. However, such approach is not effective.
Applying Tellegen's theorem similarly to the frequency domain, one can obtain the sensitivity equation in time domain [1]:

$$\int_0^T\!\!\int_V [\,\boldsymbol{J}_0^+(\tau)\cdot\Delta\boldsymbol{E}(t) - \boldsymbol{L}_0^+(\tau)\cdot\Delta\boldsymbol{H}(t)\,]\;\mathrm{d}V\;\mathrm{d}t = \int_0^T\!\!\int_V \boldsymbol{E}(\mathrm{t})\cdot\boldsymbol{E}^+(\tau)\Delta\gamma\;\mathrm{d}V\;\mathrm{d}t \tag{7}$$

The material parameters of the adjoint model ($^+$) are the same as the originals. Both models, original and adjoint, are analyzed in different times t and τ. Usually it will be assumed that τ is opposite to t : $\tau = T - t$, where T is duration time of the analysis. Excitation of the adjoint model depends on the objective function. If the sensitivity is calculated for the potential values in mesh nodes, the excitation should be assumed as Dirac's impulse for the time T:

$J_0^+(\tau) = \delta(T)$.

The geometric position of this impulse coincides with measurement area of the electromagnetic field. In the simplest case it may be a current introduced into the finite element mesh node. When the measurement proceeds with the help of a coil, the excitation current should be distributed on the area of this coil.

To fulfill the sensitivity analysis on the basis of Eq.(7) one should follow two steps:

1. Analyze the adjoint model in opposite time τ, saving the values of $\boldsymbol{E}^+$ and $\boldsymbol{H}^+$ for each node, where the sensitivity will be evaluated,
2. Analyze the original model in time period from 0 to T, and in parallel evaluate the sensitivity.

It is very convenient to apply a constant value of the time increment. Then the stiffness matrix of both models is the same and remains unchanged during the time-analysis. For this reason, the algorithm is very effective, since the matrix factorization should be carried out only once.

4. Iterative algorithm based on TSVD

Solution of equations (1) or (6) provides corrections of conductivity γ . While the corrections are relatively large, the iterative approach should be taken. The equations (1) and (6) containing measurement data are usually ill-posed, so the solution of inverse task, in opposite to field analysis, may be ambiguous. To ensure good convergence of iterations, the excessing number of measurements is provided. In this case arises the over-determined equation system, while $j >> i$. The effective and superior tool, which is very important for analysis of discrete ill-posed problems, is Singular Value Decomposition (SVD) [4]. This method in combination with the traditional Gauss-Newton algorithm may constitute easy in the implementation for solving the identification problem. Then SVD of $\boldsymbol{S} \in \mathbb{R}^{j \times i}$ is a decomposition of the form:

$$\boldsymbol{S} = \boldsymbol{U\Sigma V}^{\mathrm{T}} = \sum_{l=1}^{i} \boldsymbol{u}_l \sigma_l \boldsymbol{v}_l \ , \tag{8}$$

where: $\boldsymbol{U} = (\boldsymbol{u}_1, ..., \boldsymbol{u}_n)$ and $\boldsymbol{V} = (\boldsymbol{v}_1, ..., \boldsymbol{v}_n)$ are matrices with orthonormal form $\boldsymbol{UU}^{\mathrm{T}} = \boldsymbol{V}^{\mathrm{T}}\boldsymbol{V} = \boldsymbol{I}_n$, and where $\boldsymbol{\Sigma} = \mathrm{diag}(\sigma_1, ..., \sigma_n)$ has non-negative diagonal elements appearing in non-increasing order such that:

$$\sigma_1 \geq ... \geq \sigma_n \geq 0 \, , \tag{9}$$

which are so called the singular values of the matrix $\boldsymbol{S}$. If one assumes, that matrix $\boldsymbol{S}$ indicates the Jacobian goal function, the equation (1)or (6) in matrix notation for each iteration might describe as:

$$\boldsymbol{S}^{<k>} \Delta \boldsymbol{\xi}^{<k>} = \boldsymbol{U}^{<k>} \, . \tag{10}$$

Moreover, solving the equations system (10) by means of SVD, for example in the first iteration the index k was omitted because of notice clarity, one may define such as:

$$\boldsymbol{\Delta\xi} = \sum_{l=1}^{i} \frac{\boldsymbol{u}_l^{\mathrm{T}} \boldsymbol{U}}{\sigma_1} \boldsymbol{v}_l \,. \tag{11}$$

Thus, in the simplest case in order to introduce the regularization method, it is enough to define the filter factors of following form:

$$f_{TSVD}(\sigma) = \begin{cases} 0 & \sigma \le \delta \\ 1 & \sigma > \delta \end{cases} \tag{12}$$

where: δ is the chosen threshold of smoothness. In the numerical implementation the regularization parameter δ was chosen in indirect way, t.i. through assuming the minimal conditioned coefficient κ, which ought to characterize the matrix $\boldsymbol{S}$.

5. Numerical example

Let us consider the following model of eddy-current equipment for testing of heat exchanger tubes of steam generator in nuclear plants. The eddy-current sensor consisting of three coils moves inside the long, conducting tube. The coil in the middle is used for magnetic field excitation, other two are differential measurement coils.

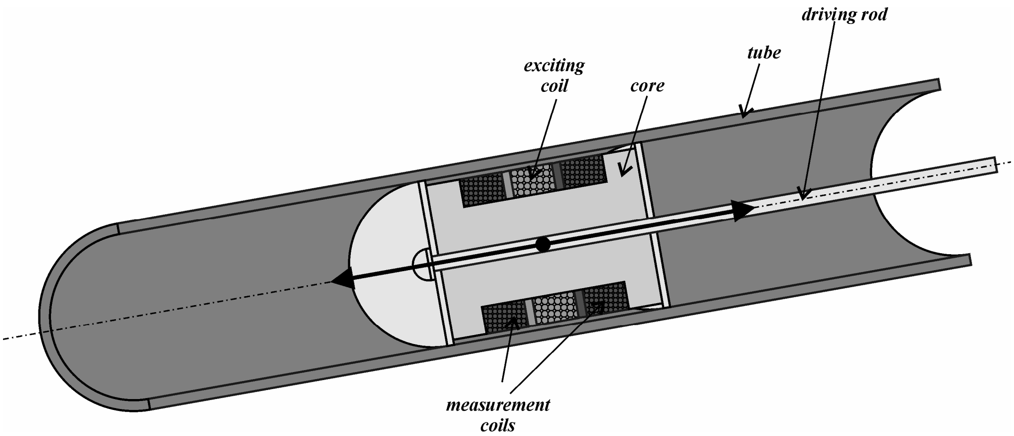

Fig.1. Differential eddy-current sensor inside tube.

In each position the sensor is excited with exponential current and the voltage impulse is registered. The model exhibits cylindrical symmetry and can be analyzed using a 2D formulation. For the aim of measurement simulation the area was divided into 189 696 finite elements with 95 409 nodes. The transient field analysis was carried out with the backward Euler scheme, with n = 150 constant time steps, each of Δt = 10 ns. For d = 97 positions of the sensor the induced voltage shapes were registered. As measurement data are obtained the inverse job of conductivity recognition can start. Three exemplary cracks are shown in Fig.2, their electrical conductivities differing from tube wall are described in Table 1.

For recognition process one thinner mesh was used with 128 700 elements and 64 775 nodes. From this reason only 61 sensor positions were used and the measurement data had to be interpolated for some positions. Hence, the sensitivity matrix (6) consisted of 9150 rows and 360 columns (number of finite elements in search area).

Table 1. Conductivity in area of cracks

Crack	I	II	III
Conductivity	0.1 MS/m	0.3 MS/m	0.3 MS/m

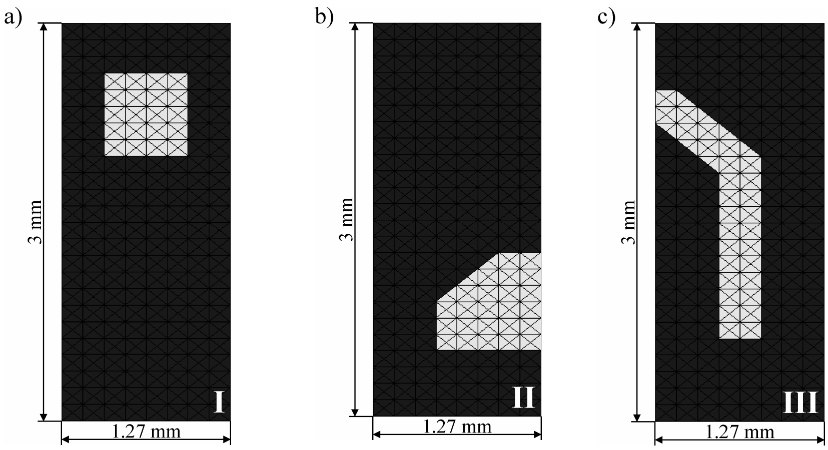

Fig.2. Predicted shapes of cracks (search area shown black).

The conductivity distributions (crack shapes) and crack positions on search area were correctly recognized after 20 iterations (Fig.3).

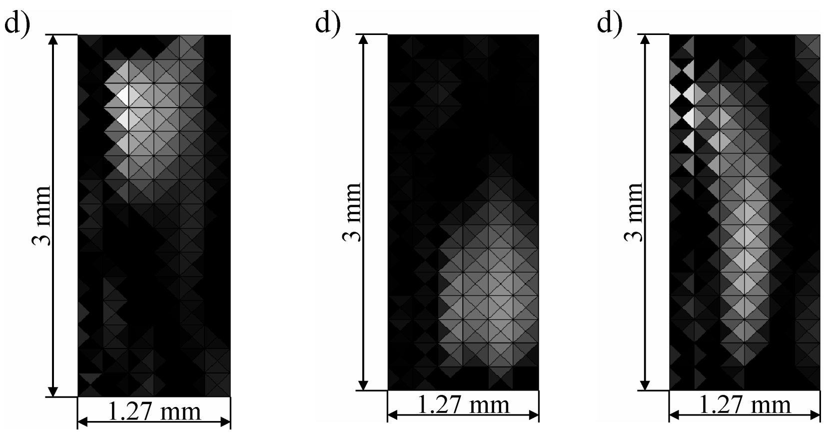

Fig.3. Recognized shapes of cracks (search area shown black).

6. Conclusions

The examples show the reliability of proposed methods. The success of numerical evaluation of conductivity distribution depends mainly on the exact measurement of the magnetic flux. The error of sensitivity evaluation has a secondary meaning and influences only the manner in which the result is obtained. In the examples shown above, instead of the measurements, the models with cracks were analyzed by an FEM based on another discretization, providing data for further iterative process. Then, the cracks were removed, and the algorithm tried to reconstruct the nodal potentials based on sensitivity values of the nodes. If the real data containing measurement errors were used, the results could be worse. Disadvantage of sensitivity algorithms in the time domain is long computation time with personal computers, but the calculations can be easily parallelized.

Further scientific work on this area should lead to three-dimensional algorithms, which allow modeling of wider class of cracks. For 3D-sensitivity analysis the different formulations [10] should be tested. Although the sensitivity equation (7) remains the same, it's terms would be evaluated in quite different manner, as in 2D. For recognition of real cracks the application of data filtering and new methods for regularization of equations system are necessary.

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