Ilmenauer Beiträge zur Wirtschaftsinformatik

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Bottom-Up Simulation of a Stock Market Using a Thousand Fuzzy Agents

Working Paper 2009-01, February 2009

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Ilmenauer Beiträge zur Wirtschaftsinformatik (Working Paper) 2009-01, Technische Universität Ilmenau, February 2009

ISSN 1861-9223

ISBN 978-3-938940-22-8

URN urn:nbn:de:gbv:ilm1-2009200057

Anschrift: Technische Universität Ilmenau, Fakultät für Wirtschaftswissenschaften, Institut für Wirtschaftsinformatik, PF 100565, D-98684 Ilmenau.
http://www.tu-ilmenau.de/fakww/Ilmenauer_Beitraege.1546.0.html

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Abstract: *In this research the focus is on emergent behaviour in large groups of stock market participants. In contrast to established views of the capital market, we do not assume that market participants make rational investment decisions based on full information about important macroeconomic figures such as interest or exchange rates. Nor is the market modelled top-down with mathematical equations. Instead, trading decisions and market behaviour are the result of individual participants reacting to stock price and price changes. Each participant is modelled as a fuzzy agent that behaves according to simple trading rules. This is sufficient to create regular and complex macroscopic stock price patterns.*

Key words: *simulation, stock market, fuzzy agent, swarms, emergence*

1 Introduction

Traditional models of financial markets take a macroscopic perspective, modelling market behaviour with mathematical equations. The underlying assumption in the classical case is the efficient market hypothesis (EMH), which suggests that all relevant information is instantly available to all market participants, who then make well-informed, rational decisions about their investments. From this, market price is a result of the distribution of expectations in the market regarding future yield [1]. Frequently, a normal distribution of these expectations is assumed and only random influence can lead to a short-term above-average performance of individual market participants. As a result stock prices display a random path, and it becomes impossible to forecast future prices (Random Walk Theory) [2].

Reality is apparently different. Empirical studies have confirmed correlations between historic and current price levels for stocks [3]. Moreover, results in fractal mathematics have underlined that the random walk theory does not apply and structures can be found in stock price graphs [1] [4]. The question arises of how financial markets can be modelled more adequately.

More recent models, such as the synergetic capital market model from Landes/Loistl [15], support our way of thinking that a micro-level examination of the entities and processes in share trading is required. However, the synergetic capital market model is quite complex in order to create the most realistic depiction possible of the macroscopic “capital market” system [cf. 15, especially pp. 319 – 345]. Moreover, fundamental influences on markets play an important role, which is not the assumption underlying our work.

In this paper we argue for a bottom-up model that focuses on individual market participants and draws a remote parallel to biological swarms. The general assumption here is that at least the (very large number of) non-commercial private traders act more like members of a herd rather than analyzing fundamental influences on the market (such as changes in interest rates or economic climate) to arrive at their trading decisions. Market behaviour then arises as an emergent phenomenon at the macroscopic level. The research goal we pursue is to increase our understanding of the relationship between simple, swarm-like individual trading behaviour and its macroscopic effects at the market level.

Section 2 gives a short introduction to concepts central to our research such as swarms and emergent behaviour. In section 3 the experimental setting is highlighted. Section 4 includes experimental results and their interpretation. We conclude with lessons learned and implications for future research in section 5.

2 Swarms and Stock Markets

Many animals in nature, such as birds, fish and ants, operate in swarms, i.e. they form large groups of individuals that display coordinated movements. While the animals themselves are quite limited in their abilities and the rules of coordination are simple, complex swarm behaviour can be generated at the macroscopic level since each individual generates an indirect force on all other individuals and is itself under multiple influences from other swarm members.

This effect may be associated with what has been called "relative emergence" [9], meaning that with current knowledge and techniques, it is impossible to deduce the macroscopic behaviour of a system from an analysis of its components and microstructure.[1]

Swarms have recently received much attention in the context of artificial life and artificial intelligence [5] [6] [7] [8] due to the emergent behaviour they display. Robustness and the ability for self-organisation are important characteristics of swarms. These features make artificial swarm-like systems interesting for practical applications, where autonomy and fault tolerance are important themes, such as in multi-agent systems research. Moreover, swarms may be seen as an example for bottom-up simulations that model real systems not at the macroscopic level using mathematical equations but focus on the microscopic parts of the system, their characteristics and microstructure. This is the view on stock markets adopted in this paper. It is, in fact, typical for agent-based computational economics [14] [16] [17]. For an overview of agent-based models of financial markets see [18].

We take up the position of Schleis [10], who describes the population of stock market participants as a social being in its own right, whose members influence and control each other particularly through the pricing mechanism. Combining this viewpoint with the idea of a bottom-up market model as discussed in [15], one arrives naturally at the research goal to construct a micro-level model of a capital market containing a large number of trader entities with simple behaviour, influencing each other indirectly through their trading decisions and the resulting stock price level. Our focus is on the relationship between individual trading behaviour and macroscopic stock price movements. It is important to underline that we do not aim to forecast stock prices nor do we attempt to generate the exact historical curve of a particular stock. The goal pursued here is more moderate. By concentrating on market participants (trader agents) and their simple swarm-like behaviour, thus modelling a stock market from the bottom up, the aim is to generate complex system behaviour in the sense that stock price patterns are created that resemble those of real stock markets. We believe it is unnecessary to assume the typical BDI-agents and complex reasoning and cooperation processes of multi-agent simulations to achieve this goal. This, however, would give a hint that trading decisions in real markets are often taken in a much simpler fashion than is commonly thought. We further believe it is not required to assume external market forces or changes of fundamental data to create major changes of stock prices. If this was so, then forecasting stock market developments would be proven a hopeless task.

So, while design decisions and parameterisations are only concerned with the micro-level of trader agents, the focus of interest is the results at the macroscopic level of stock prices and their connection with the micro-level.

3 A Bottom-Up Simulation of a Stock Market

3.1 Structure of a Fuzzy Trader Agent

Our stock market is populated by a large number of simple trader agents that buy and sell a single stock. 1000 agents are employed in the simulation. This quantity was determined empirically during initial test runs. It is sufficiently large to achieve a well-distributed diversity in the agent population, but also avoids very excessive computational runtime requirements.

A basic assumption is that market participants do not perform complex analyses of macroeconomic figures before they act. Rather, they evaluate stock price and the trend in

[1] This, however, does not exclude its being possible in the future. Practically speaking, it may suffice that the analysis would be extremely complex and time-consuming at present.

prices over some period of time and then decide whether to trade based on simple rules.[2] Market participants are modelled as stimulus-response agents. They receive information and react immediately, based on behaviour that is coded in a fuzzy rule base and inference mechanism. Figure 1 gives the structure of a trader agent, which is essentially the structure of a Mamdani-style technical fuzzy controller.

Crisp (not fuzzy) input information (price level, trend in prices) is fuzzyficated, i.e. mapped on fuzzy sets. This allows the compatibility of the inputs (facts) with the conditions of rules in the knowledge base. The result is in each case a real number in the interval [0,1]. For each rule, the compatibility results of individual conditions are aggregated to an overall compatibility for the condition part of the rule. A rule is activated when its condition part is fulfilled to an extent greater than zero. Several rules may be activated in parallel.

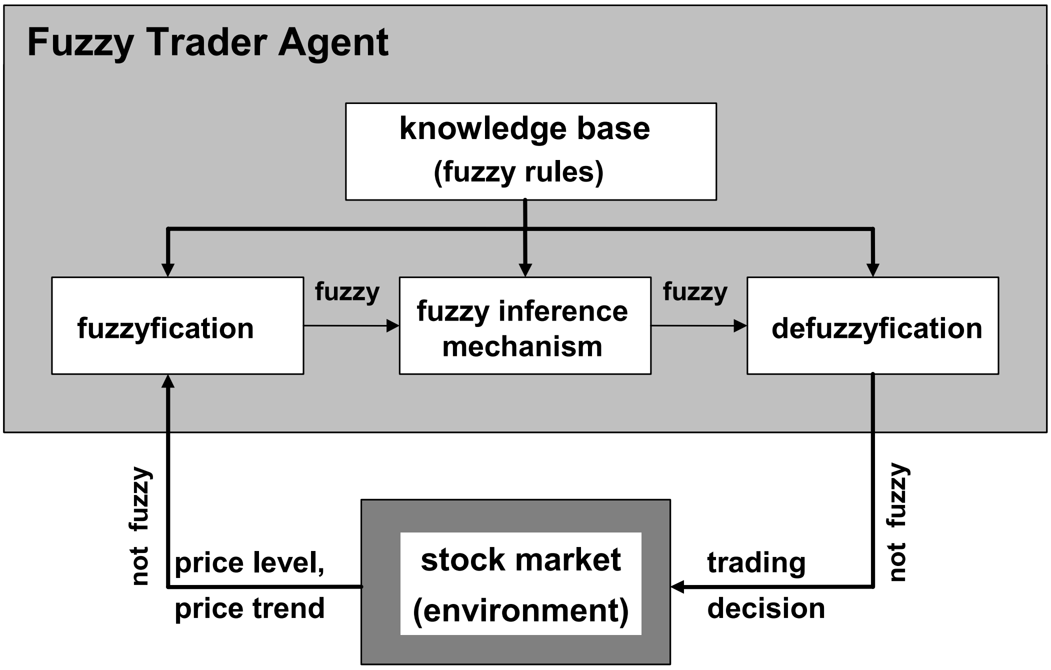

Fig. 1 Structure of a fuzzy trader agent. Crisp information on price level and the trend in prices is the input while a decision to buy, hold or sell a certain amount of stock is the output after some fuzzy inference process that uses fuzzy rules from the agent's knowledge base.

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2 Agents could also make use of trading volume information, but for the experiments here we abstract from volume figures, as the fundamental principles of trading remain the same.

A fuzzy inference mechanism then maps the input to the output variables of the fuzzy trader agent by simple feedforward reasoning. We have chosen the well-established max-min inference to arrive at the fuzzy output results.[3] In a last step, the fuzzy output set concerning the trading decision (buy, hold, sell) is converted to a crisp decision by applying standard centre-of-gravity defuzzyfication. The direction and the amount of the trading are fully determined through the defuzzyficated output. Figure 2 demonstrates the principle mechanism of fuzzy inference using two input values and one output value.

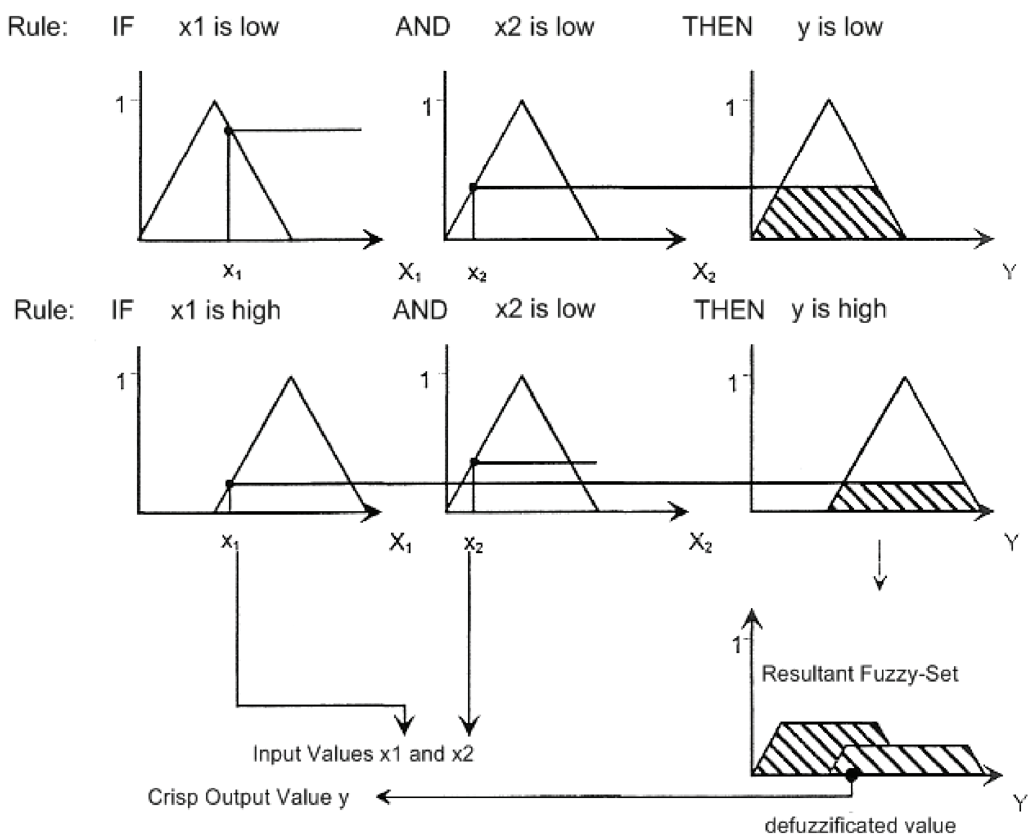


Fig. 2 Fuzzy Inference Mechanism (Fuzzy Control Logic).

Using fuzzy set theory to model the trader agents has several advantages. First, fuzzy rules offer a simple yet elegant way to explicitly code human knowledge. Second, only a few rules suffice to generate sensible agent response in all relevant contexts of our model.

3 For more details on fuzzy set theory and fuzzy control, see for instance [12] [13].

Third, fuzzy systems are generally robust and display smooth behaviour patterns in dynamic environments [11]. Fourth, the behaviour of agents can be changed individually by adapting fuzzy sets or rules in the knowledge base.

3.2 Further Model Details at the Microscopic Level

The input variables of each fuzzy trader agent in our model are price level and trend in prices, both normalized to the range [-1, 1]. The defuzzyficated output value is interpreted as the decision to buy, hold or sell stock. Figure 3 displays the fuzzy sets for input and output.

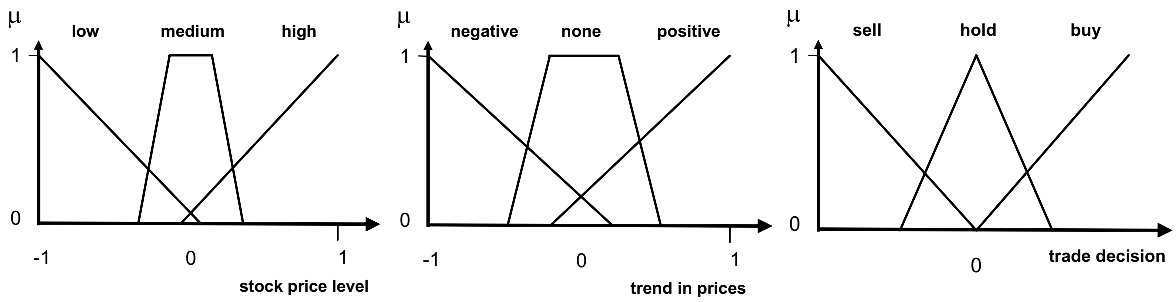


Fig. 3 Fuzzy sets (linguistic terms) for input and output variables of fuzzy trader agents.

The real valued function $\mu_{\tilde{A}} : X \rightarrow [0,1]$ is the membership function. Herein, a value $\mu_{\tilde{A}}(x) = 0$ means that x does not belong to the fuzzy set $\tilde{A}$, while a value $\mu_{\tilde{A}}(x) = 1$ indicates full membership. Values in the interval $0 < \mu_{\tilde{A}}(x) < 1$ indicate a partial membership of x in the set $\tilde{A}$. Based on the fuzzyficated inputs the inference mechanism of each fuzzy agent generates a crisp value in the interval [-1, 1] for the output. This value forms the basis for the trading decision of the respective agent.

The precise geometry of each input fuzzy set can vary between agents to emulate different market assessments as will be described later. However, all agents have an identical knowledge base of rules as given in table 1. Agents generally show market-conforming behaviour, thus displaying swarm-like coordination through the stock price. On an abstract level, in a swarm and in a stock market individuals influence their environment through individual actions while in the next step the environment feeds information back to them and influences their next actions.

IF Price	AND Trend in Prices	THEN Trade Decision
low	Negative	buy
medium	Negative	sell
high	Negative	sell
low	None	hold
medium	None	hold
high	None	hold
low	Positive	buy
medium	Positive	buy
high	Positive	sell

Table 1. Rule base of fuzzy trader agent (rules identical for all agents).

This is in accordance with empirical studies about human behaviour in complex, non-linear systems where test subjects frequently followed an ad-hoc hypothesis of linear trend [1, pp.126]. Moreover, Schleis argues from a psychological position that crowds of people tend to display self-enforcing feedback processes of euphoria or panic [10, pp.29]. In general, the stock boom and subsequent crash at the turn of the century supports his argument.

A trader will only change his trading decision against the current market trend when new information with sufficient intensity is available that indicates a probable change in the stock price pattern. So, when the trend in prices is still positive but the price level is considered “high” by the agent, it will start selling. Contrary, it will start buying when the trend in prices is still negative but the price level is already considered “low”.

It is assumed that trading is always possible. This is, of course, a simplification over the real stock market where limits, such as stop-loss, play an important role and one might not be able to sell stock at a given time because of a lack of demand. However, such additional constraints would only complicate the model without adding value to pursuing our current research goals.

No central market maker is employed in our model who decides about the stock price. However, the basic market mechanism is present, i.e. the stock price will change in proportion to direction and volume of the overall trading decisions of all agents in the market.

3.3 Simulation at the Macroscopic Level

The stock market simulation is processed in discrete time steps. These steps are conceptually comparable to the regular time interval for the price determination in real stock markets. For example, this price determination happens every second for the *Deutsche Aktienindex* DAX. At the macroscopic level, the inputs (price level, trend in prices) for all agents are calculated, using the current status of the stock market system and the outputs (trading decisions) of the agents. The trend in prices Δk of trading cycle t is calculated at

$$\Delta k_t = \frac{M_t^{buy} - M_t^{sell}}{M_t^{buy} + M_t^{sell}} \qquad \textbf{(1)}$$

where M_t^{buy} is the total trade volume of stock bought in period t while M_t^{sell} is the total trade volume sold in period t. At this stage, we relax the requirement that both volumes must be identical, thus avoiding the necessity for bookkeeping of individual stock volumes for one thousand trade agents. Instead, we assume some trading institution as part of the market that is always prepared to buy and sell stock at the current price in the required amount. This simplifying assumption will be removed in future research, while it currently helps to focus on the aspects of interest. It should be noted that in real markets such asynchronous trade situations can indeed occur, for instance in the case of short sales by hedge funds. The initial Δk_0 is set to the value of zero (neutral). The stock price k_t at the end of trading period t is then given as

$$k_t = k_{t-1} + \Delta k_{t-1} \qquad \textbf{(2)}$$

The calculation of the relevant stock price level p_t as one of the input variables for the trading agents varies between the different simulation experiments. However, it is not identical to the stock price k_{t-1} at the end of the previous trading period. Instead, agents individually calculate an average price over some historic period and use this in their trading decision. The length of this historic period s is a random variable that is determined independently once for each agent at the start of the simulation, just as individual traders in real markets take individual perspectives on past prices. In our experiments it is assumed that this random variable is normally distributed. If s is small, then an agent ignores most of the price history in its calculation of the input "stock price level".

The 1,000 fuzzy trader agents communicate only indirectly through the stock price k_t that is the result of aggregated trading decisions of the individual agents. Trading does not necessarily occur in each period. Instead, trading frequency f is a normally distributed random variable, determined once for each agent. Thus, different trading attitudes, such as day traders and long-term investors can be emulated.

In addition to the historic period s and the trading frequency f, a third parameter is of major importance for the simulation: the diversity d of market assessments in the population of agents. In real markets, some traders are more risk-averse than others, thus their individual assessment of a given market situation can differ significantly. This is emulated by varying the geometry (position and width) of the fuzzy sets for each of the two input variables of a trader agent within predefined ranges. Thus, the assessment of stock price level and trend in prices varies between different agents, influencing their individual decision to buy, hold or sell stock. This variation of fuzzy sets is done at the start of the simulation individually and once for each agent using a $(0,\sigma)$-normally distributed random variable. The standard deviation of the random variable can be used to create different amounts of behavioural diversity in the population of trader agents.

With s, f and d some useful parameters are available to influence the individual behaviour of the fuzzy trader agents and analyse the relation of microscopic behaviour patterns and macroscopic effects. When looking at the effects in section 4 it is worth mentioning that random numbers are only created and applied during initialisation, but the simulation itself is deterministic. Please note that the utilisation of the afore-mentioned parameters and fuzzy sets does not mean that the trader agents adjust themselves to the system output through learning. All parameters are set to initialise a simulation run and thereafter remain static during the course of the run.

4 Experiments with the Bottom-Up Simulation

For each experimental setting 20 runs with different random number seeds were performed. A single run consists of 300 trading periods. As averaging over different runs is not meaningful in this context, we present graphs and discuss results of individual runs that display 'typical' behaviour for the respective parameterisation. It is acknowledged that this choice is somewhat subjective. The initial price level and trend in prices in all experiments are set to zero (neutral), serving as a baseline reference for the following price movements.

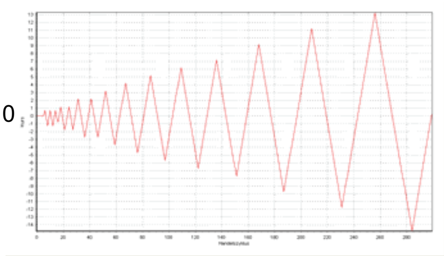
It should be kept in mind that the regular patterns described below are all created without exogenous shocks to the stock exchange system. Moreover, agents do not coordinate themselves directly through communication or a central coordination agent. They only observe other agents behaviour indirectly through the price level and trend in prices and apply simple trading rules, representing their market assessments.

4.1 Experiment 1

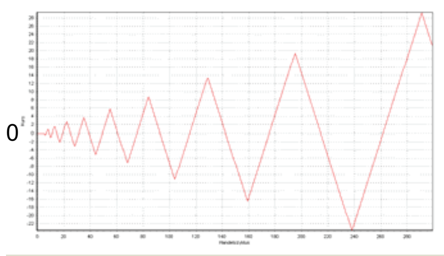
In the first experiment, agents calculate the price level input p_t as the average stock price within their relevant historic period s ($s \in [1 \ldots t-1]$). k_i denotes the stock price in period i.

$$p_t = \frac{\sum_{i=t-s-1}^{t-1} k_i}{s} - k_0 \qquad \textbf{(3)}$$

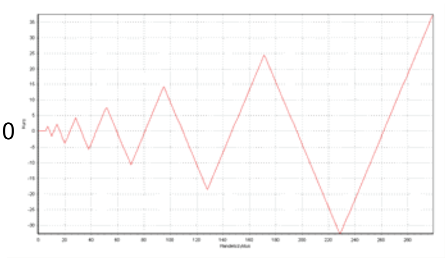
Furthermore, trading occurs deterministically in every period and the diversification of market assessments in the population is initially low. This allows us to better isolate and analyse the influence of s on the macroscopic level. Figures 4 (a)-(d) demonstrate the effect of raising the expectation of s with its standard deviation remaining at zero, meaning that agents include more historic price values in their price level calculation, but do so in an identical way. The resulting macroscopic effects are an increase in amplitude of the stock price movement with time and a proportional prolongation of each price cycle. As s is raised, more and more values enter the calculation of the agents' stock price level. Thus, the influence of short-term price movements diminishes and the inertia of the price level rises, leading to later reactions of agents to current stock price movements and, thus, the visible macroscopic effects.



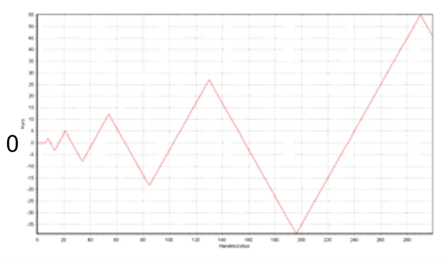
(a)



(b)



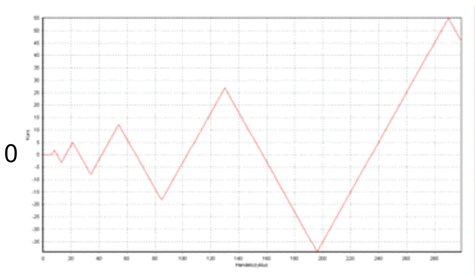
(c)



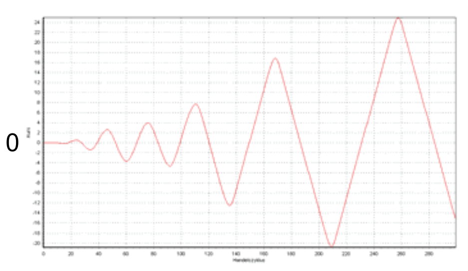
(d)

Fig. 4 Results with behavioural agent diversification $d = 0.1$ (low), trading in each period and *s*-value expectations of 10% (a), 20% (b), 30% (c) and 50% (d) of simulation periods. The curves display stock price k_t over time during a complete simulation run.

When the diversity of market assessments in the agent population is raised *ceteris paribus (c.p.)* (meaning more behavioural diversification), then this smoothens the stock price curve at its turning points and simultaneously softens the price cycles (figure 5). Now, as the market assessments at each point in time are more diverse, more agents may act in opposition to the general trend in the population, thus indirectly convincing other agents to leave their current trading positions. This leads to less abrupt changes of stock price at the turning points of the price pattern and also shortens corresponding price cycles by lowering the amplitudes.

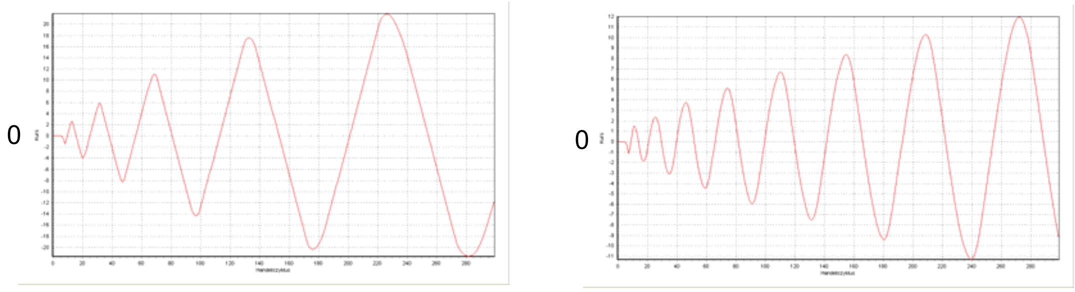


(a)



(b)

Fig. 5 Results for *s*-value expectation of 50% and behavioural agent diversification $d = 0.1$ (a) and $d = 0.5$ (b). Rest of parameters as in figure 4.



(a) (b)

Fig. 6 Results for *s*-value expectation of 50% with a standard deviation of 10% (a) and 30% (b). Rest of parameters as in figure 4.

A similar effect can be achieved *c.p.* by introducing a standard deviation for *s*, as can be seen in figure 6. Now, the behavioural diversification in the population is low again, but agents apply their trading rules to different price levels, resulting again in different market assessments, thus smoothing the stock price curve at the turning points, but also shortening the price cycles.

4.2 Experiment 2

In the second experiment, agents calculate the price level input p_t as the average stock price within their historic period *s* (s∈[1…t−1]), but with reference to the stock price at the beginning of this period. k_i again denotes the stock price in period *i*.

$$p_t = \frac{\sum_{i=t-s-1}^{t-1} k_i}{s} - k_{t-s-1} \qquad (4)$$

Here, an interpretation of results is much more difficult, as each agent deduces an *individual* stock price from the average price over its historic period, leading to very different inputs, and consequently trading decisions, for agents.

The first series of simulation runs has identical parameterisation as in the previous experiment with focus on the length *s* of the historical price period. Comparing the results of figure 4(a) and figure 7(a), it becomes evident that with a short relevant price history the modified price level calculation of agents forces the system to search an initial high stock price level before it starts oscillating. The initial rise in figure 7(a) is due to the deduction made from the average price ($k_{t-s-1} > k_0$) that pushes agents to continue buying stock, thus raising the stock price. However, as the parameter s is raised and the relevant historic period becomes longer in price level calculations, this initial search phase diminishes and the curve now resembles the result in our first experiment – but with a lower mean stock price due to the deduction of negative stock prices in the agents' price level calculations.

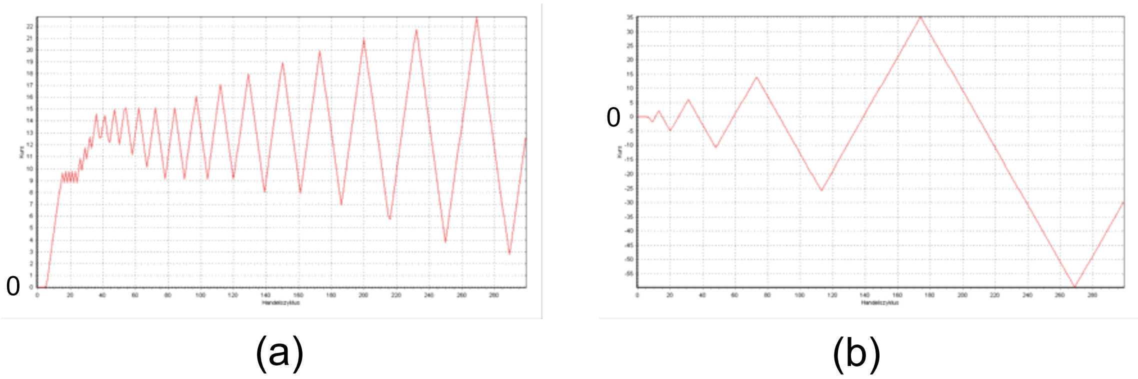


Fig. 7 Results with behavioural agent diversification $d = 0.1$, trading in each period and s-value expectation of 10% (a) and 50% (b), standard deviation of s is zero. The curves again display the stock price k_t over time during a complete simulation run.

A very significant influence, not seen in experiment 1, comes through the introduction of a standard deviation of s (figure 8).

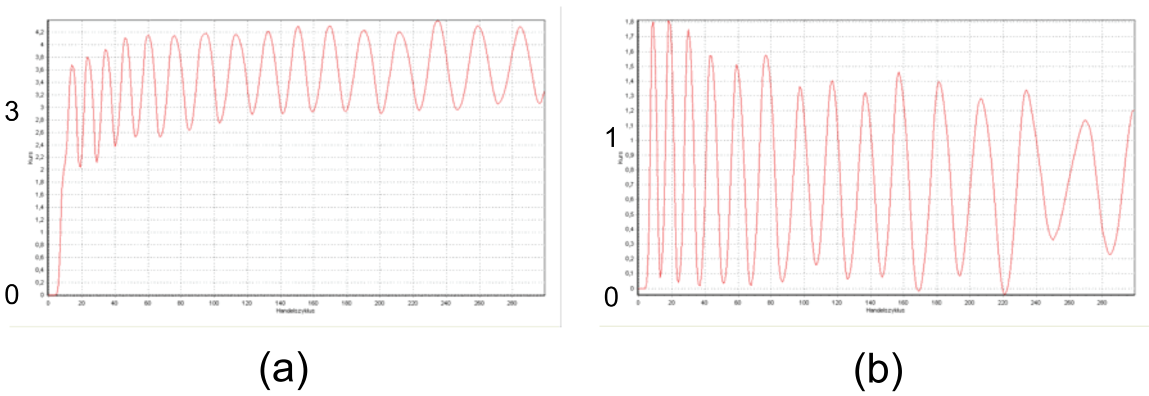


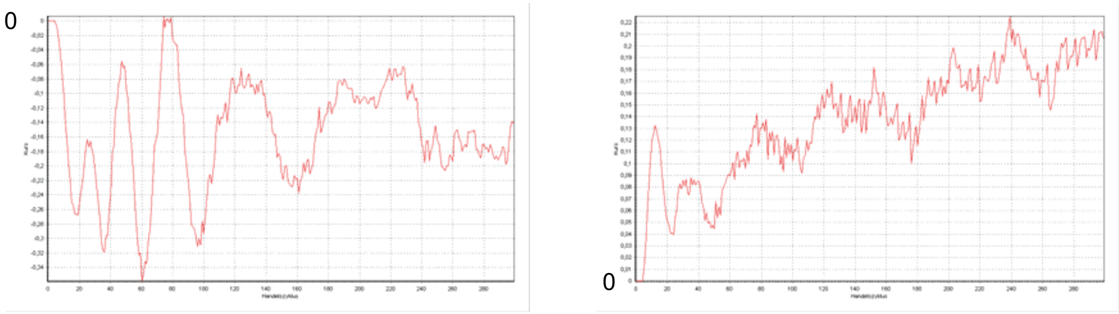
Fig. 8 Results for behavioural agent diversification $d = 0.1$, trading in each period and s-value expectation of 10% with a standard deviation of 20% (a) and 30% (b).

With greater diversification of s-values in the agent population, the agents' calculated price level input now varies widely, leading to different price interpretations. The results are a significantly reduced amplitude of the stock price movement, shorter oscillation cycles and a lower predictability of the price at the macroscopic level.

4.3 Experiment 3

In this last set of empirical investigations, the influence of a more varied trading frequency is analysed. So far, it was assumed that agents make trading decisions in all periods during the simulation. This is of course not in accordance with real markets, where traders may act more or less frequently, with the extremes of day traders and long-term investors. To emulate such situations we start by introducing an expectation for the random trading frequency *f* that is identical for all agents. This means, all agents continue to trade simultaneously, but not in every period of the simulation. Figure 9 displays the results for this experiment. The curve is less smooth, because of longer intervals without trading, followed by trading periods where the stock price is abruptly corrected. This is particularly pronounced at the turning points. However, the oscillation of the stock price is less distinctive. The averaging price level calculation of the agents combined with the periods without trading now leads to a stretching of the price oscillation pattern.

Fig. 9 Results for behavioural agent diversification $d = 0.3$, trading frequency of 20% with a standard deviation of zero, an *s*-value expectation of 30% with a standard deviation of zero. The price level calculation is as in section 4.2.



(a) (b)

Fig. 10 Results for behavioural agent diversification $d = 0.5$ (high), a trading frequency of 0% with a standard deviation of 10%, an s-value expectation of 10% with a standard deviation of 30% (a) and 40% (b). The price level calculation is as in section 4.2.

The experiments so far have shown that with the assumption of regular trading it is possible to isolate and interpret the influence of individual model parameters on the macroscopic level. The final simulation now serves to demonstrate the effects of more irregular trading, combined with a large behavioural diversity (different market assessments) within the population of agents, when the length s of historic periods used in the price level calculation by the agents is different (figure 10). Stock price now displays a high volatility and resembles real-world stock price patterns. However, the resulting graphs of similar experiments are so different that it seems impossible to draw definite conclusions about the influence of s on the macroscopic level here. The interactions of system parameters now become very complex and hard to understand. One arrives at a system which displays regularities and complex behaviour at the same time.

5 Conclusion and Future Work

The main objective of this paper was a better understanding of the relationship between certain behavioural parameters of the participants of an artificial stock market and the emergent macro-level stock price patterns created through the trading decisions of individual agents. The experiments are certainly preliminary and to be extended, but it is nevertheless believed that some useful lessons can be learned from the simulations.

First, regular oscillations and complex stock price patterns can be created bottom-up without the necessity of a mathematical top-down model of the simulated market. Moreover, exogenous (economic) shocks to the system are not required to change the direction of the stock price. Nor is it required that agents can communicate directly or use a central coordination instance to achieve aligned behaviour. Focusing on modelling individual traders, now gives the opportunity to directly implement and test assumptions about the influence of decision parameters and agent behaviour as part of a bottom-up simulation model.

Second, the trading rules of market participants may be captured with only a few rules using the analogy of technical fuzzy control applications where crisp input combined with a simple model of human knowledge is sufficient to create robust system behaviour. By changing the geometry of fuzzy sets, behavioural variation can be introduced in otherwise identical agents. Thus, we have a straightforward way of modelling some elementary aspects of the psychology of trading behaviour.

Third, seasonal price patterns occur in our simulation just because of price speculations by the individual agents. This would suggest that in real markets oscillations can occur, even if traders ignore all fundamental data. Thus, it does not seem mandatory to refer to interest rates, overall economic situation or exchange ratios – all commonly used to explain trading patterns, but agents just watching historical price data and the trend in prices can create such macroscopic patterns. This does not mean that fundamental economic data has no influence on individual trading. However, this influence may be overestimated, particularly when it comes to non-professional traders who might behave more swarm-like with indirect coordination through the market price and simple plausibility-based trading rules instead of making rational choices based on full economic information and deep analysis.

Fourth, averaging historic prices in making a trading decision is a fundamental reason for dynamic stock prices, as is behavioural variety amongst traders. A low diversity among traders and similar appreciation of price history suggests regular and very pronounced oscillations of the stock price while greater behavioural variation of agents in combination with a varied appreciation of price history should create very volatile and complex price patterns through reinforcement effects. It follows that stock price forecasting at the macroscopic level remains a very difficult (if not hopeless) task, even if one could assume that *no* change in fundamental economic data or exogenous shock occurs.

The research outlined here may be extended in various directions. For instance, in our model the stock price is determined in a simplified way that could be improved to more strongly resemble the pricing mechanism of an actual stock exchange. This would include the introduction of a central market maker. The rule base of a fuzzy trader agent is currently very small and focuses on stock price data. It could be extended to integrate further inputs or decision parameters. Moreover, the rule bases of agents could be different. Also, agents currently display only stimulus-response behaviour but are unable to learn. Changing this, though, would require a paradigm shift away from swarm-like agents to complex, intelligent agents, while the swarm metaphor currently does not seem to be fully explored. Finally, it would be interesting to simulate exogenous shocks to the stock market system and analyse their effects.

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