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COMPUTER SCIENCE AND AUTOMATION

COMPUTER SCIENCE MEETS AUTOMATION

VOLUME I

Session 1 - Systems Engineering and Intelligent Systems

Session 2 - Advances in Control Theory and Control Engineering

Session 3 - Optimisation and Management of Complex Systems and Networked Systems

Session 4 - Intelligent Vehicles and Mobile Systems

Session 5 - Robotics and Motion Systems

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Preface

Dear Participants,

Confronted with the ever-increasing complexity of technical processes and the growing demands on their efficiency, security and flexibility, the scientific world needs to establish new methods of engineering design and new methods of systems operation. The factors likely to affect the design of the smart systems of the future will doubtless include the following:

- As computational costs decrease, it will be possible to apply more complex algorithms, even in real time. These algorithms will take into account system nonlinearities or provide online optimisation of the system's performance.
- New fields of application will be addressed. Interest is now being expressed, beyond that in "classical" technical systems and processes, in environmental systems or medical and bioengineering applications.
- The boundaries between software and hardware design are being eroded. New design methods will include co-design of software and hardware and even of sensor and actuator components.
- Automation will not only replace human operators but will assist, support and supervise humans so that their work is safe and even more effective.
- Networked systems or swarms will be crucial, requiring improvement of the communication within them and study of how their behaviour can be made globally consistent.
- The issues of security and safety, not only during the operation of systems but also in the course of their design, will continue to increase in importance.

The title "Computer Science meets Automation", borne by the 52nd International Scientific Colloquium (IWK) at the Technische Universität Ilmenau, Germany, expresses the desire of scientists and engineers to rise to these challenges, cooperating closely on innovative methods in the two disciplines of computer science and automation.

The IWK has a long tradition going back as far as 1953. In the years before 1989, a major function of the colloquium was to bring together scientists from both sides of the Iron Curtain. Naturally, bonds were also deepened between the countries from the East. Today, the objective of the colloquium is still to bring researchers together. They come from the eastern and western member states of the European Union, and, indeed, from all over the world. All who wish to share their ideas on the points where "Computer Science meets Automation" are addressed by this colloquium at the Technische Universität Ilmenau.

All the University's Faculties have joined forces to ensure that nothing is left out. Control engineering, information science, cybernetics, communication technology and systems engineering – for all of these and their applications (ranging from biological systems to heavy engineering), the issues are being covered.

Together with all the organizers I should like to thank you for your contributions to the conference, ensuring, as they do, a most interesting colloquium programme of an interdisciplinary nature.
I am looking forward to an inspiring colloquium. It promises to be a fine platform for you to present your research, to address new concepts and to meet colleagues in Ilmenau.

Professor Peter Scharff
Rector, TU Ilmenau

Professor Christoph Ament
Head of Organisation

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H. Toossian Shandiz / A. Hajipoor

Chaos in the Fractional Order Chua System and its Control

Abstract: In this paper, we study the chaotic behaviors in the fractional order Chua system. We found that chaos exists in the fractional order Chua system with order less than 3. The lowest order we found to have chaos in this system is 2.7. Linear feedback control of chaos in this system is also studied.

1. Introduction

Fractional calculus is one of the classical mathematical topics in recent years. According to [1,2], more attentions have been paid to the application of fractional calculus in physics, engineering systems and financial analysis.

The fractional-order dynamics of a system known to us include viscoelastic systems [3,4], dielectric polarization [5], electrode–electrolyte polarization [6], electromagnetic waves [7], quantitative finance [8], and quantum evolution of complex systems [9]. Moreover, the control of fractional-order dynamic systems is also performed by various researchers [10–15].

Zaslavsky [16] conducted a comprehensive review for the existing models of fractional kinetics and their connection to dynamical models, phase space topology, and other characteristics of chaos. Many researchers have found that the chaotic attractors indeed exist in fractional-order systems according to [17–24]. In 2004, Li and Chen [25] found that the hyper chaos in fractional order Rossler equations has an order as low as 3.8.

In this paper, we study the chaotic behaviors in the fractional order Chua system [24]. A linear feedback control is also presented for this fractional order system.

2. Approximation of Fractional Derivative

There are several definitions of fractional derivatives [1]. Perhaps the best known is the Riemann–Liouville definition, which is given by

$$\frac{d^{\alpha} f(t)}{dt^{\alpha}} = \frac{1}{\Gamma(n-\alpha)} \frac{d^{n}}{dt^{n}} \int_{0}^{t} \frac{f(t)}{(t-\tau)^{\alpha-n+1}} d\tau \qquad (1)$$

where $\Gamma(.)$ is the gamma function and $n-1 \le \alpha < n$. This definition is significantly different from the classical definition of derivative.

Fortunately, the Laplace transform is still applicable and works as one would expect. Upon considering all the initial conditions to be zero, the Laplace transform of the Riemann–Liouville fractional derivative satisfies the following equation.

$$L\left\{\frac{d^{\alpha} f(t)}{dt^{\alpha}}\right\} = s^{\alpha} L\{f(t)\} \qquad (2)$$

Thus, the fractional integral operator of order α can be represented by the transfer function $F(s)=1/s^{\alpha}$. The standard definition of fractional differential does not allow direct implementation of fractional operators in time-domain simulations. An efficient method to circumvent this problem is to approximate fractional operators by using standard integer order operators. In [26], an effective algorithm is developed to approximate fractional order transfer functions. Basically, the idea is to approximate the system behavior in the frequency domain. By utilizing frequency domain techniques based on Bode diagrams, one can obtain a linear approximation of the fractional order integrator, the order of which depends on the desired bandwidth and discrepancy between the actual and the approximate magnitude Bode diagrams. This approximation approach was adopted in [15], [18], [21–23]. In Table 1 of [15], approximations for $1/s^q$ with $q=0.1-0.9$ in steps 0.1 are given, with errors of approximately $2\,dB$. We also use these approximations in the following simulations.

3. The Fractional Order Chua System

We consider the fractional order Chua system .The standard derivative [24] is replaced by fractional derivatives as follows:

$$\begin{aligned}\frac{d^q x}{dt^q}&=\alpha\left[y+\frac{x-2x^3}{7}\right]\\ \frac{d^q y}{dt^q}&=x-y+z\\ \frac{d^q z}{dt^q}&=-\frac{100}{7}y\end{aligned}\qquad (3)$$

where q is the fractional order. When $q=1$, system (3) is the original integer order Chua system. Simulations are performed for $q=0.9$, $q=1.1$ The simulation results demonstrate that chaos indeed exist in the fractional order Chua system with order less than 3. When $q=0.9$, $q=1.1$ chaotic attractors are found and the phase portraits are shown in Figs. 1, 2 and 3, respectively. When $q=.8$ no chaotic behavior is found, which indicates that the lowest limit of the fractional order for this system to be chaotic is $q=0.8-0.9$. Thus, the lowest order we found for this system to yield chaos is 2.7

4. Stability and Controller Design

In this section, stability of the fractional order Chua system is discussed. Then a controller is proposed to meet the stability criteria.

4.1. Stability Region of Fractional Order Systems

Stability of fractional systems has been thoroughly investigated. The necessary and sufficient conditions have been derived in [25]. It has been shown that the stability region of a linear set of fractional order equations with order q, is bounded by a cone, with vertex at the origin, and hat extends into the right half of the s-plane such that it encloses an angle of $\pm q\pi/2$ as shown in Fig. 1. For example, the stability region of the linearized part of equation (3) when $q=0.5$ is the entire s-plane less the area enclosed by the cone making $\pm 45^{\circ}$. Thus, when $q=1$, we get the all-familiar stability region of the integer order system, i.e. the left half-plane where the imaginary axis becomes the border of stability region. Hence, if the eigenvalues of the system Jacobian matrix are placed anywhere outside the cone in Fig. 1, the fractional order system will be stable. Moreover, a controller that stabilizes the integer order system stabilizes the fractional order system. Therefore, a controller that places the characteristic roots in the left half-plane will stabilize both the integer order model as well as all of its fractional versions. However, from performance standpoint, it may be necessary to place the characteristic roots of the fractional system in the right half-plane but outside the stability cone. In this case, the fractional order system is stable whereas the integer order system is unstable.

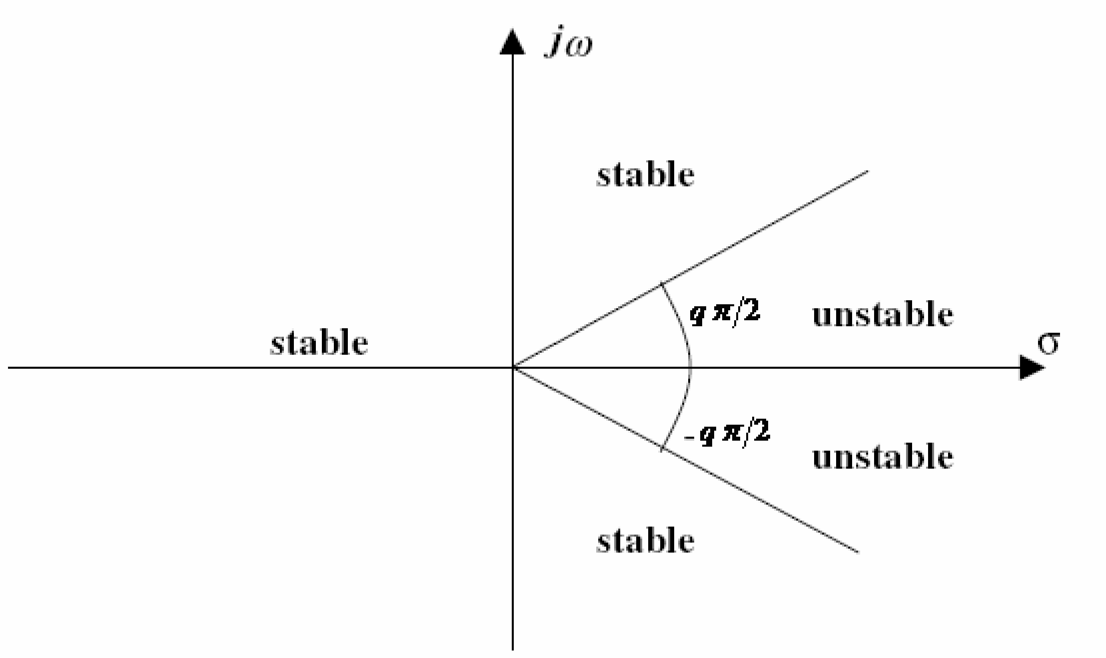

Figure 1. Stability region of fractional order system for order q

4.2. State feedback controller design

State feedback controllers will be proposed to stabilize the fractional chaotic system described by equation (3). The controller design may be based on placing the eigenvalues of the Jacobian system matrices for (3) when $q=1$ in the left half of the s-plane. Alternatively, we will demonstrate how the fractional system can be stabilized by using static gains that place the eigenvalues in the right half-plane but outside the cone described in $\theta = \pm q\pi/2$. The composite fractional system models with a control law are described by:

$$\frac{d^q X}{dt^q} = AX + B_1 f(X) + B_2 u \qquad (4)$$

Where $X^q = [x^q \; y^q \; z^q]$, $f(x) = \alpha x - 2x^3/7$, the matrix $B_1 = [0\;0\;1]^T$ for system (3), and where $[.]^T$ is the transpose of [.]. The input matrix B_2 is chosen so that the pair (A, B_2) for the corresponding system is controllable.

The static gain controller takes the form $u = -Kx$ where $K = [k_1, k_2, k_3]$. It can be seen that with $B_2 = [1\;1\;1]^T$, the Chua model is completely controllable as indicated by the controllability matrix $Q = [B \;\; AB \;\; A^2B]$. The dynamics of the controlled fractional chaotic Chua describe by:

$$\begin{bmatrix} \frac{d^q x}{dt^q} \\ \frac{d^q y}{dt^q} \\ \frac{d^q z}{dt^q} \end{bmatrix} = \begin{bmatrix} -k_1 & -k_2+\alpha & -k_3 \\ -k_1+1 & -k_2-1 & -k_3+1 \\ -k_1 & -k_2-\frac{100}{7} & -k_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix} \left(\frac{x-2x^3}{7}\right) \qquad (5)$$

The controller gains k_1, k_2 and k_3 are chosen such that the eigenvalues of $[A - B_2K]$ are placed outside the cone of angle $\theta = \pm q\pi/2$.

5 Conclusion

In this paper, we have studied the chaotic dynamics of the fractional order Chua system. We found that chaos exists in this system with order less than 2.7. A simple, but effective, linear feedback controller is also designed to stabilize the fractional order chaotic Chua system.

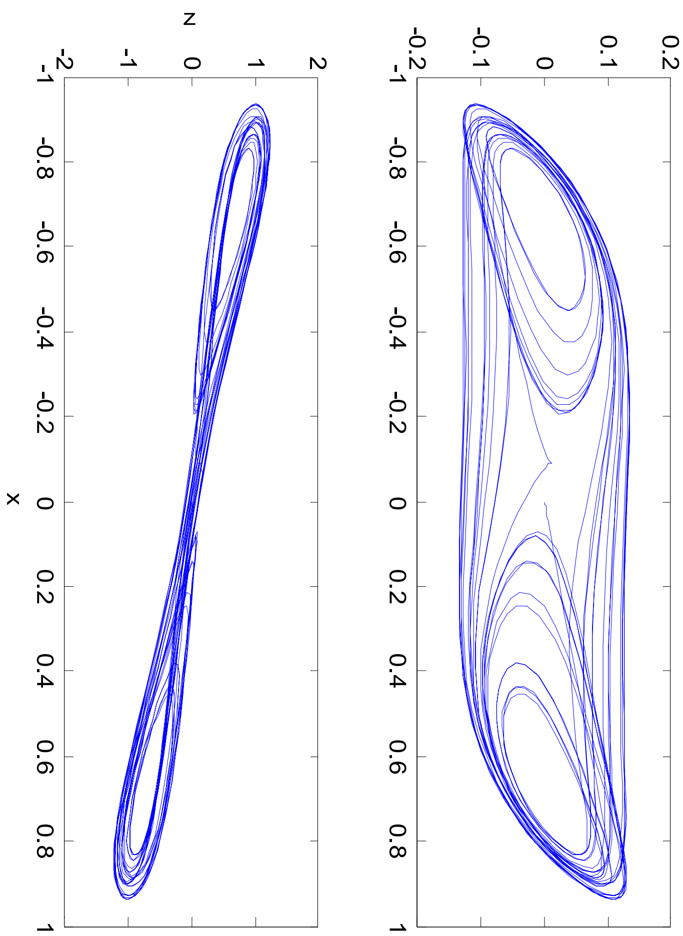

Figure 2. Chaotic attractor of the fractional order Chua system with order $q = .9$ q=0.9 and $\alpha = 12.75$

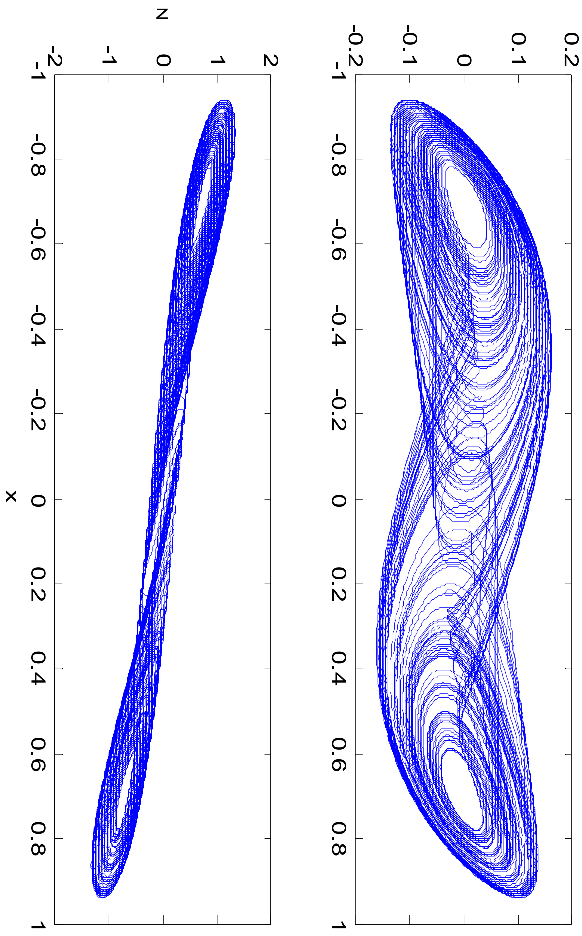

Figure 3. Chaotic attractor of the integer order Chua system with order $q = 1$ and $\alpha = 9.5$

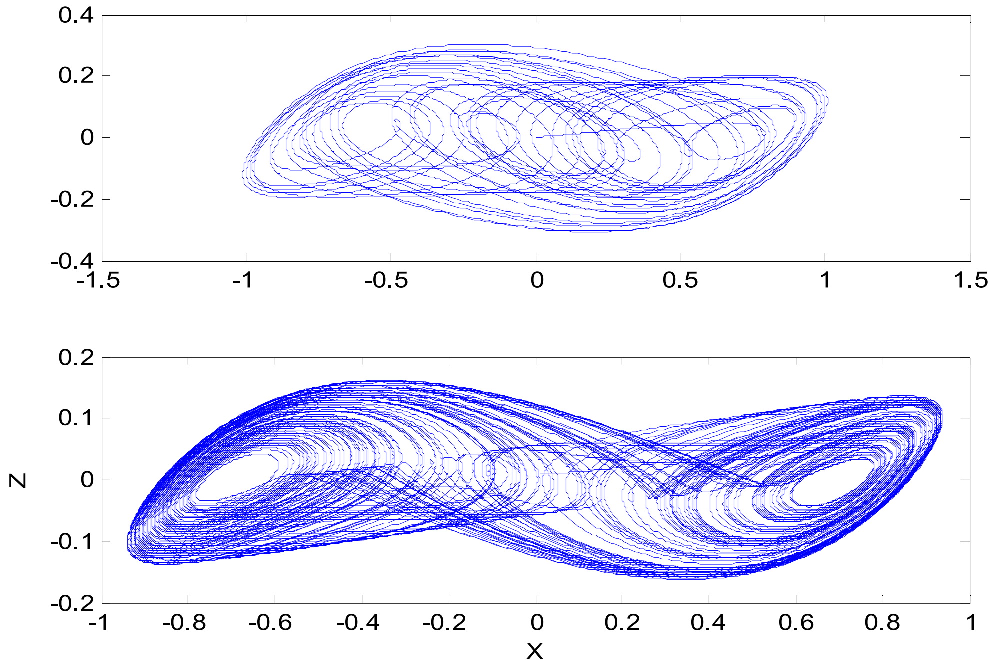

Figure 4. Chaotic attractor of the fractional order Chua system with order $q = 1.1$ and $\alpha = 7$

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