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Multivariable adaptive λ–tracking for nonlinear chemical processes

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Abstract

It is shown that incorporation of a simple 'dead–zone' into the known adaptive high–gain control strategy $u(t) = -k(t)y(t)$, $\dot{k}(t) = \|y(t)\|^2$ for linear systems also yields λ–tracking or λ–stabilization in the presence of output corrupted noise for a large class of reference signals and a large class of *multivariable nonlinear* 'minimum phase' systems with relative degree one and known sign of the high-frequency gain. These results are applied to a chemical reactor showing the practical usefulness of these control laws.

1 Introduction

Chemical processes are characterized by a number of typical properties that imply specific demands on controller design schemes. Besides the fact that usually not the whole state can be measured and that most processes are of multivariable nature, process nonlinearities and large model uncertainties must be considered in this context. In this paper we propose an adaptive high-gain control scheme that specifically addresses these properties and is thus well suited for the control of many chemical processes.

The field of high-gain adaptive control of minimum phase systems has been initiated by [8,9,10,13]. Ubiquitous in the area is the following simple output feedback and adaptation strategy

$$\begin{array}{|rcll|} u(t) & = & -k(t)\,y(t) & \\ \dot{k}(t) & = & \|y(t)\|^2 & ,k(0) = k_0 \in \mathbb{R}. \end{array} \qquad (1.1)$$

This approach has been successfully applied to various classes of minimum phase systems, see [3] for a comprehensive bibliography.

There are only few papers available where the nominal system is assumed to be nonlinear rather than linear with nonlinear uncertainties: [1,2,6,7,11].

The concept of λ-tracking, introduced by [4], is only slightly different to (1.1): λ-stabilization or λ-tracking means, that the output is no longer controlled to a setpoint but into a λ-neighbourhood of the setpoint (or the reference trajectory to be tracked), where $\lambda > 0$ is prespecified and may be arbitrarily small. The main advantage of this control objective as opposed to the standard one, is that a rather general class of nonlinear systems can be treated and that a serious robustness problem of previous control laws will be overcome. This is achieved by a 'dead–zone' which is incorporated into the gain adaptation.

The present paper extends the results of [1]. In [1] we proved that the simple adaptation strategy (1.1) also works for *nonlinear* systems which are multivariable, strong relative degree one, minimum phase with unkown 'sign of the high-frequency gain'. The second goal was to apply the concept of λ-tracking to nonlinear single-input/single-output systems. In the present paper, adaptive λ-tracking is generalized to nonlinear *multi-input/multi-output* systems with known sign of the high-frequency gain.

2 Problem description

Throughout this paper we consider multivariable nonlinear systems in input affine form

$$\left.\begin{array}{rcl} \dot{y}(t) & = & f(t,y(t),z(t)) + g(t,y(t),z(t))\,u(t) \\ \dot{z}(t) & = & h(t,y(t),z(t)) \end{array}\right\} \qquad (2.1)$$

where, for $n, m \in \mathbb{N}$ with $n > m$,

$$\begin{array}{rcl} f & : & \mathbb{R}\times\mathbb{R}^m\times\mathbb{R}^{n-m} \to \mathbb{R}^m, \\ g & : & \mathbb{R}\times\mathbb{R}^m\times\mathbb{R}^{n-m} \to \mathbb{R}^{m\times m}, \quad \text{and} \\ h & : & \mathbb{R}\times\mathbb{R}^m\times\mathbb{R}^{n-m} \to \mathbb{R}^{n-m} \end{array}$$

are assumed to be *Carathéodory functions* [1] with an equilibrium point $(y_e, z_e, u_e) \in \mathbb{R}^m \times \mathbb{R}^{n-m} \times \mathbb{R}^m$, i.e.

$$\begin{aligned} 0 &= f(t, y_e, z_e) + g(t, y_e, z_e)\, u_e \quad \text{and} \\ 0 &= h(t, y_e, z_e) \quad \text{for all } t \in \mathbb{R}. \end{aligned}$$

The state dimension n needs not to be known.

As usual $u(t)$ is considered as manipulated variable and $y(t)$ is the output to be controlled.
System (2.1) is in the so-called Byrnes-Isidori normal form, see [2], where $\dot{z} = h(t, y_e, z)$ is the zero dynamics. Starting out in this form implies that a) the nonlinear system has a relative degree of one and b) that the inputs $u(\cdot)$ do not enter the internal dynamics.

In addition to requiring a relative degree of one we have to demand that the following assumptions on the nonlinear system will hold:

(A1) f is globally Lipschitz at (y_e, z_e), i.e. for some unknown constant $M_f > 0$, independent of $t \in \mathbb{R}$, we have, $\forall\, (t, y, z) \in \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^{n-m}$,

$$\|f(t, y, z) - f(t, y_e, z_e)\| \le M_f \left\| \begin{smallmatrix} y - y_e \\ z - z_e \end{smallmatrix} \right\|.$$

(A2) h is continuously differentiable and globally Lipschitz at y_e, i.e. for some unknown constant $M_h > 0$, independent of (t, z), we have, $\forall\, (t, y, z) \in \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^{n-m}$,

$$\|h(t, y, z) - h(t, y_e, z)\| \le M_h \|y - y_e\|.$$

(A3) g is uniformly bounded away from zero and from above, i.e. there exist positive–definite $P = P^T \in \mathbb{R}^{m \times m}$ and $\sigma_1, M_g > 0$ such that $\forall\, (t, y, z) \in \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^{n-m}$,

$$\begin{aligned} 2\sigma_1 I_m &\le P g(t, y, z) + g(t, y, z)^T P \\ \text{and} & \qquad\qquad\qquad\qquad (2.2) \\ \|g(t, y, z)\| &\le M_g. \end{aligned}$$

P, σ_1, M_g are unknown, only existence is ensured.

(A4) The zero dynamics are uniformly exponentially converging towards η_e, i.e. there exist (unknown) $M, \varepsilon > 0$ such that the solution of

$$\dot{\eta}(t) = h(t, y_e, \eta(t)), \qquad \eta(0) = \eta_0$$

satisfies for all $t \ge 0$, $\eta_0 \in \mathbb{R}^m$

$$\|\eta(t) - \eta_e\| \le M\, e^{-\varepsilon t} \|\eta_0\|.$$

From an application point of view, assumptions (A1) and (A2) can be considered as "technical assumptions".

[1] $\alpha : \mathbb{R} \times \mathbb{R}^q \to \mathbb{R}$ is called a Carathéodory function, if $\alpha(\cdot, x) : t \mapsto \alpha(t, x)$ is measurable on $\mathbb{R}$ for each $x \in \mathbb{R}^q$, and $\alpha(t, \cdot) : x \mapsto \alpha(t, x)$ is continuous on $\mathbb{R}^q$ for all $t \in \mathbb{R}$.

If the system is single-input/single-output, i.e. $m = 1$, then (A3) simplifies to the assumption that $g(\cdot)$ is uniformly bounded away from zero and uniformly bounded from above, i.e.

(A3)' there exist $\sigma_1, \sigma_2 > 0$ such that, for all $(t, y, z) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}^{n-1}$,

$$\sigma_1 \le g(t, y, z) \le \sigma_2.$$

The strongest assumption, and the one that is probably the most difficult to show for an application, is assumption (A4), that requires the system to be globally minimum phase. It is clear, that these assumptions, together with the implied assumption, that the relative degree is one and the inputs do not enter the internal dynamics, are not met by many practical control problems. On the other hand many important practical control problems, like for example many chemical reactors, will meet the requirements. In Section 4 we will demonstrate results from a chemical reactor for which the assumptions can be shown to hold.

3 λ–stabilization and λ–tracking

The control objective is as follows: For a prespecified but arbitrary $\lambda > 0$, guarantee that $y(t) + n(t)$ (where $y(\cdot)$ denotes the output of the system and $n(\cdot)$ may be viewed as a reference signal or noise) asymptotically tends to the ball

$$\mathcal{B}_\lambda(0) := \{x \in \mathbb{R}^m \mid \|x\| < \lambda\}$$

as t tends to ∞. For this modified control objective the control law (1.1) is modified by incorporating a 'dead-zone' into the gain adaptation, see (3.1) below. Moreover, the closed–loop system becomes robust with respect to measurement noise belonging to

$$\mathcal{W}^{1,\infty} := \{f : [0, \infty) \to \mathbb{R} \text{ absolutely continuous on compact intervals and } f(\cdot), \dot{f}(\cdot) \in L_\infty(0, \infty)\}.$$

The result is formulated in a general way so that it also solves the problem of λ–tracking of $\mathcal{W}^{1,\infty}$–signals.

3.1 Theorem

Suppose $\gamma, \lambda, f_e > 0$, $\delta \in \mathbb{R}$. For any multivariable system (2.1) satisfying (A1)–(A4), and $\|f(t, y_e, z_e)\| \le f_e$ for all $t > 0$, the application of the feedback

$$\begin{aligned} u(t) &= -\,[y(t) + n(t)] + \delta \\ \dot{k}(t) &= \begin{cases} \gamma\, (\|y(t) + n(t)\| - \lambda)\, \|y(t) + n(t)\| & \text{, if } \|y(t) + n(t)\| \ge \lambda \\ 0 & \text{, if } \|y(t) + n(t)\| < \lambda \end{cases} \end{aligned} \qquad (3.1)$$

where $n(\cdot) \in \mathcal{W}^{1,\infty}$ is arbitrary, permits, for arbitrary initial conditions $y_0 = y(0) \in \mathbb{R}^m$, $z(0) = z_0 \in \mathbb{R}^{n-m}$, a solution $(y(\cdot), z(\cdot), k(\cdot)) : [0,\omega) \to \mathbb{R}^{n+1}$ to the closed-loop system, and any solution satisfies on its maximal interval of existence, $[0,\omega)$, $\omega \in (0,\infty]$, the properties:

(i) $\omega = \infty$;

(ii) $\lim_{t\to\infty} k(t) = k_\infty$ exists and is finite;

(iii) $y(\cdot), z(\cdot) \in L_\infty(0,\infty)$;

(iv) $y(t) + n(t)$ approaches the ball $\mathcal{B}_\lambda(0)$ as $t \to \infty$.

Theorem 3.1 states that the desired closed-loop objective can be achieved with the simple control strategy (3.1). Note that not much information about the plant to be controlled is needed. The only knowledge required is that the plant satisfies the assumptions — which is, admittandly, not always easy to show, however. Also note that the structure of control law (3.1) is independent of the plant to be controlled. There are three tuning parameters, γ, δ and λ, that can be used to "customize" the feedback for the application at hand and to improve its performance. Guidelines for a suitable choice of values for these tuning parameters are given in [1].
The function $n(\cdot)$ can be interpreted in different ways, e.g. as a reference signal or as a measurement noise. See [1] for details.

Proof of Theorem 3.1:
We use the following notation: For $\rho > 0$ and $P = P^T \in \mathbb{R}^{m\times m}$ positive definite let

$$q := \mu_{\min}(P)^{1/2} \quad \text{and} \quad p := \|P\|^{1/2}$$

$$\begin{aligned} \|x\|_P &:= \sqrt{<x, Px>} \\ D_\rho(w) &:= \begin{cases} \gamma(\|w\|_P - \rho) , & \|w\|_P \geq \rho \\ 0 , & \|w\|_P < \rho \end{cases} \\ d_\rho(w) &:= \begin{cases} \gamma(\|w\| - \rho) , & \|w\| \geq \rho \\ 0 , & \|w\| < \rho \end{cases} \end{aligned}$$

Then, see [4], for all $w \in \mathbb{R}^m$,

$$q\|w\| \leq \|w\|_P \leq p\|w\| \tag{3.2}$$
$$D_{p\lambda}(w) \geq p\, d_\lambda(w) \tag{3.3}$$
$$D_{q\lambda}(w) \leq q\, d_\lambda(w). \tag{3.4}$$

We now proceed in several steps.
(a): The closed-loop system (2.1), (3.1) posesses a solution $(y(\cdot), z(\cdot), k(\cdot)) : [0,\omega) \to \mathbb{R}^{n+1}$, which is maximally extended over $[0,\omega)$ for some $\omega \in (0,\infty)$. This follows from the classical theory of ordinary differential equations.
(b): We shall prove boundedness of $k(\cdot)$ on $[0,\omega)$. Seeking a contradiction, suppose that $k(\cdot) \notin L_\infty(0,\omega)$. Let $t_0 \in [0,\omega)$ such that $k(t) > 0$ for all $t \in [t_0,\omega)$. Set

$$\begin{aligned} w(t) &:= y(t) + n(t) \\ V_\rho(w) &:= \begin{cases} \frac{\gamma}{2}(\|w\|_P - \rho)^2 , & \|w\|_P \geq \rho \\ 0 , & \|w\|_P < \rho . \end{cases} \\ \Theta_\rho(w) &:= \begin{cases} \gamma \frac{\|w\|_P - \rho}{\|w\|_P}\, w , & \|w\|_P \geq \rho \\ 0 , & \|w\|_P < \rho . \end{cases} \end{aligned}$$

Differentiation of the Lyapunov-like candidate $V_\rho(\cdot)$ along the solution of

$$\begin{aligned} \dot w(t) = f(t, w(t) - n(t), z(t)) + g(t, w(t) - n(t), z(t))\,\delta \\ + \dot n(t) - k(t) g(t, w(t) - n(t), z(t))\, w(t) \end{aligned}$$

yields, by (A1), (A3), and (3.2), for all $t \in [t_0,\omega)$,

$$\begin{aligned} &\frac{d}{dt} V_\rho(w(t)) \\ &= D_\rho(w(t)) \frac{1}{\|w(t)\|_P} \langle w(t), P\dot w(t)\rangle \\ &\leq D_\rho(w(t)) \frac{\|P\, w(t)\|}{\|w(t)\|_P} \|f(t, w(t) - n(t), z(t))\| \\ &\quad + D_\rho(w(t)) \frac{\|P\, w(t)\|}{\|w(t)\|_P} \|\dot n(t)\| \\ &\quad - k(t)\, D_\rho(w(t)) \frac{1}{\|w(t)\|_P} \langle w(t), P\, g(t, w(t) - n(t), z(t))\, w(t)\rangle \\ &\quad + D_\rho(w(t)) \frac{1}{\|w(t)\|_P} \|w(t)\|\, \|P\|\, \|g(t, w(t) - n(t), z(t))\,\delta\| \\ &\leq D_\rho(w(t)) \frac{p^2}{q} M_f (\|w(t)\| + \|z(t)\| + M_1) \\ &\quad - k(t)\, D_\rho(w(t)) \frac{\sigma_1}{\|w(t)\|_P} \|w(t)\|^2 + D_\rho(w(t)) \frac{p^2}{q} M_g \delta \\ &\leq D_\rho(w(t)) \frac{p^2}{q} M_f \left(\|w(t)\| + \|z(t)\| + M_1 \frac{p}{\rho} \|w(t)\|\right) \\ &\quad - k(t) \frac{\sigma_1}{p} D_\rho(w(t))\, \|w(t)\| + \frac{p^2}{q} M_g \delta \frac{p}{\rho} D_\rho(w(t))\, \|w(t)\| \\ &\leq M_3\, D_\rho(w(t))\, \|z(t)\| + \left[M_2 - \frac{\sigma_1}{p} k(t)\right] D_\rho(w(t))\, \|w(t)\| \end{aligned} \tag{3.5}$$

where

$$M_1 := \sup_{t\in[0,\infty)} \{M_f (\|n(t) + y_e\| + \|z_e\|) + f_e + \|\dot n(t)\|\} ,$$
$$M_2 := \tfrac{p^2}{q} M_f + \tfrac{p^2}{q} M_f\, M_1 \tfrac{p}{\rho} + \tfrac{p^2}{q} M_g \delta \tfrac{p}{\rho} .$$

Applying Variation-of-Constants to

$$\frac{d}{dt}\tilde z(t) = h(t, y_e, \tilde z(t) + z_e) + h(t, y(t), \tilde z(t) + z_e) - h(t, y_e, \tilde z(t) + z_e) ,$$

where $\tilde{z}(t) = z(t) - z_e$, and using (A4) and (A2), gives

$$\begin{aligned}
\|\tilde{z}(t)\| &\le M e^{-\varepsilon(t-t_0)} \|\tilde{z}(t_0)\| \\
&\quad + \int_{t_0}^{t} M e^{-\varepsilon(t-s)} M_h \|y(s) - y_e\| \, ds \\
&\le M e^{-\varepsilon(t-t_0)} \|\tilde{z}(t_0)\| \\
&\quad + M M_h \int_{t_0}^{t} e^{-\varepsilon(t-s)} \Big[\tfrac{1}{\gamma} \|\Theta_\rho(w(s))\| + \\
&\quad \left(\|y(s)\| - \tfrac{1}{\gamma} \|\Theta_\rho(w(s))\| \right) + \|y_e\| \Big] \, ds ,
\end{aligned}$$

and since, for $\|w\| \ge \rho$,

$$\begin{aligned}
\|y\| - \tfrac{1}{\gamma}\|\Theta_\rho(w)\| &= \|w - n\| - (\|w\|_P - \rho) \tfrac{\|w\|}{\|w\|_P} \\
&\le \|n\| + \tfrac{\rho}{q},
\end{aligned}$$

and

$$\|y\| - \frac{1}{\gamma}\|\Theta_\rho(w)\| \le \|n\| + \frac{\rho}{q},$$

we obtain

$$\|\tilde{z}(t)\| \le M_3 + M_3 \mathcal{L}(\|\Theta_\rho(w(\cdot))\|)(t),$$

whence

$$\|z(t)\| \le M_4 + M_4 \mathcal{L}(\|\Theta_\rho(w(\cdot))\|)(t) \tag{3.6}$$

where

$$\begin{aligned}
M_3 &:= M\|\tilde{z}(t_0)\| + M M_h \tfrac{1}{\gamma} \\
&\quad + M M_h \left[\|n(\cdot)\|_{L_\infty(0,\infty)} \tfrac{\rho}{q} + \|y_e\| \right] \cdot \sup_{t \ge t_0} \int_{t_0}^{t} e^{-\varepsilon(t-s)} \, ds \\
\mathcal{L}(v(\cdot))(t) &:= \int_0^t e^{-\varepsilon(t-\tau)} v(\tau) \, d\tau, \\
M_4 &:= M_3 + \|z_e\| .
\end{aligned}$$

(3.6) together with the fact that $\|\Theta_\rho(w(\cdot))\| \le \frac{1}{q} D_\rho(w)$ and $\|D_\rho(w)\| \le \gamma p \|w\|$, yields

$$\begin{aligned}
&\int_{t_0}^{t} D_\rho(w(s)) \|z(s)\| \, ds \\
&\le M_4 \int_{t_0}^{t} D_\rho(w(s)) \left[1 + \mathcal{L}(\|\Theta_\rho(w(\cdot))\|)(s) \right] ds \\
&\le M_4 \int_{t_0}^{t} D_\rho(w(s)) \left[\frac{p}{\rho} \|w(s)\| + \mathcal{L}(\|\Theta_\rho(w(\cdot))\|)(s) \right] ds \\
&\le M_4 \frac{p}{\rho} \int_{t_0}^{t} D_\rho(w(s)) \|w(s)\| \, ds \\
&\quad + M_4 \|D_\rho(w(\cdot))\|_{L_2(t_0,t)} \, \|\mathcal{L}(\|\Theta_\rho(w(\cdot))\|)\|_{L_2(t_0,t)} \\
&\le M_4 \frac{p}{\rho} \int_{t_0}^{t} D_\rho(w(s)) \|w(s)\| \, ds + M_4 \frac{\|\mathcal{L}\|}{q} \|D_\rho(w(\cdot))\|^2_{L_2(t_0,t)} \\
&\le M_5 \int_{t_0}^{t} D_\rho(w(s)) \|w(s)\| \, ds ,
\end{aligned} \tag{3.7}$$

where

$$M_5 := M_4 \frac{p}{\rho} + M_4 \frac{\|\mathcal{L}\|}{q} \gamma p .$$

Choose $t_1 \in [t_0, \omega)$ such that

$$M_6 - \frac{\sigma_1}{p} k(t) < 0 \qquad \text{for all} \quad t \in [t_1, \omega),$$

where

$$M_6 := M_3 M_5 + M_3 .$$

Then, integration of $V_\rho(w(\cdot))$ over $[t_1, t)$ yields, by (3.5), (3.7) and (3.2), that

$$\begin{aligned}
&V_{p\lambda}(w(t)) - V_{p\lambda}(w(t_1)) \\
&\le M_3 M_5 \int_{t_1}^{t} D_{p\lambda}(w(s)) \|w(s)\| \, ds \\
&\quad + \int_{t_1}^{t} \left[M_3 - \frac{\sigma_1}{p} k(s) \right] D_{p\lambda}(w(s)) \|w(s)\| \, ds \\
&= \int_{t_1}^{t} \left[M_6 - \frac{\sigma_1}{p} k(s) \right] p \dot{k}(s) \, ds \\
&= \int_{k(t_1)}^{k(t)} \left[M_6 - \frac{\sigma_1}{p} \mu \right] p \, d\mu .
\end{aligned} \tag{3.8}$$

Since $k(\cdot)$ is assumed to be unbounded, the right hand side of (3.8) tends to ∞, thus contradicting non-negativeness of $V_{p\lambda}(w(t))$. Therefore, $k(\cdot) \in L_\infty(0, \omega)$ is proved.

(c): Since $k(\cdot) \in L_\infty(0,\omega)$, (3.8) yields boundedness of $V_{p\lambda}(\cdot)$ and hence $w(\cdot), y(\cdot) \in L_\infty(0,\omega)$. Since $\dot{\eta}(t) = h(t, y_e, \eta(t))$ is exponentially stable, an application of Variation-of-Constants to

$$\dot{z}(t) = h(t, y_e, z(t)) + h(t, y(t), z(t)) - h(t, y_e, z(t))$$

and using (A2) yields, for some $M, \varepsilon > 0$,

$$\|z(t)\| \le M e^{-\varepsilon t} \|z(0)\| + M \int_0^t M_h e^{-\varepsilon(t-s)} \|y(s) - y_e\| \, ds ,$$

and therefore $z(\cdot) \in L_\infty(0,\omega)$. By maximality of ω it follows that $\omega = \infty$. Hence (i)–(iii) are proved.

(d): It remains to prove (iv). From (3.5), boundedness of $z(\cdot)$ and $k(\cdot)$, and (3.4) we deduce, for some $M_7 > 0$,

$$\frac{d}{dt} V_{q\lambda}(e(t)) \leq M_7 D_{q\lambda}(w(t))\|w(t)\| \leq M_7 q\dot{k}(t).$$

Therefore, the derivative of the C^1–function

$$W(\cdot) : \mathbb{R}^{m+1} \to \mathbb{R}, \qquad (w,k) \mapsto V_{q\lambda}(w) - [M_7 q + 1] k$$

along the solution components w and k satisfies

$$\frac{d}{dt} W(w(t), k(t)) \leq -\dot{k}(t) = -d_\lambda(w(t))\|w(t)\| \leq 0.$$

Now LaSalle's Invariance Principle for non-autonomous systems, see [7], proves that the ω-limit set of the bounded solution $(y(\cdot), w(\cdot), k(\cdot))$ is contained in $\{(y,w,k) \in \mathbb{R}^{n+1} \mid \|w\| \leq \lambda\}$. This proves (iv) and completes the proof. □

A 'generalization' of Theorem 3.1 to single-input/single-output systems with unknown sign of the high-frequency gain is presented in [1]. For multivariable systems, asymptotic tracking has also been solved for nonlinear systems. However, even for linear systems it is unclear, if a similar result holds for the multivariable case, see [4].

4 Application of multivariable adaptive λ-tracking to CSTR control

In order to demonstrate the practicality of controller (3.1), we consider control of an exothermic continuous stirred tank reactor (CSTR), in which the reaction

$$A \xrightarrow{r_1} B \xrightarrow{r_2} P, \quad B \xrightarrow{r_3} X \tag{4.1}$$

takes place. The desired product P is produced by a consecutive reaction from initial reactant A. Intermediat product B does not only react to product P, but also forms an unwanted isomer X in a parallel reaction.

The flow q that is fed to the reactor contains only substance A with concentration c_{A0} and temperature ϑ_0. The energy flow j_Q is used for cooling the reactor.

The following nonlinear differential equations describe the dynamics of the reactor:

$$\begin{aligned}
\dot{c}_A &= q(c_{A0} - c_A) - k_1(\vartheta) \cdot c_A \\
\dot{c}_B &= -q\, c_B + k_1(\vartheta) \cdot c_A - k_2(\vartheta) \cdot c_B - k_3(\vartheta) \cdot c_B \\
\dot{c}_P &= -q \cdot c_P + k_2(\vartheta) \cdot c_B \\
\dot{\vartheta} &= q(\vartheta_0 - \vartheta) - \tfrac{1}{\rho_P}\Big[k_1(\vartheta) c_A \Delta H_{r_1} + k_2(\vartheta) c_B \Delta H_{r_2} \\
&\quad + k_3(\vartheta) c_B \Delta H_{r_3}\Big] + \tfrac{1}{\rho_P} \cdot j_Q \,.
\end{aligned} \tag{4.2}$$

c_A, c_B, c_P are the concentrations of substances A, B and P respectively and are always positive. The reactor temperature is denoted by ϑ. The reaction velocities k_i are assumed to follow the Arrhenius law

$$k_i(\vartheta) = k_{i0} \cdot \exp\left\{\frac{Ei}{\vartheta}\right\}, \qquad i = 1,2,3, \tag{4.3}$$

where E_i are the activation energies and k_{i0} are the collision factors. The physico–chemical parameters, together with the steady-state data, are given in Table 1.

operating point		parameter data	
q_S	$0.15\,[min^{-1}]$	k_{10}	$1.169 \cdot 10^{10}\ [min^{-1}]$
j_{QS}	$-4.5\,[\frac{KJ}{\ell \cdot min}]$	k_{20}	$1.445 \cdot 10^{11}\ [min^{-1}]$
c_{A0S}	$5\,[\frac{mol}{\ell}]$	k_{30}	$1.689 \cdot 10^{11}\ [min^{-1}]$
ϑ_{0S}	$243.15[K]$	E_1	$-9000\,[K]$
c_{AS}	$3\,[\frac{mol}{\ell}]$	E_2	$-9500\,[K]$
c_{BS}	$0.5\,[\frac{mol}{\ell}]$	E_3	$-9800\,[K]$
c_{PS}	$1.0\,[\frac{mol}{\ell}]$	ρ_P	$1\ [\frac{KJ}{\ell \cdot K}]$
ϑ_S	$253.15[K]$	ΔH_{r_1}	$-40\ [\frac{KJ}{mol}]$
		ΔH_{r_2}	$-20\ [\frac{KJ}{mol}]$
		ΔH_{r_3}	$120\ [\frac{KJ}{mol}]$

Table 1: Physico–chemical parameter and steady–state data for the CSTR.

The reactor feed is assumed to be the output of some upstream unit. Therefore feed concentration c_{A0} and feed temperature ϑ_0 will vary with time and are hence considered as disturbances. Using a proper controller we want to maintain both, the product concentration c_P and the reactor temperatur ϑ, at the steady state values given in Table 1 despite these disturbances. The demanded steady–state control tolerances are

$$\begin{aligned} |c_P - c_{PS}| &\leq 0.02\,\tfrac{mol}{\ell} \\ |\vartheta - \vartheta_S| &\leq 1K. \end{aligned} \tag{4.4}$$

The flow rate q and the cooling power j_Q are available as manipulated variables. We have thus a two–input/two–output control problem.

The open–loop response of the reactor to a disturbance in the feed temperature ϑ_0 by only $-0.5\,K$ is given in Figure 1.

Without control, the temperature goes unstable and drops to a new steady–state value where the reaction extinguishes, even though the Jacobi-linearized reactor is stable.

In order to apply adaptive λ–tracking a number of assumptions have to be satisfied (Compare Sec. 2). Equations (4.2) are given in Byrnes–Isidori–normal form, hence it can be seen immediately that the reactor has a well–defined vector relative degree $r = [1, 1]^T$. Exponential stability of the 2nd order zero–dynamics can be shown for the relevant operating region. In addition, the system equations need to have form (2.1) where the inputs do not enter the internal dynamics. Nonlinear coordinate transformation

$$\begin{aligned} \tilde{c}_A &= \frac{c_P}{c_{A0} - c_A}, & \tilde{c}_B &= -\frac{c_P}{c_B}, \\ \tilde{c}_P &= c_P, & \tilde{\vartheta} &= \vartheta, \end{aligned} \tag{4.5}$$

that is nonsingular for the relevant operating regime, where $c_P > 0$ holds, brings eqs. (4.2) to the required form. Assumption (A3) is however not met for the reactor. It is easy to show that no positive–definite $P = P^T$ exists, that satisfies eq. (2.2). A simple input transformation

$$u_1 = -q, \qquad u_2 = jQ \tag{4.6}$$

(that is a simple gain change in the first input channel), results in a transformed system for which any diagonal, positive–definite P will satisfy condition (2.2). Assumption (A1) and (A2) are also satisfied.

Strictly speaking, Theorem 3.1 cannot be applied for the reactor because the zero–dynamics is not globally exponentially stable. Extension of Theorem 3.1 to the case with non–global domains is the subject of current investigation. For practical applications controller (3.1) will nevertheless achieve closed–loop stability if the region of attraction of the stable zero–dynamics is larger than the operating region considered. This is the case here and, as all other assumptions are satified, closed–loop stability can be expected.

Appropriate scaling of the inputs and outputs is essential in the multivariable case. An even load on the manipulated variables can be achieved by input scaling

$$u = \begin{bmatrix} 1/10 & 0 \\ 0 & 1/3 \end{bmatrix} \overline{u}. \tag{4.7}$$

Because the control tolerances for ϑ and c_P vary by a factor of 50 (compare (4.4)), the outputs are scaled by

$$\overline{y} = \begin{bmatrix} 50 & 0 \\ 0 & 1 \end{bmatrix} y \tag{4.8}$$

in order that a choice of $\lambda = 1$ will guarantee (4.4) asymptotically in a least conservative way. Controller parameter γ is chosen as $\gamma = 100$.

The excellent control performance of the adaptive λ–tracker can be seen from Figs. 2–5. Fig. 2 shows the response of the product concentration c_P and reactor temperature ϑ to a constant disturbance in the feed temperature ϑ_0 by $-20\,K$. This is a severe disturbance. Note that the open–loop reactor goes unstable for a disturbance in ϑ_0 of only $-0.5\,K$. The product concentration c_P stays within the control tolerance for all times. The temperature leaves the required tolerance for only less than half a minute. Fig. 3 shows the manipulated variables, that reach their new steady–state values without excessive action. The high gain parameter is depicted in Fig. 4 The closed–loop behaviour for a joint disturbance in the feed concentration by $-3\,\frac{mol}{\ell}$ and $+10\,K$ is depicted in Fig. 5

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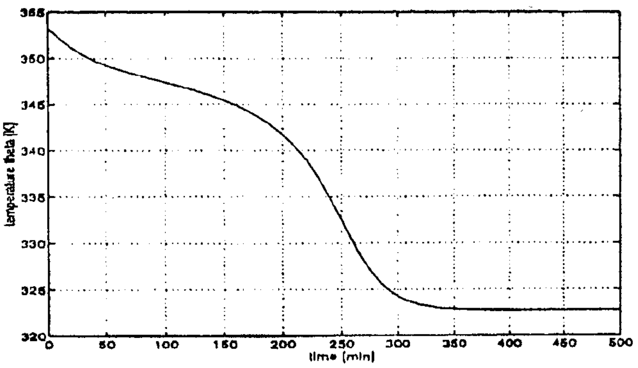

Figure 1: Open-loop response of the reactor temperature ϑ to a disturbance in the feed temperature ϑ_0 by $-0.5\,K$.

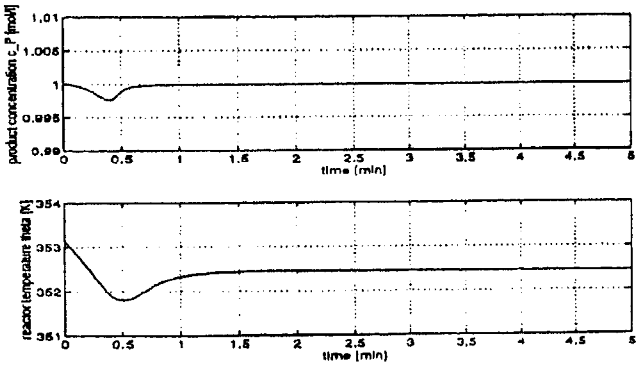

Figure 2: Closed-loop behaviour of the product concentration c_P and reactor temperature ϑ to a disturbance in the feed temperature ϑ_0 by $-20\,K$.

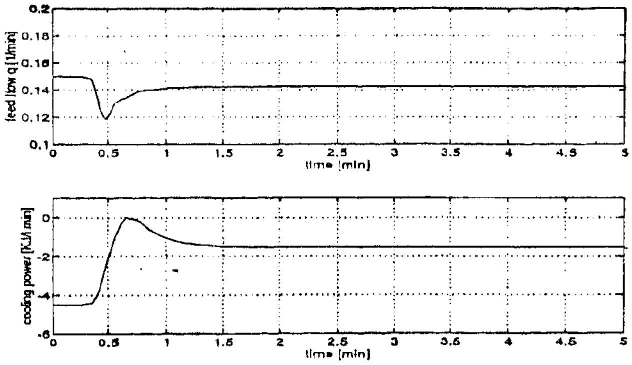

Figure 3: Closed-loop behaviour of the manipulated inputs q and j_Q for a disturbance in the feed temperature by $-20\,K$.

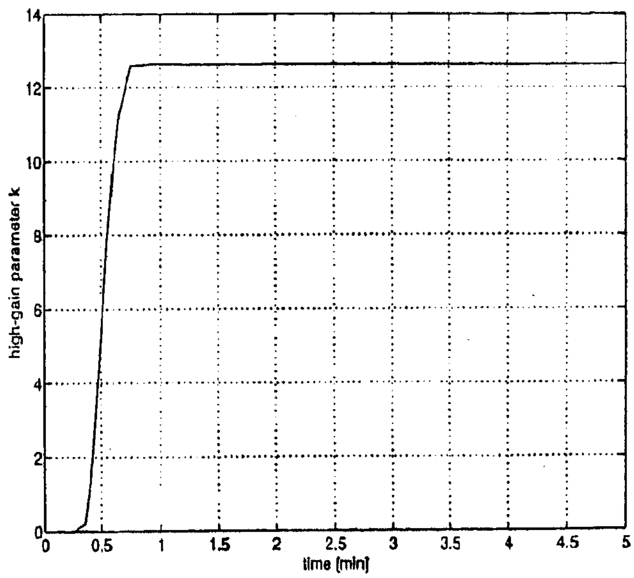

Figure 4: Closed-loop behaviour of the high-gain parameter ϑ to a disturbance in the feed temperature ϑ_0 by $-20\,K$.

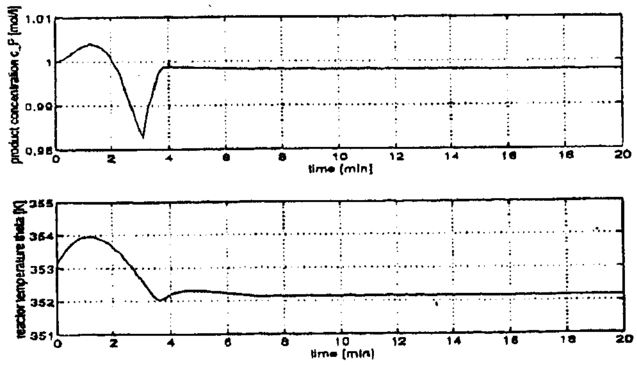

Figure 5: Closed-loop behaviour of the product concentration c_P reactor temperature ϑ to a joint disturbance in ϑ_0 by $+10\,K$ and c_{A0} by $-3\,\frac{mol}{l}$.