

Error model to determine the reliability of parametric measurements

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Abstract

No exact resolution can be assigned to technical parametric measurements owing to their function principle. However, it is often inevitable to analyse the precision of these measurements, e.g. for comparison of methods or for choosing proper measurement parameters. This paper presents a calculation algorithm based on likelihood-theory where both the regular and the random errors will be taken into account to determine the confidence intervals. Thus, the reliability of parametric measurements can be characterized by a single quantity.

Keywords: measurement reliability, probabilities, error model

1. Introduction

Parametric measurements become important especially for fast, in-process applications. It is characteristic for them that the measurands are determined indirectly by a parameter that varies proportional to it. Generally, the measurand and the measuring parameter represent different physical dimensions (concentration/conductibility, roughness/light scattering), their relation is unknown or very complex. Thus, they can not be connected directly and we are not able to assign a certain resolution to our measurement technique. However, it is often necessary to analyse the precision of these methods i.e. for selecting the adequate measuring parameter or parameter-combination.

In this case we can make further tests with the calibrated system; the error resulting from them characterizes the precision that forms a feedback for the choice of suitable parameters or for the improvement of regression model (see figure 1). Conventionally, we calculate two sorts of errors from a test series: the regular (systematic) error that is given by the absolute deviation from the correct value and the random error that can be characterized by the dispersion of measured results [1,2]. It is obvious that both types of errors have influence on the reliability of a parametric measurements. In the followings we present a likelihood calculation where both the regular and the random errors will be taken into account to determine the confidence intervals. Thus, we have a safe quantity for further optimization of the measurement.

2. Interpretation of measurement reliability for parametric processes

We take the calculation of confidence radius generally used in measurement technology as a basis for the precision analysis of parametric methods. Accordingly, the goal is to determine an interval around the measured value, in which the correct value falls with a predefined probability (significance level, generally 95%). Conventionally, for indication of measurement results it is assumed that the average of measured values and the correct value are equal and the confidence interval is calculated from the standard deviation of measurement results. Since the correct values are also known in our cases (we calibrate on this basis), the analysis precision can be further improved. See the figure 2 as an example.

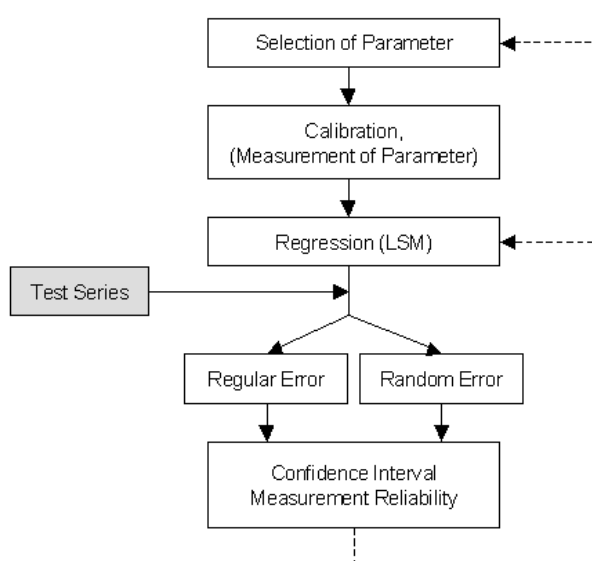


Figure 1. Calibration process of parametric measurements

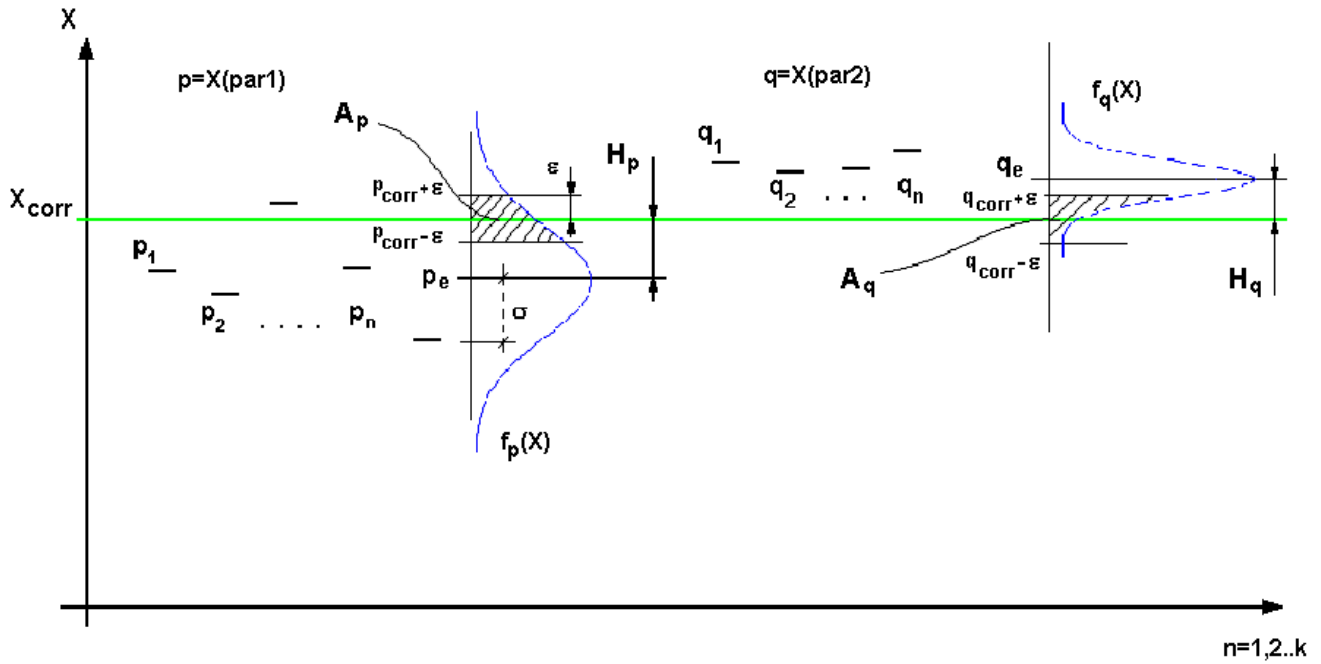


Figure 2. Interpretation of regular and random errors for parametric processes

Let us take two different measuring parameter p and q . We want to determine the same quantity X by both parameters for which we know the correct value: X_{corr} (green horizontal line in diagram 2). However, instead of the correct value we measure the values p_1, p_2, p_n for p and q_1, q_2, q_n for q as a result. The question is, for which parameter is the measurement uncertainty lower?

Suppose the averages of the measurement results are p_e and q_e (expected values) around which the values p_n and q_n form normal distributions. The corresponding density functions are drawn as blue Gauss curves: $f_p(x)$ and $f_q(x)$. The expected values and the correct values are visibly not equal, the deviations are H_p for p and H_q for q that also represent the absolute errors (the both parameters describe the quantity X with different absolute errors and standard deviations). It is obvious: the higher the probability that we measure the correct value the more reliable the parameter is. These probabilities can also be expressed mathematically as follows:

$$P_p(X_{corr} - \varepsilon < X < X_{corr} + \varepsilon) = \int_{X_{corr} - \varepsilon}^{X_{corr} + \varepsilon} f_p(X) dX = A_p \quad (1)$$

$$P_q(X_{corr} - \varepsilon < X < X_{corr} + \varepsilon) = \int_{X_{corr} - \varepsilon}^{X_{corr} + \varepsilon} f_q(X) dX = A_q \quad (2)$$

where $\pm \varepsilon$ denotes an arbitrary small interval around the correct value. The values P_p and P_q correspond to the areas A_p and A_q hatched in the diagram. It can occur that although $H_q < H_p$, for the above-mentioned probabilities applies: $P_q < P_p$. That means, although parameter q operates with a lower absolute error, this deviation emerges „stable“ because of the low dispersion and the probability that we measure the correct value is lower.

It points out that the parameter reliability might be defined by the probability that the measured value falls into an interval around the correct value. If we want to describe the measurement precision by a confidence radius generally used in the practice we have to reverse the above statement: we set the probability to the usual significance level of 95% and try to calculate the corresponding confidence interval $\pm r$. This case is shown in figure 3: X denotes an arbitrary measurand, $F_{m,s}(X)$ is the distribution function with the expected value m and dispersion s ; $f_{m,s}(X)$ is the corresponding density function.

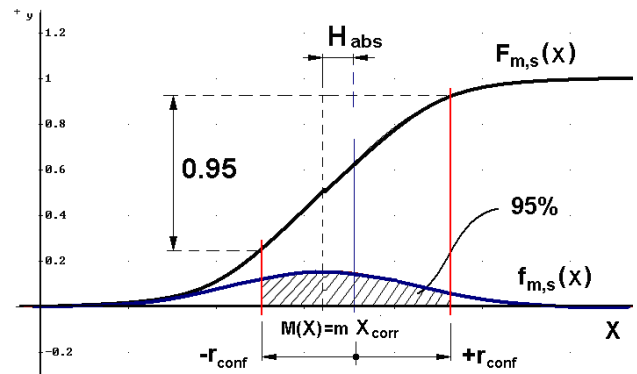


Figure 3. Confidence interval for parametric processes

Now it is to calculate which radius $\pm r$ is to take around a single (one-shot) measured value in order that the correct value will fall in this interval with a probability of 95%. Mathematically expressed:

$$r = ? \quad \text{so that} \quad P(X - r < X_{corr} < X + r) = 0.95 \quad (3)$$

After consideration it is obvious: since X is a random variable and X_{corr} is a constant value, the following relation applies [3]:

$$P(X - r < X_{corr} < X + r) = P(X_{corr} - r < X < X_{corr} + r) \quad (4)$$

On this basis the probability (3) can be calculated from the density function of the measured parameter as:

$$\begin{aligned} P(X - r < X_{corr} < X + r) &= \int_{X_{corr}-r}^{X_{corr}+r} f_{m,s}(X) dX = \\ &= F_{m,s}(X_{corr} + r) - F_{m,s}(X_{corr} - r) = 0.95 \end{aligned} \quad (5)$$

That means that the confidence radius searched around the measured value is equal to an interval around the correct value where the area under the density function is 0.95 (see the hatched region in diagram 3).

3. The computing algorithm

The calculation method presented previously is suitable to express the measurement uncertainty produced by all errors with a single quantity: with the confidence interval calculated by formula (5). According to this the precision of a parametric measurement is to determine as follows:

1. Completion of a test series by the calibrated parametric method with measurements as many as possible ($n > 10$). Determination of measured values by the regression model.
2. Calculation of expected values, empirical dispersions and absolute errors from the measured values. Fitting of normal distribution to the measured values and determination of density function (if we are using sample means for the measurement we should calculate the dispersion accordingly: $s_{\bar{x}} = s_x / \sqrt{n}$)

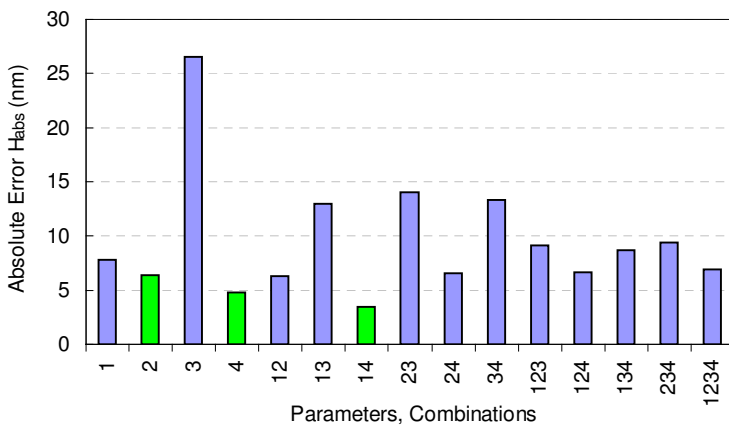


Figure 4. Absolute errors for each parameter combination

3. Setting of a small start interval $\pm r$ around the correct value. Enlarging the interval in small steps Δr and calculation of probabilities $P(X_{corr} - r < X < X_{corr} + r)$. Continuation of the iteration procedure up to the value $P=0.95$. The confidence radius for $P=0.95$ provides the precision of the parametric measurement on the given significance level.

We should choose the start interval and the step wide of iteration procedure Δr according to the measuring parameter. In step 2. we can calculate the probabilities P according to formula (5) by integration of the fitted density function or by the error function as follows:

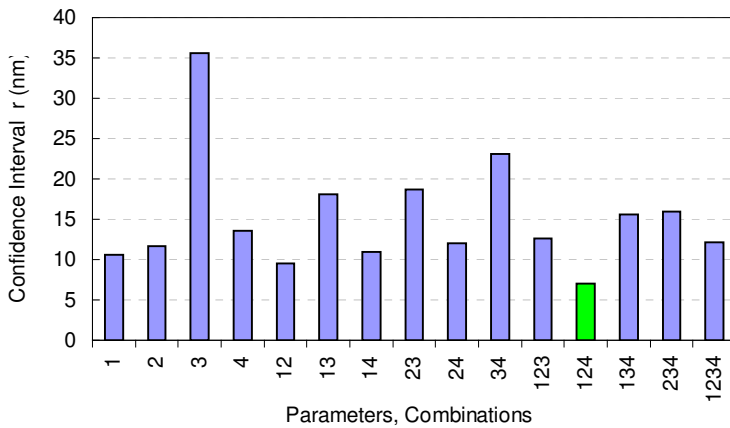
$$P = \frac{1}{2} \operatorname{erf}\left(\frac{\sqrt{2}(\Delta + r)}{2s}\right) - \frac{1}{2} \operatorname{erf}\left(\frac{\sqrt{2}(\Delta - r)}{2s}\right) \quad (6)$$

where $\Delta = X_{corr} - m$

4. An example: comparison of parameters

In the followings we present the evaluation method through an example. We would like to calibrate a surface analysing process where the roughness will be determined from the statistical properties of coherent intensity pattern reflected from the surface (so called laser speckle method [4]). We selected four characteristic properties as measuring parameters: 1. intensity 2. contrast 3. 2D standard deviation and 4. sum of pixel differences (deviation of intensity patterns). By experiences, all the four quantities are dependent on the roughness, they change according to it. In the first step we calibrated the setup by a probe series and we calculated the regression model to each parameter.

P	Habs (nm)	P	Habs (nm)
1	7.8	24	6.5
2	5.3	34	13.2
3	26.5	123	9.1
4	2.7	124	4.9
12	6.3	134	8.6
13	12.9	234	9.3
14	4.3	1234	6.8
23	14		



P	r (nm)	P	r (nm)
1	10.6	24	12
2	11.7	34	23.1
3	35.6	123	12.6
4	13.5	124	7
12	9.5	134	15.6
13	18.1	234	15.9
14	11	1234	12.1
23	18.6		

Figure 5. Calculated confidence intervals for each parameter combination

Now we wish to choose a parameter or parameter combination that results the most reliable measurement. Thus, we measured every probe 25 times again with the calibrated setup without doing any changes to the settings. By this test series we calculated the absolute errors for each parameter and parameter combination. The results are shown in figure 4.

Note that parameters 2, 4 and the combination 14 provided the lowest absolute errors. If we carry out the presented evaluation process in these cases we obtain the confidence intervals as plotted in figure 5 (averages over the whole roughness range).

If we compare the results of figures 4 and 5 it appears that the orders set by the absolute errors and by the confidence intervals are different. The lowest confidence interval (± 7 nm) is to reach by the combination 124, therefore this case provides the highest reliability for our measurement.

5. Conclusions

In this study we presented a computing algorithm by which the reliability of parametric measurements can be determined by the results of test series after the calibration. The computing is based on the likelihood theory and focuses on the probability that we measure the correct value under the given conditions. The measurement reliability is characterized by the confidence interval, but since also the correct values are known, both the regular and the random errors can be taken into account. Hence we have a safe quantity available for the comparison of reliabilities of measuring parameters or for the optimization of measuring models.

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