λ-TRACKING FOR EXOTHERMIC CHEMICAL REACTIONS WITH SATURATING INPUTS

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Abstract

We consider adaptive regulation of the temperature for exothermic chemical reactors. These reactors may exhibit multiple steady states, and the relevant operating point could be an unstable open-loop equilibrium. The adaptive controller combines the so called λ-tracking approach with a feedback which obeys a saturation; it is simple in its design, does not invoke an observer, and can cope with measurement noise. Tracking is achieved up to any pre-specified accuracy, the choice of which is left to the designer. The class of exothermic chemical reactors to which the controller is applicable is considerably larger than the classes to which robust (non-adaptive) global stabilisers, previously studied in the literature, can be applied. The adaptive control strategy does not require any knowledge of the systems parameters and does not invoke an internal model. It is only assumed that the reference temperature to be tracked is feasible. The problem of temperature tracking is solved both locally, i.e. the temperature must lie in a bounded interval, and globally.

1 Introduction

In this paper, we consider input-constrained adaptive control for a class of nonlinear systems arising as models for controlled exothermic chemical reactors. Our objective is set-point control of temperature. Since the rate of conversion should be economically profitable, the set-point is often close to a hyperbolic equilibria of the open loop system.

In chemical engineering, an analysis of the dynamics and the control of exothermic continuous stirred tank reactors (ExCSTRs) originated in the work by Aris and Admudson [2]. More recently, and of relevance here, Viel et al. [10] proposed a state feedback controller including an observer which can globally stabilise the temperature of ExCSTRs. These controllers require full or partial knowledge of the state and since some states are difficult to measure these controllers can be difficult to realize. Jadot et al. [8]. have derived adaptive dynamic output feedback controllers by modifying standard PI type controllers. This controller, which also uses an anti-windup scheme, is more complicated than our control strategy. Similar results have been obtained by Alvarez-Ramirez and Fermat [1]. But there, unlike our approach, the proposed controller requires a minimum phase assumption.

We combine adaptive λ-tracking, as introduced by Ilchmann and Ryan [5], with the results on non-adaptive global stabilisation of ExCSTRs under input constraints achieved by Viel et al. [9]. Here λ stands for an arbitrarily small but prespecified accuracy of the bounded tracking error. The adaptive controllers introduced in this paper generalise - in the sense that tracking is not achieved asymptotically but for arbitrary small and prespecified accuracy λ - and extend the robust global stabilisation results obtained in [9]: a considerably bigger class of reactor models is allowed (e.g. conservation of mass is not necessary for the model but a dissipativity condition suffices), "small" noise corrupting the temperature measurement is tolerated, and parameters of the system model are not invoked in the controller. The present approach differs in an essential way from the standard λ-tracking approach, see for example [5]: Whilst in the latter work the difficult part lies in proving the boundedness of the closed-loop trajectories, for reactors of the form (1.1) this boundedness is a consequence of the properties of reactor models. However, we encounter new difficulties because of the input saturation.

We consider the class of systems

$$\left.\begin{aligned} \dot{x}(t) &= C\, r\big(x(t),T(t)\big) + d\,[x^{\mathrm{in}} - x(t)] \\ \dot{T}(t) &= b^T r\big(x(t),T(t)\big) - q\,T(t) \;+\; u(t), \end{aligned}\right\} \tag{1.1}$$

where, for $n \in \mathbb{N}$ and $n > m \in \mathbb{N}$, the constants and variables denote

$x(t)$ - concentrations of the n chemical species,

$x^{\text{in}} \in \mathbb{R}^n_{\geq 0}$ - constant feed concentrations,

$C = [c_1, \ldots, c_m] \in \mathbb{R}^{n\times m}$ - stoichiometric matrix,

$b \in \mathbb{R}^m_{\geq 0}$ - coefficients of the exothermicity,

$d > 0$ - dilution rate,

$q > 0$ - heat transfer rate between heat exchanger and reactor,

$T(t)$ - temperature of the reactor,

and the function

$$r(\cdot,\cdot) : \mathbb{R}^n_{\geq 0} \times \mathbb{R}_{>0} \to \mathbb{R}^m_{\geq 0} \quad \text{is locally Lipschitz with } r(0,T) = 0 \text{ for all } T > 0 \tag{1.2}$$

and represents the reaction kinetics. Note that the volume of the culture is constant since the inflow rate and outflow rate are equal.

The following structural assumptions of (1.1) are assumed.

(A1) $\mathbb{R}^{n+1}_{>0}$ is positively invariant under (1.1) for any bounded and locally integrable $u(\cdot) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$.

(A2) There exists $\gamma \in \mathbb{R}^n_{>0}$ such that $\gamma^T c_i \leq 0$ for all columns $c_1, \ldots, c_m$ of the stoichiometric matrix C.

(A3) For $T^* > 0$ there exist $0 < \underline{T} < T^* < \overline{T}$, $\rho > 0$, $0 < \underline{u} < \overline{u}$, such that $0 < \underline{u} + \rho < qT - b^T r(x,T) < \overline{u} - \rho$ for all $(x,T) \in \bar{\Omega}_\gamma \times [\underline{T}, \overline{T}]$,

where

$$\Omega_\gamma := \{x \in \mathbb{R}^n_{>0} \,|\, \gamma^T x < \gamma^T x^{\text{in}}\},$$

and $\bar{\Omega}_\gamma$ denotes its closure. The assumption (A1) is "natural" for reactors – concentrations and temperature should not become zero once they are positive. In Section 2 we show that the special form of a single reaction reactor (2.1), so that $n = 2$, satisfies this assumption automatically. For reactors with higher multiple coupled reactions, sufficient conditions for (A1) are given in terms of specific rates (see e.g. Proposition 6 in Ilchmann and Weirig [6]). If (1.1) satisfies the law of conservation of mass, then there exists some $\gamma \in \mathbb{R}^n_{>0}$, so that $\gamma^T c_i = 0$ for all the column vectors c_i in the stoichiometric matrix C, and so in particular (A2) is satisfied. This result can already be found implicitly in the standard monograph by Gavalas [4], and it is also assumed. However, if C does not represent the *exact* stoichiometric relationships between the different species, then the model does not satisfy the conservation of mass, but it might still be realistic since all "essential" reactions are obeyed. For this approach, which was taken in Bastin and Dochain [3], [6] developed the concept of 'non-cyclic processes', which ensures dissipativity of mass. Non-cyclic processes satisfy (A2), and therefore in this paper we assume (A2) as a relaxation of the conservation of mass assumption. Note that by continuity of $r(\cdot,\cdot)$, (A3) is a consequence of the following weaker assumption:

(A3′) For $T^* > 0$ there exist $0 \leq \underline{u} < \overline{u}$, such that, $0 \leq \underline{u} < qT^* - b^T r(x,T^*) < \overline{u}$ for all $x \in \Omega_\gamma$.

Our reasons for working with (A3) are two-fold: The introduction of ρ makes the exposition in the proofs clearer; in some of the results we need to use explicit knowledge of $[\underline{T}, \overline{T}]$ so that (A3) holds for some ρ. (A3′) is simply a feasibility assumption arising because of the saturation of the non-negative input $u(\cdot)$ at $\underline{u}$ and $\overline{u}$. The assumption (A3′) coincides with (H3) in [9].

The control objective is to regulate the temperature $T(t)$ towards a prespecified neighbourhood of a given reference temperature T^*. The actual error

$$\hat{e}(t) = T^* - T(t)$$

may be corrupted by noise $n(\cdot)$, so that the measured error becomes

$$e(t) = T^* - T(t) + n(t).$$

The noise signal $n(\cdot) : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is a bounded and continuous function.

The control objective will be achieved by the so called λ-tracker, i.e.

$$\left.\begin{aligned} e(t) &= T^* - T(t) + n(t), \\ u(t) &= \text{sat}_{[\underline{u},\overline{u}]}\big(\beta(t)\, e(t) + u^*\big), \\ \dot{\beta}(t) &= \gamma \begin{cases} (|e(t)| - \lambda)^l, & \text{if} \quad |e(t)| > \lambda \\ 0, & \text{if} \quad |e(t)| \leq \lambda \end{cases} \end{aligned}\right\} \tag{1.3}$$

where $l \geq 1$, $\lambda, \gamma, \beta(0) > 0$ are design parameters, $u^* \in (\underline{u}, \overline{u})$ is a constant offset, and

$$\text{sat}_{[\underline{u},\overline{u}]}(\eta) := \begin{cases} \underline{u}, & \text{if} \quad \eta < \underline{u} \\ \eta, & \text{if} \quad \eta \in [\underline{u}, \overline{u}] \\ \overline{u}, & \text{if} \quad \eta > \overline{u}. \end{cases}$$

Note the simplicity of the adaptive λ-tracker. It consists of a proportional error feedback with saturation, and the time-varying proportional gain $\beta(\cdot)$ is determined adaptively by the error measurement alone.

We also consider a non-adaptive version of (1.3) where the feedback law is replaced by

$$u(t) = \text{sat}_{[\underline{u},\overline{u}]}\big(\beta^*\, e(t) + u^*\big), \tag{1.4}$$

for some $\beta^* > 0$. Although this non-adaptive strategy is possibly conservative since β^* needs to be sufficiently large, it is useful because it simplifies (1.3). Note that we give explicit lower bounds for β^* in terms of weak systems data.

Finally, we consider (1.3) but with a so called σ-modification of the gain adaptation, i.e., for some $\sigma, \beta^* > 0$,

$$\begin{aligned} \dot{\beta}(t) = \; & -\sigma[\beta(t) - \beta^*] \\ & + \gamma \begin{cases} (|e(t)| - \lambda)^l, & \text{if} \quad |e(t)| > \lambda \\ 0, & \text{if} \quad |e(t)| \leq \lambda. \end{cases} \end{aligned} \tag{1.5}$$

The advantage of (1.3) over (1.4) is that the gain is found adaptively. However, temporary disturbances will clearly lead to an increase in the gain. Whilst this higher gain will produce faster rejection of the disturbance, to retain this higher gain, as (1.3) would, when the temporary disturbances have passed is not satisfactory. The σ-modification (1.5) has the effect of decreasing the gain back towards β^*.

Throughout the paper we assume that the saturation bounds, the offset, the temperature setpoint, and λ satisfy

$$\begin{array}{l} 0 \leq \underline{u} < u^* < \overline{u}, \qquad 0 < \lambda < \overline{T} - T^*, \\ 0 < \underline{T} < T^* < \overline{T}. \end{array} \tag{1.6}$$

For the proofs of the results in Sections 2, 3 and 4 see Ilchmann et al. [7].

2 Local λ-setpoint control – single reaction

In this section, we consider a *first order* exothermic chemical reactor and regulation of its temperature by λ-tracking without noise corrupting the output measurement, i.e. $n(\cdot) \equiv 0$. An application of the λ-tracker (1.3) will force the temperature $T(t)$ towards a λ-strip around the setpoint T^*. The dynamics of a first order exothermic reaction may be described by mass and energy balances of the reaction process. Specifically, we assume a model for a single reaction of the form

$$\left.\begin{array}{rcrcl} \dot{r}(t) &=& -k\big(T(t)\big)\, r(t) &+& d\,[r^{\rm in} - r(t)] \\ \dot{p}(t) &=& k\big(T(t)\big)\, r(t) &-& d\,p(t) \\ \dot{T}(t) &=& b\,k\big(T(t)\big)\, r(t) &-& q\,T(t) \;+\; u(t). \end{array}\right\} \tag{2.1}$$

Here $b > 0$ denotes the exothermicity of a reaction $A \longrightarrow B$, $r^{\rm in}$ is the constant feed rate, and the reaction kinetics are given by a locally Lipschitz function $k(\cdot) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ with $k(0) = 0$. A typically example of $k(\cdot)$ is the Arrhenius law $k(T) = k_0 e^{-\frac{E}{RT}}$, where k_0 is a constant factor, E is the activation energy, and R is the Joule constant.

The function $k(\cdot)$ and the positive constants d, q and b are unknown.

For the class of systems (1.1), the feasibility assumption (A3) becomes:

(A3″) There exist $\rho > 0$ and $0 < \underline{T} < T^* < \overline{T}$ such that, $0 < \underline{u} + \rho < q\,T - b\,k(T)\,r < \overline{u} - \rho$ for all $(r, T) \in [0, r^{\rm in}] \times [\underline{T}, \overline{T}]$.

We show that the λ-tracker (1.3) with $n(\cdot) \equiv 0$ is universal for the class of systems (2.1) satisfying (A3″) in the sense that the closed-loop system (1.3), (2.1) has bounded trajectories, the gain adaptation converges to a finite limit and, most importantly, the temperature converges to a λ-neighbourhood of the setpoint T^*. This result is achieved locally in the sense that the initial temperature must lie inside $(0, \overline{T})$ and a condition imposed on the initial gain in terms of the feasibility assumption ensures that $(0, \overline{T})$ is positively invariant:

Theorem 2.1. Suppose (1.6) and (A3″). Define

$$\Omega_{r^{\rm in}} := \big\{(r,p) |\; r > 0, p > 0, r + p < r^{\rm in}\big\}.$$

Then the application of the λ-tracker (1.3) with $n(\cdot) \equiv 0$ to any system (2.1) yields, for any initial data

$$\begin{array}{l} \big(r(0), p(0)\big) \in \Omega_{r^{\rm in}}, \qquad T(0) \in (0, \overline{T}), \\ \beta(0) \geq [u^* - \underline{u}]/[\overline{T} - T^*], \end{array} \tag{2.2}$$

a closed-loop system with unique solution

$$\big(r(\cdot), p(\cdot), T(\cdot), \beta(\cdot)\big) : \mathbb{R}_{\geq 0} \longrightarrow \Omega_{r^{\rm in}} \times (0, \overline{T}) \times \mathbb{R}_{>0}$$

on the whole time axis $\mathbb{R}_{\geq 0}$ and, moreover,

(i) $\lim\limits_{t\to\infty} \beta(t) = \beta_\infty \in \mathbb{R}_{\geq 0}$, i.e. the gain adaptation converges,

(ii) $\lim\limits_{t\to\infty} \operatorname{dist}\Big(|T^* - T(t)|,\, [0, \lambda]\Big) = 0$, i.e. the temperature $T(t)$ tends to the λ-neighbourhood of the setpoint T^*. □

The non-adaptive controller suggested in [9] for the system (2.1) has the form

$$u(t) \;=\; \operatorname{sat}_{[\underline{u}, \overline{u}]}\Big(\beta^*(T^* - T(t)) + qT(t) - bk(T(t))r(t)\Big).$$

Here $\beta^* \geq 0$ needs to be sufficiently large, the controller requires explicit knowledge of the reaction kinetics and the feasibility assumption (A3″) is assumed. Note that the λ-tracker does not require any knowledge of the typically uncertain reaction kinetics nor any other system parameters, and that a sufficiently large gain is found adaptively. Note that we only achieve λ-tracking whilst in [9] asymptotic tracking of the setpoint T^* is achieved. However, $\lambda > 0$ can be chosen arbitrarily small and since there is usually uncertainty in the measurement anyway, λ-setpoint control is appropriate.

3 Local tracking – multiple reactions

In this section, the local tracking result in Theorem 2.1 is extended to multiple reactions, i.e. reactors of the form (1.1) satisfying (A1)–(A3). Significantly we also allow for noise corrupting the measurement.

Theorem 3.1. Suppose (1.6), (A1)–(A3), and the continuous noise satisfies

$$\sup_{t\geq 0} \{|n(t)|\} =: \|n\|_\infty \;<\; \min\big\{\overline{T} - T^*,\; \lambda/2\big\}. \tag{3.1}$$

Then the application of the λ-tracker (1.3) to any system (1.1) yields, for any initial data

$$\begin{array}{l} x(0) \in \Omega_\gamma, \quad T(0) \in (0, \overline{T}), \\ \beta(0) \geq [u^* - \underline{u}]/[\overline{T} - T^* - \|n\|_\infty], \end{array} \tag{3.2}$$

a closed-loop system with unique solution

$$\big(x(\cdot), T(\cdot), \beta(\cdot)\big) \;:\; \mathbb{R}_{\geq 0} \quad \longrightarrow \quad \Omega_\gamma \times (0, \overline{T}) \times \mathbb{R}_{>0}$$

defined on the whole time axis $\mathbb{R}_{\geq 0}$ and, moreover,

(i) $\lim_{t\to\infty} \beta(t) = \beta_\infty \in \mathbb{R}_{\geq 0}$, i.e. the gain adaptation converges,

(ii) $\lim_{t\to\infty} \operatorname{dist}\Big(|T^* - T(t)|,\ [0, \lambda + \|n\|_\infty]\Big) = 0$, i.e. the temperature $T(t)$ tends to the $[\lambda + \|n\|_\infty]$-strip $\big[T^* - [\lambda + \|n\|_\infty],\ T^* + [\lambda + \|n\|_\infty]\big]$ as $t \to \infty$. □

Note that the only information needed for the λ-tracker to work is sufficiently large initial condition of the gain parameter $\beta(0)$ in terms of the upper feasibility bound $\overline{T}$ and $\|n\|_\infty$. This is not a restriction since $\|n\|_\infty$ is known and in the local case we assume $T(0) < \overline{T}$, so that knowledge of $\overline{T}$ is needed.

4 Global tracking – multiple reactions

The main result of the previous section, Theorem 3.1, has the shortcoming that it is local in the sense that the initial temperature must lie inside $(0, \overline{T})$. This shortcoming can, under adverse temporary disturbances to the reaction, lead to a problem of thermal runaway in that the reaction dynamics are attracted to another stable, but undesirable, equilibrium. See the simulations in Figure 1. Due to the given input saturations, it may be impossible to reduce the temperature of the reaction from such equilibria by control of the temperature alone. This problem already arises in the non-adaptive case, see the example and simulations in [9]. To overcome this problem in [9] an additional input action is introduced which has a cooling effect if the temperature is too large. For example in the single reaction case, they consider the model

$$\left.\begin{array}{rcrcl} \dot r(t) &=& -k\big(T(t)\big)\, r(t) &+& d\,[v(t) - r(t)] \\ \dot p(t) &=& k\big(T(t)\big)\, r(t) &-& d\,p(t) \\ \dot T(t) &=& b\,k\big(T(t)\big)\, r(t) &-& q\,T(t) \;+\; u(t), \end{array}\right\} \tag{4.1}$$

and set the additional input $v(\cdot)$ to

$$v(t) \;=\; \begin{cases} r^{\rm in}, & \text{if} \quad T(t) \in (0, \overline{T}) \\ 0, & \text{if} \quad T(t) \in [\overline{T}, \infty). \end{cases} \tag{4.2}$$

The effect of (4.2) is that, whenever $T(t) \geq \overline{T}$, then $\dot r(t) \leq -d\,r(t)$ and hence $r(\cdot)$ decreases. If $T(t) \geq \overline{T}$ is maintained, then eventually $r(t)$ is small enough to yield a decrease in temperature.

It is not clear to us whether the resulting discontinuous closed-loop system has a solution. Our approach replaces (4.2) by a simple piecewise linear control for $v(\cdot)$ which connects 0 and $r^{\rm in}$. A similar approach works for multiple reactions if we specify the model as follows.

The state is divided into two substates $x_1(t)$ and $x_2(t)$, so that all reactants are collected in x_1. Applying a permutation to (1.1) yields a system of the form

$$\begin{array}{rcrcl} \dot x_1(t) &=& C^1\, r\big(x(t), T(t)\big) &+& d\,[x_1^{\rm in} - x_1(t)] \\ \dot x_2(t) &=& C^2\, r\big(x(t), T(t)\big) &+& d\,[x_2^{\rm in} - x_2(t)] \\ \dot T(t) &=& b^T\, r\big(x(t), T(t)\big) &-& q\,T(t) \;+\; u(t), \end{array}$$

where $C^1 \in \mathbb{R}^{(n-m)\times m}$, $C^2 \in \mathbb{R}^{m\times m}$, $x_1^{\rm in} \in \mathbb{R}^{n-m}_{\geq 0}$, $x_2^{\rm in} \in \mathbb{R}^m_{\geq 0}$. Since x_1 represents the reactants of the chemical reactor, it follows that each entry of C^1 is non-positive, i.e. $C^1 \in \mathbb{R}^{(n-m)\times m}_{\leq 0}$. The assumption (1.2) on the reaction kinetics must be strengthened for multiple reactions as follows:

(A4) There exists a continuous function $\kappa(\cdot) : \Omega_\gamma \to \mathbb{R}_{\geq 0}$ such that, $\|r(x,T)\| \leq \kappa(x)\, T$ for all $(x,T) \in \Omega_\gamma \times \mathbb{R}_{>0}$, and $\lim_{x\to\hat x} \kappa(x) = 0$ uniformly for any $\hat x \in \mathbb{R}^n_{\geq 0}$ with $\hat x_i = 0$ for some $i \in \{1, \ldots, n\}$.

Note that (A4) encompasses the following class of systems considered in [9].

$$b^T\, r(x,T) \;=\; \sum_{i=1}^m b_i\, k_i(T)\, \varphi_i(x)$$

where each $b_i > 0$, each function $T \mapsto k_i(T)$ is positive, bounded and globally Lipschitz, and each function $x \mapsto \varphi_i(x)$ is non-negative and vanishes if any component of x is zero.

The constant concentration of reactants in the feed flow $x_1^{\rm in}$ is replaced by a feedback term $v(\beta(\cdot)e(\cdot))$. The overall model then becomes

$$\left.\begin{array}{rcl} \dot x_1(t) &=& C^1\, r\big(x(t), T(t)\big) + d\,[v(\beta(t)e(t)) - x_1(t)] \\ \dot x_2(t) &=& C^2\, r\big(x(t), T(t)\big) + d\,[x_2^{\rm in} - x_2(t)] \\ \dot T(t) &=& b^T\, r\big(x(t), T(t)\big) - q\,T(t) \;+\; u(t). \end{array}\right\} \tag{4.3}$$

In order to reduce the gradient of the exothermic reaction when the temperature is too large, so as to produce a cooling effect on the system, motivated by (4.2) we choose, for arbitrary $\delta \in (0, u^* - \underline{u})$, a continuous and $(n-m)$-dimensional $v(\cdot)$ given by

$$v(\beta e) \;=\; \begin{cases} 0 \quad \text{if} \quad \beta e \in (-\infty, \underline{u} - u^*] \\ [\beta e + u^* - \underline{u}]\, x_1^{\rm in}/\delta, \quad \text{if} \quad \beta e \in (\underline{u} - u^*, \underline{u} - u^* + \delta) \\ x_1^{\rm in}, \quad \text{if} \quad \beta e \in [\underline{u} - u^* + \delta, \infty). \end{cases} \tag{4.4}$$

This cooling action by $v(\cdot)$ could be driven by the error instead of βe. But since βe is used in the control action anyway and is more sensitive, we use (4.4).
We are now in a position to state the main result of this paper.

Theorem 4.1. (Adaptive tracking in the presence of noise.) Suppose (1.6), (A1)–(A4), and that the continuous noise satisfies

$$\sup_{t\geq 0} \{|n(t)|\} \;=:\; \|n\|_\infty \;<\; \lambda/2. \tag{4.5}$$

Then the application of the λ-tracker (1.3) combined with (4.4) to any system (4.3) yields, for any initial data $x(0) \in \Omega_\gamma$, $\beta(0) > 0$, a closed-loop system with unique solution

$$\big(x(\cdot), T(\cdot), \beta(\cdot)\big) \;:\; \mathbb{R}_{\geq 0} \quad \longrightarrow \quad \Omega_\gamma \times \mathbb{R}^2_{>0}$$

defined on the whole time axis $\mathbb{R}_{\geq 0}$ and, moreover,

(i) $\lim_{t\to\infty} \beta(t) = \beta_\infty \in \mathbb{R}_{\geq 0}$, i.e. the gain adaptation converges,

(ii) $\lim_{t\to\infty} \operatorname{dist}\Big(|T^* - T(t)|,\ [0, \lambda + \|n\|_\infty]\Big) = 0$, i.e. the temperature $T(t)$ tends to the $[\lambda + \|n\|_\infty]$-strip $\big[T^* - [\lambda + \|n\|_\infty],\ T^* + [\lambda + \|n\|_\infty]\big]$ as $t \to \infty$. □

The following Theorem 4.2 shows that tracking can be achieved by the non-adaptive feedback (1.4), if the constant gain parameter β^* is sufficiently large (depending on the feasibility bounds). This feedback is simpler than (19) in [9] and an explicit lower bound for the gain in terms of crude system's data is given.

Theorem 4.2. (Non-adaptive tracking in the presence of noise.)
Suppose (1.6), (A1)–(A4), (4.5), and

$$\beta^* \geq \beta' := \max\left\{ \frac{\overline{u} - u^*}{\lambda - 2\|n\|_\infty},\ \frac{u^* - \underline{u}}{\lambda - 2\|n\|_\infty} \right\}. \qquad (4.6)$$

Then the application of the non-adaptive output feedback

$$\begin{array}{rcl} u(t) & = & \operatorname{sat}_{[\underline{u},\overline{u}]}\big(\beta^*\, e(t) \,+\, u^*\big), \\ e(t) & = & T^* - T(t) + n(t), \end{array} \qquad (4.7)$$

combined with (4.4) to any system (4.3) yields, for any initial data $x(0) \in \Omega_\gamma$, $\beta(0) > 0$, a closed-loop system with a unique solution

$$\big(x(\cdot), T(\cdot)\big) \ :\ \mathbb{R}_{\geq 0} \quad \longrightarrow \quad \Omega_\gamma \times \mathbb{R}_{>0}$$

on the whole time axis $\mathbb{R}_{\geq 0}$ and, moreover, there exists a finite time $t_1 \geq t'$ such that

$$e(t) \ \in\ (-\lambda, \lambda) \qquad \text{for all} \qquad t \geq t_1. \qquad (4.8)$$

□

One could ask why to use an adaptive feedback controller when one could use a constant gain feedback controller. Indeed, in order to apply the global λ-tracker with adaptive $\beta(\cdot)$ in the presence of output noise $n(\cdot)$ we need to know $\overline{T}$ and $\underline{T}$, whilst knowing is conservative and the gain found adaptively may be much smaller. Note also that if $n(\cdot) \equiv 0$, then (4.5) holds trivially so that the adaptive gain feedback controller (1.3) applies without restriction whilst the constant gain feedback controller (4.7) would still need to verify that $\beta^* \geq \beta'$. If the system is subjected to transient disturbances which violate (4.5), then the constant gain feedback controller (4.7) would not apply. On the other hand, using the λ tracker (1.3) would clearly lead to large gain parameters which, since $\dot\beta(t) \geq 0$, would be retained as the transient disturbance pass. Such higher values of gain tend to produce a poor control action which switches rapidly between the saturation bounds. This possibility of large transient disturbances leading to unnecessarily large gain parameters motivates the following σ-modification of the gain adaptation in (1.3).

Theorem 4.3. (Adaptive tracking with σ-modification in the presence of noise.)
Suppose (1.6), (A1)–(A4), (4.5), $l \geq 1$, $\sigma, \lambda, \gamma > 0$, and $\beta^* \geq \beta'$ as defined in (4.6). Then the application of the adaptive output feedback

$$\begin{array}{rcl} e(t) & = & T^* - T(t) + n(t), \\ u(t) & = & \operatorname{sat}_{[\underline{u},\overline{u}]}\big(\beta(t)\, e(t) \,+\, u^*\big), \\ \dot\beta(t) & = & -\sigma[\beta(t) - \beta^*] \\ & & +\gamma \begin{cases} (|e(t)| - \lambda)^l, & \text{if} \quad |e(t)| > \lambda \\ 0, & \text{if} \quad |e(t)| \leq \lambda. \end{cases} \end{array} \qquad (4.9)$$

combined with (4.4), to any system (4.3) yields, for any initial data $x(0) \in \Omega_\gamma$, $\beta(0) > 0$, a closed-loop system with unique solution

$$\big(x(\cdot), T(\cdot), \beta(\cdot)\big) \ :\ \mathbb{R}_{\geq 0} \quad \longrightarrow \quad \Omega_\gamma \times \mathbb{R}^2_{>0}$$

on the whole time axis $\mathbb{R}_{\geq 0}$ and, moreover,

(i) $\lim_{t\to\infty} \beta(t) = \beta^*$,

(ii) there exists a finite time $t' \geq 0$ such that $|T^* - T(t)| \in [0, \lambda + \|n\|_\infty]$ for all $t \geq t'$,
i.e. after finite time t' the temperature $T(t)$ stays, for all $t \geq t'$, within the $[\lambda + \|n\|_\infty]$-strip $\big[T^* - [\lambda + \|n\|_\infty],\ T^* + [\lambda + \|n\|_\infty]\big]$. □

5 Simulations

The simulations are performed for the same reaction equations as in [9]. A single reaction is modelled with the Arrhenius law $k(T) = k_0 e^{-k_1 T}$. As in [9] the system parameters of (2.1) are taken to be

$$\begin{array}{l} k_0 = e^{25},\ d = 1.1,\ q = 1.25\,[\mathrm{min}^{-1}], \\ k_1 = 8700\,[\mathrm{K}],\ r^{\mathrm{in}} = 1\,[\mathrm{mol/l}],\ b = 209.2\,[\mathrm{Kl/mol}]. \end{array} \qquad (5.1)$$

The objective is to regulate the temperature in a neighbourhood of $T^* = 337.1[\mathrm{K}]$. It is easy to see that the feasibility assumption (A3″) is satisfied for

$$\begin{array}{l} \underline{u} = 295, \quad \overline{u} = 505, \quad \underline{T} = 240, \\ \overline{T} = 339.65\ [\mathrm{K}], \quad \rho = 5. \end{array} \qquad (5.2)$$

The parameters of the λ-tracker (1.3) are chosen as

$$u^* = 330, \quad T^* = 337.1\,[\mathrm{K}], \quad \lambda = 2.85, \quad l = 2. \qquad (5.3)$$

Note that the tolerated error around the reference temperature is less than 1%. In the simulations we choose $\beta(0) = 12$ and consider the two initial conditions $T(0) = 320$ and $T(0) = 390$. As in [9], we choose $r(0) = 0.02$ and $p(0) = 1.07$ for the initial conditions of the single reactor (2.1).

Adaptive λ-tracker (1.3) with cooling action (4.4), applied to the single reaction (2.1) in the presence of noise: Figure 1 il-

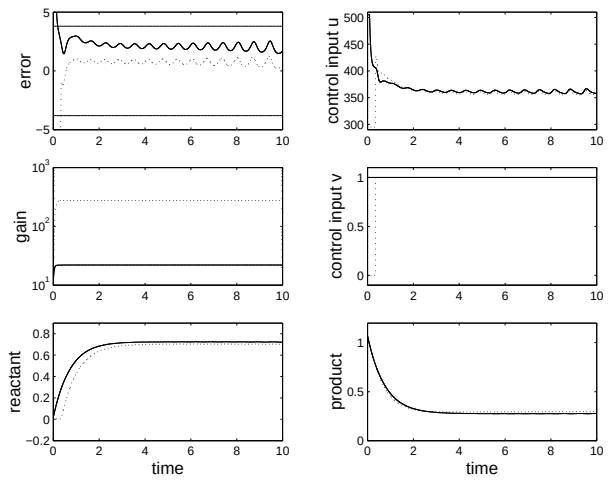

Figure 1: Closed-loop behaviour of the adaptive λ-tracker (1.3) combined with cooling action (4.4) for global setpoint control with noise with parameters (5.3) and noise signal given by (5.4), applied to the single reaction (4.1) with parameters (5.1), (5.2), $T(0) = 320$ (solid), $T(0) = 390$ (dotted).

lustrates that the controller (1.3) combined with (4.4) can cope with noise signal:

$$n(t) = \frac{1}{12} q_1(t). \tag{5.4}$$

Here $q_1(\cdot)$ is the first component of the Lorenz equation

$$\begin{array}{lcll} \dot{q}_1(t) & = & 10[q_2(t) - q_1(t)], & q_1(0) = 1 \\ \dot{q}_2(t) & = & 28q_1(t) - q_2(t) - q_1(t)q_3(t), & q_2(0) = 0 \\ \dot{q}_3(t) & = & q_1(t)q_2(t) - \frac{8}{3}q_3(t), & q_3(0) = 3 \end{array}$$

which has chaotic and but bounded solutions. In this case $|n(t)| \leq 1.42$ for all $t \geq 0$. Hence, $n(\cdot)$ satisfies (4.5) for the data given in (5.2) and (5.3). The error $T^* - T(t)$ is forced into the $[\lambda + \|n\|_\infty]$-strip $[-4.27, 4.27]$ despite the chaotic behaviour of the noise term (5.4). When $T(0) = 390$ the input v is switched off, i.e. $v(0) = 0$, and consumption of the reactant is increased. This causes the temperature to drop, there exists $t' \geq 0$ such that $v(t) = r^{\text{in}}$ for all $t \geq t'$ and λ-tracking is achieved. On the other hand, if $T(0) = 320$, $v(\cdot) \equiv r^{\text{in}}$ and there is no need for a cooling action.

7 Conclusion

In this paper we have applied λ–tracking techniques in the setpoint control of the temperature for a class of nonlinear systems arising as models in chemical reactor control. Their application requires only limited information concerning the system and they also quite readily tolerate bounded noise. In many respects they generalise the controllers developed by [9]. It is worth noting that the minimum phase assumptions usually needed for λ-tracking are not needed here. Instead, we exploit the natural property of chemical reactors that the internal state, i.e. the concentrations, is bounded.

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